Unit 9 Test Review
Pre-Calculus Honors

Name: Key

PARABOLAS

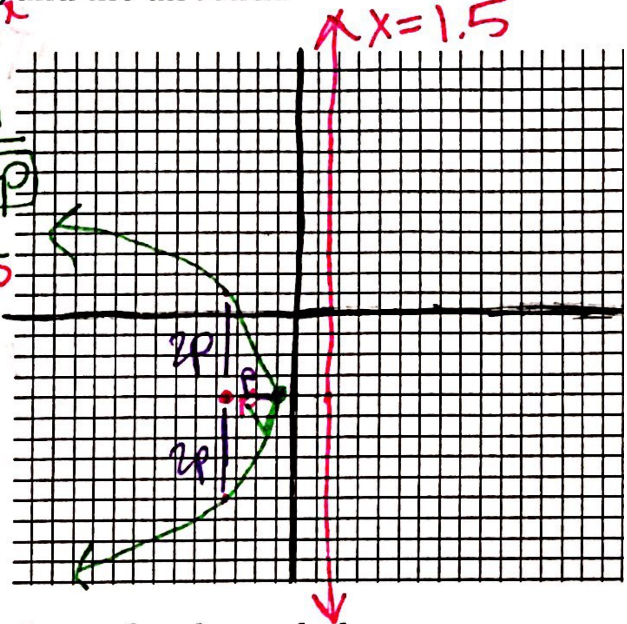
Sketch & then find the vertex equation, the focus, and the directrix.

1. Vertex: (-1,-4) $h\ k$
   Opens to the left
   $4p=$ Focal width: 10

Directrix $x=$

$|a| = \frac{1}{4p} = \frac{1}{10}$

$x = 1.5$



Equation: $x = -\frac{1}{10}(y+4)^2 - 1$

k h

Focus: $(-3.5, -4)$

Directrix: $x = 1.5$

$4p = 10$
$p = 2.5$
$2p = 5$

Use the information provided to write the vertex form of each parabola.

2. $0 = 2x^2 + 36x - y + 170$

$+y \qquad +y$

$y = 2x^2 + 36x + 170$

$y = 2(x^2 + 18x + 81) + 170$

$\left(\frac{18}{2}\right)^2 + 162$

$9^2 = 81$

$y = 2(x+9)^2 + 170 - 162$

Vertex form: $\boxed{y = 2(x+9)^2 + 8}$

$a=2 \quad h=9 \quad k=8$

3. $y + \cancel{60x} + \cancel{294} = -3x^2$

$-60x \; -294 \qquad -60x - 29$

$y = -3x^2 - 60x - 294$

$y = -3(x^2 + 20x + 100) - 2$ $+3$

$y = -3(x+10)^2 - 294 + 3$

Vertex form: $\boxed{y = -3(x+10)^2 +}$

ELLIPSES

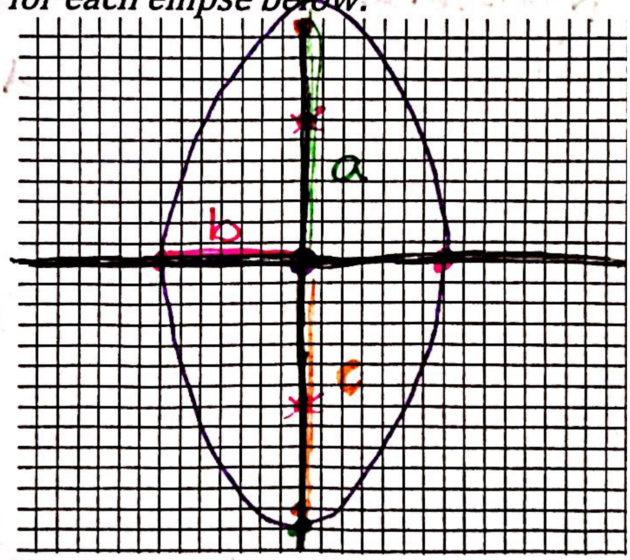
Sketch & then identify the center, vertices, & foci for each ellipse below:

