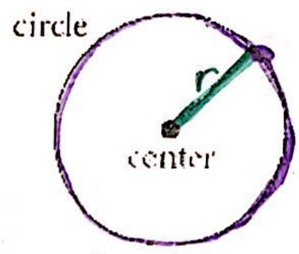
Lesson 6.6 – Equations of Circles

A circle is the set of all points in a plane equidistant from a given point (center).



Equation of Circle: $(x-h)^2 + (y-k)^2 = r^2$

(x, y) is any point on the circle

(h, k) is the center

r = radius length

Examples:

Find the Center and the Radius

1. $(x+1)^2 + (y-3)^2 = 25$ $\sqrt{r^2} = \sqrt{25}$

$\boxed{\text{Center: } (-1, 3),\ r = 5}$

2. $x^2 + (y+3)^2 = 9$

$(0, -3),\ r = 3$

3. $(x-6)^2 + y^2 = 5$

$\boxed{\text{Center: } (6, 0),\ r = \sqrt{5}}$

4. $x^2 + y^2 = 144$

$(0, 0),\ r = 12$

Given the center and the radius, write an equation of a circle.

1. Center (-4,3), radius 4 $r=4$, $r^2=16$

$\boxed{(x+4)^2 + (y-3)^2 = 16}$

2. Center (0, 0), radius = 3

$\boxed{x^2 + y^2 = 9}$

3. Center (-1, 0), radius = 6

$\boxed{(x+1)^2 + y^2 = 36}$

4. Center (0, -3) radius = 5

$\boxed{x^2 + (y+3)^2 = 25}$

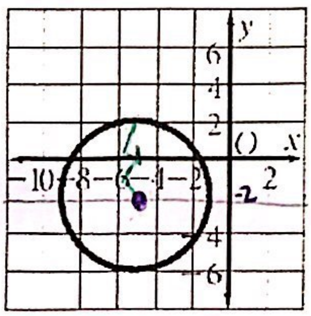
5. Center (-2, 6), radius = 4

$\boxed{(x+2)^2 + (y-6)^2 = 16}$

6. Center (1, -5), radius = 2.5

$\boxed{(x-1)^2 + (y+5)^2 = 6.25}$

1. Writing an Equation from a graph:



Center $(-5, -2)$

$r = 4$

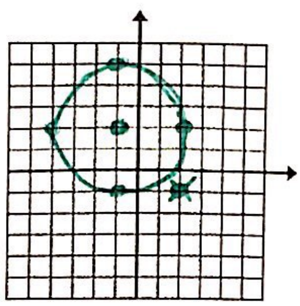
$(x+5)^2 + (y+2)^2 = 16$

2. Graph the following circle.

$(x+1)^2 + (y-2)^2 = \sqrt{9}$

Center $(-1, 2)$

$r = \sqrt{9} = 3$



1

$(x \quad)^2 + (y \quad)^2 = \#$

Use Completing the Square to find the center and the radius!

Steps:

- Move the constant to the right side of the equation.
- Complete the square for x and y. (Remember to add the same value to the right side!)
- Write in factored form on the left and combine the values on the right.

1. $x^2 + y^2 + 6y - \cancel{27} = 0$

$+\cancel{27} \quad +27$

$\left(\frac{b}{2}\right)^2$

$x^2 + y^2 + 6y = 27$

$x^2 + y^2 + 6y + \underline{9} = 27 + \underline{9}$

$b/2$

$x^2 + (y + \underline{3})^2 = 36$

$\boxed{\text{Center: } (0, -3),\ r = 6}$

2. $x^2 + y^2 - 8x - 4y + 19 = 0$

3. $(x^2 + 2x) + (y^2 + 14y) - 31 = 0$

$x^2 + \underline{2}x + \underline{1} + y^2 + \underline{14}y + \underline{49} = 31 + \underline{1} + \underline{49}$

$\frac{2}{2} \quad \left(\frac{2}{2}\right)^2 \quad \frac{14}{2} \quad \left(\frac{14}{2}\right)^2$

$(x + 1)^2 + (y + 7)^2 = 81$

$\boxed{\text{Center: } (-1, -7),\ r = 9}$

4. $x^2 - 10x + y^2 + 4y - 7 = 0$

6.6 Exercises – Equations of Circles

Find the Center and the Radius of each circle.

1. $x^2 + (y-4)^2 = 49$ $(0,4)$ $r=7$

2. $(x+5)^2 + (y-3)^2 = 81$ $(-5,3)$ $r=9$

3. $(x-3)^2 + (y+6)^2 = 64$ $(3,-6)$ $r=8$

Given the following centers and radii, write the equation for the circle.

4. Center: (-3, 2) Radius: 3

$(x+3)^2 + (y-2)^2 = 9$

6. Center (1, -3) Radius: 4

$(x-1)^2 + (y+3)^2 = 16$

5. Center: (-4, 0) Radius: 5

$(x+4)^2 + y^2 = 25$

7. Center (0,0) Radius: 8

$x^2 + y^2 = 64$

Find the center and radius of each circle.

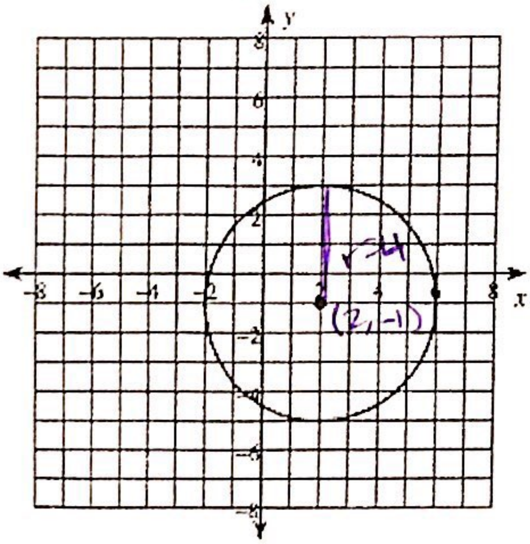
8. $x^2 + 10x + y^2 - 6y + 18 = 0$

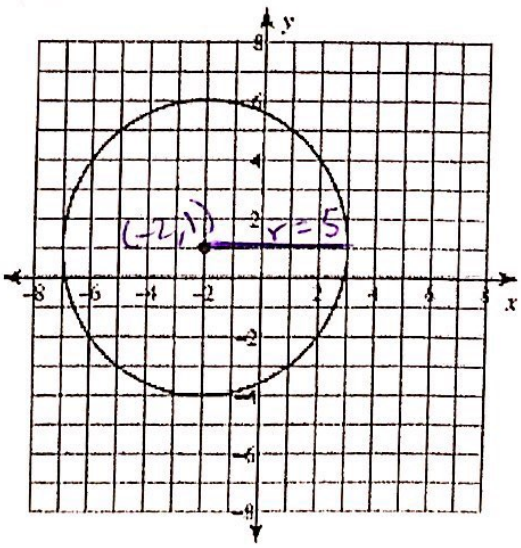
$(x+5)^2 + (y-3)^2 = 16$ $(-5,3)$ $r=4$

9. $x^2 - 4x + y^2 + 6y - 3 = 0$

$(x-2)^2 + (y+3)^2 = 16$ $(2,-3)$ $r=4$

10. Write an equation of the following circle





$(x-2)^2 + (y+1)^2 = 16$

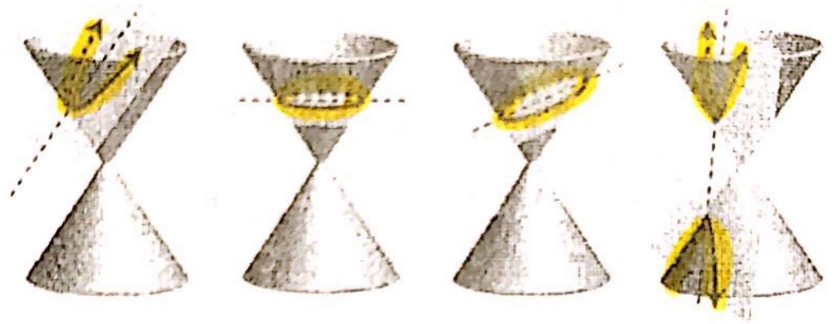
$(x+2)^2 + (y-1)^2 = 25$

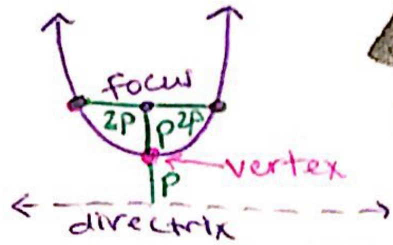
Precalculus: Parabolas

Name:

Conic: A conic is the cross section of a ___cone___!

Consider the following graphic.





* The vertex is halfway btw. the focus & directrix!

4p: Focal width!