4. $\frac{x^2}{49} + \frac{y^2}{169} = 1$ ← Major Axis

$b^2 = 49$, $b = 7$

$169 = a^2$, $a = 13$

$c^2 = a^2 - b^2$
$c^2 = 169 - 49$
$c^2 = 120$
$c = \sqrt{120}$
$c = 2\sqrt{30}$



0 ± 13

Center: $(0,0)$

Vertices: $(0,-13), (0,13)$

Foci: $(0, 2\sqrt{3}), (0, -2\sqrt{3})$

$0 \pm 2\sqrt{3}$

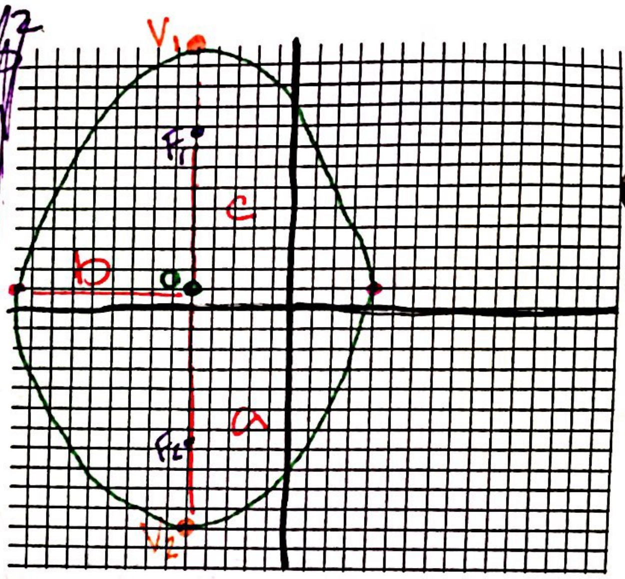
5. $\dfrac{(x+5)^2}{81} + \dfrac{(y-1)^2}{144} = 1$

$h = -5$, $k = 1$

$b^2 = 81$, $b = 9$

$144 = a^2$, $a = 12$

~~$c^2 = a^2 + b^2$, $c^2 = 12 - 9$, $c^2 = 3$, $c = \sqrt{3}$~~



(O) Center: $(-5, 1)$

1 ± 12

$\pm a$ → Vertices: $(-5, 13)$, $(-5, -11)$

$\pm c$ → Foci: $(-5, 1+3\sqrt{7})$, $(-5, 1-3\sqrt{7})$

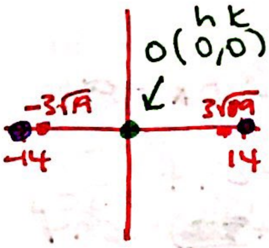
$1 \pm 3\sqrt{7}$

$c^2 = 144 - 81 \rightarrow c^2 = 63 \rightarrow c = \sqrt{63}$

$c = \sqrt{9 \cdot 7} = 3\sqrt{7}$

Use the information provided to write the standard form of each ellipse.

6. Vertices: $(14, 0)$, $(-14, 0)$
Foci: $(3\sqrt{19}, 0)$, $(-3\sqrt{19}, 0)$



$a = 14 \rightarrow a^2 = 196$

$c = 3\sqrt{19} \rightarrow c^2 = 171$

$171 = 196 - b^2$

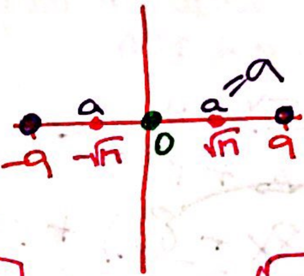
$b^2 = 25$

Standard form: $\boxed{\dfrac{(x+0)^2}{196} + \dfrac{(y+0)^2}{25} = 1}$

$= \dfrac{x^2}{196} + \dfrac{y^2}{25} = 1$

7. Foci: $(\sqrt{17}, 0)$, $(-\sqrt{17}, 0)$
Endpoints of major axis: $(9, 0)$, $(-9, 0)$

x-axis   vertices



$a = 9$ $a^2 = 81$

center: $(0, 0)$ ← h, k

Standard form: $\boxed{\dfrac{(x+0)^2}{81} + \dfrac{(y+0)^2}{64} = 1}$

$c^2 = a^2 - b^2$

$17 = 81 - b^2 \rightarrow b^2 = 64$

HYPERBOLAS

Sketch & then identify the center, vertices, foci, & asymptotes for each hyperbola below:

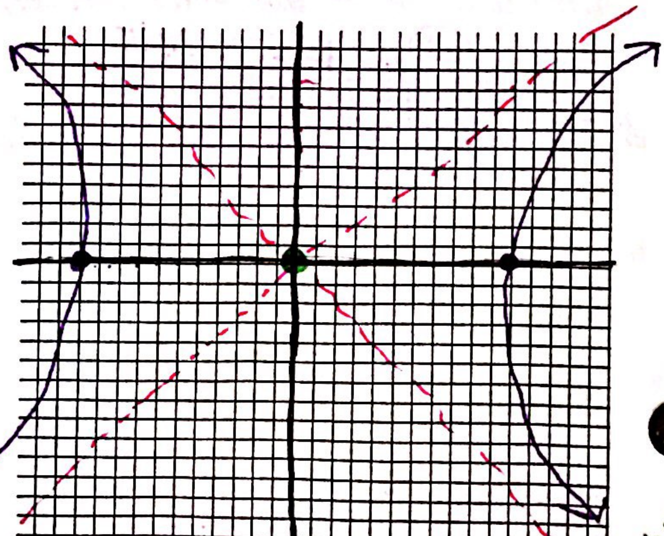
transverse axis = x

8. $\dfrac{x^2}{121} - \dfrac{y^2}{81} = 1$

$a^2 = 121$, $81 = b^2$

$a^2 = 121$

$a = 11$



Center: $(0, 0)$

0 ± 11

Vertices: $(11, 0)$, $(-11, 0)$

Foci: $(\sqrt{202}, 0)$, $(-\sqrt{202}, 0)$

Asymptotes: $y = \pm \frac{9}{11} x$

$c^2 = a^2 + b^2$

$c^2 = 121 + 81 \rightarrow c^2 = 202$

$c = \sqrt{202}$

$y - 0 = \pm m(x - 0)$ $m = \pm \frac{b}{a} = 9/11$

9. $h=-2$ $\quad k=-8$

$$\frac{(x+2)^2}{169=a^2} - \frac{(y+8)^2}{4=b^2} = 1$$

$a=13$ $\quad b=2$

$c^2=169+4$

$c^2=173$

$c=\sqrt{173}$

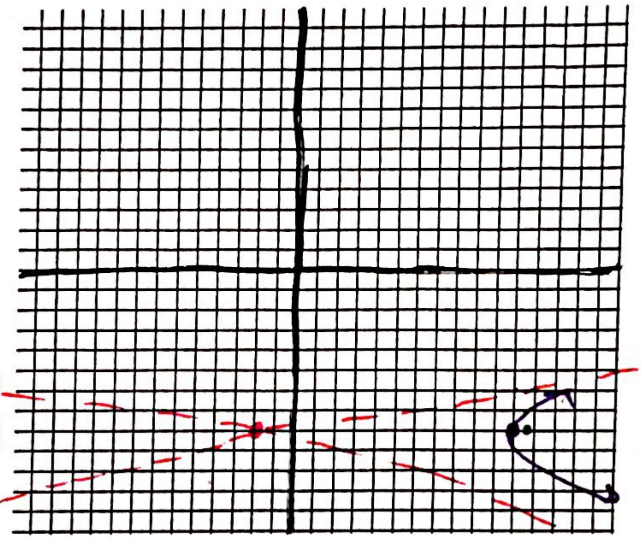
Center: $(-2,-8)$ $\quad -2 \pm 13$

Vertices: $(-15, -8)$, $(11, -8)$

Foci: $(-2-\sqrt{173}, -8)$, $(-2+\sqrt{173}, -8)$

$m=\pm\frac{b}{a} = \pm\frac{2}{13}$

Asymptotes: $y+8 = \pm 2/13\,(x+2)$



Use the information provided to write the standard form of each hyperbola.

10 $-16x^2+9y^2+32x+144y-16=0$

$-16x^2+32x+9y^2+144y=16$

$-16(x^2-2x+1)+9(y^2+16y+64)=16$

$\frac{-16(x-1)^2}{9\cdot 64}+\frac{9(y+8)^2}{9\cdot 64}=\frac{\cancel{16}-\cancel{16}+9\cdot 64}{9\cdot 64}$

Standard form: $\boxed{\frac{(y+8)^2}{64}-\frac{(x-1)^2}{36}=1}$

11 $-2x^2+3y^2+4x-60y+268=0$

$-2x^2+4x+3y^2-60y=-268$ $\quad +300 \quad -2$

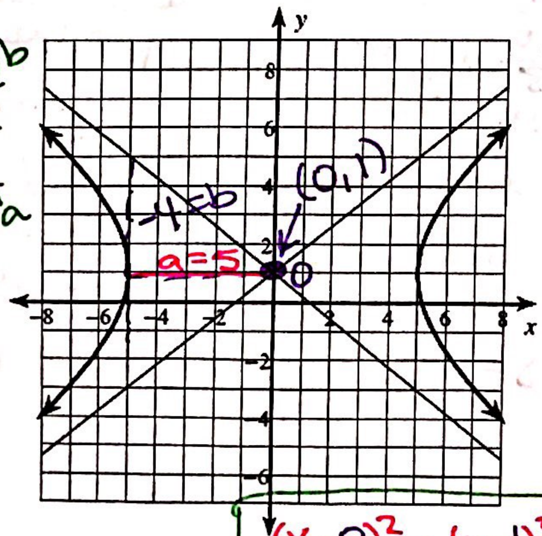
$-2(x^2-2x+1)+3(y^2-20y+100)=-268$

$\frac{-2(x-1)^2}{30}+\frac{3(y-10)^2}{30}=\frac{30}{30}$

Standard form: $\boxed{\frac{(y-10)^2}{10}-\frac{(x-1)^2}{15}=1}$

12

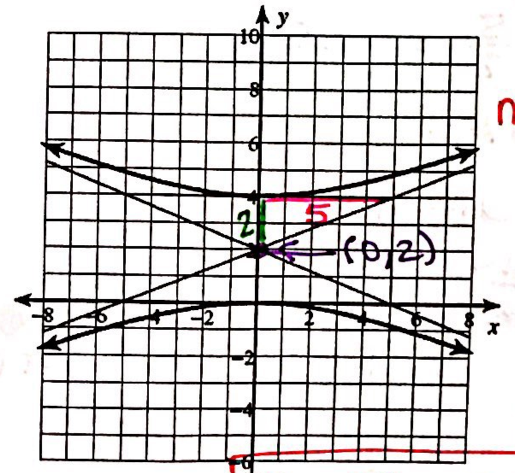
$m=\pm\frac{4}{5}$ $\quad 4 \leftarrow b$, $\quad 5 \leftarrow a$



$-4=b$ $\quad a=5$ $\quad (0,1)$

Standard form: $\boxed{\frac{(x-0)^2}{25}-\frac{(y-1)^2}{16}=1}$

13



2 $\quad$ 5 $\quad (0,2)$

$m=\pm\frac{a}{b}$ $\quad a \leftarrow 2$, $\quad b \leftarrow 5$

$a^2=4$

$b^2=25$

Standard form: $\boxed{\frac{(y-2)^2}{4}-\frac{(x-0)^2}{25}=1}$

14 Vertices: $(-5, 1)$, $(-5, -7) \rightarrow O(-5,-3)$

Foci: $(-5, -3+\sqrt{97})$, $(-5, -3-\sqrt{97})$

$a=4$

$a^2=16$

$c^2=97$

Standard form: $\boxed{\frac{(y+3)^2}{16}-\frac{(x+5)^2}{81}=1}$

$97=16+b^2$

$81=b^2$

15 Vertices: $(7, 4)$, $(7, -24) \rightarrow O(7,-10)$ $\quad a=14$, $a^2=196$

Distance from Center to Focus $= 7\sqrt{5}$ $\leftarrow c$, $\quad c^2=245$

Standard form: $\boxed{\frac{(y+10)^2}{196}-\frac{(x-7)^2}{49}=1}$

$245=196+b^2$

$49=b^2$

16. A bridge is in the shape of a semi-ellipse. If it spans 100 feet, and 30 feet from one end is 25 feet above the raging Neuse, what's the tallest boat that can fit under the bridge?

17. Consider a hyperbola where the slope of an asymptote is $\frac{3}{7}$, a vertex is (3, 8) and the center is (3, 11). Write the equation of the hyperbola, find the other vertex, the foci, and the equations of the asymptotes.

For the following problems: a)identify the type of conic, b) find the eccentricity, and c) identify the position of the directrix with respect to the pole.

18. $r = \dfrac{5/2}{\frac{2}{2}-\frac{3\sin\theta}{2}} = \dfrac{5/2}{1-3/2\sin\theta}$

$\boxed{e=3/2} > 1$

$\boxed{\text{Hyperbola}}$

$\frac{3}{2}p = \frac{5}{2} \rightarrow p = 5/3$

$-e\sin\theta \Rightarrow \boxed{y=-5/3}$

The directrix is below the pole.

19. $r = \dfrac{10/3}{\frac{3}{3}-\frac{\cos\theta}{3}} = \dfrac{10/3}{1-\frac{1}{3}\cos\theta}$

$\boxed{e=\frac{1}{3}} < 1$

$\boxed{\text{Ellipse}}$

$\frac{p}{3} = \frac{10}{3} \rightarrow p=10$

$-e\cos\theta \Rightarrow \boxed{x=-10}$

The directrix is left of the pole.

20. $r = \dfrac{1/4}{\frac{4}{4}+\frac{4\sin\theta}{4}} = \dfrac{1/4}{1+\sin\theta}$

$\boxed{e=1}$

$\boxed{\text{parabola}}$

$p = 1/4$

$+e\sin\theta$

$\Rightarrow \boxed{y = + 1/4}$

The dir. is above the pole.

~~21. Convert 18 – 20 from polar to rectangular.~~

OMIT THIS SECTION.

#18

$r(2-3\sin\theta)=5$

N

#19

A

#20

H.