Parabola: The set of all points an equidistant from a fixed point ___focus___ and a fixed line ___directrix___

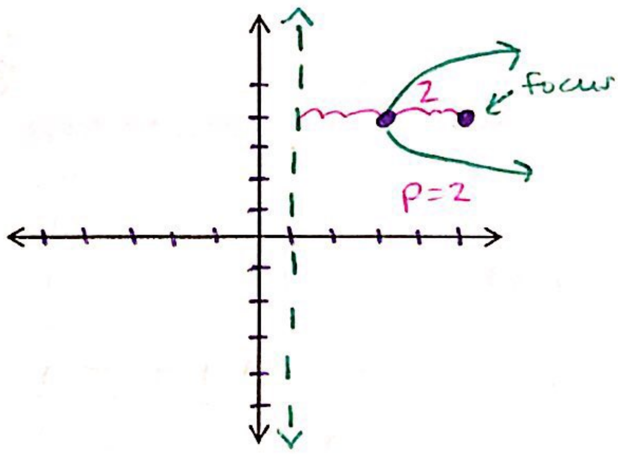
Parabolas with Vertex (h, k)

	up / down	left / right
Standard Form	$(x-h)^2 = 4p(y-k)$	$(y-k)^2 = 4p(x-h)$
Vertex	(h, k)	(h, k)
Opens	$4p>0$ up!! ; $4p<0$ down	$4p>0$ right ; $4p<0$ left
Focus	$(h, k+p)$	$(h+p, k)$
Directrix	$y = k-p$	$x = h-p$
Axis of Symmetry	$x = h$	$y = k$

Example 1: Write an equation for a parabola with vertex (3, 4) and focus (5, 4). Then, graph it.

$(y-k)^2 = 4p(x-h)$

$$\boxed{(y-4)^2 = \overset{4(2)}{8}(x-3)}$$

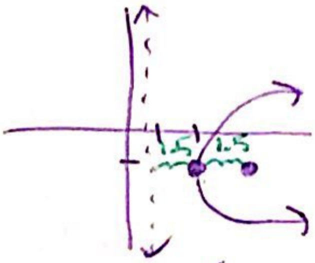


Example 2: Graph the parabola $(y-4)^2 = 8(x-3)$ using your graphing calculator.

$$y-4 = \pm\sqrt{8(x-3)} \quad +4$$

$$y = 4 \pm \sqrt{8(x-3)} \rightarrow \begin{cases} y_1 = 4 + \sqrt{8(x-3)} \\ y_2 = 4 - \sqrt{8(x-3)} \end{cases}$$

Example 3: Prove the graph of $y^2 - 6x + 2y + 13 = 0$ is a parabola and find its vertex, focus, and directrix.



$$y^2 + 2y + \underline{1} = 6x - 13 + 1$$

$$(y+1)^2 = 6x - 12$$

$$(y+1)^2 = 6(x-2)$$

4p > 0 Right! $4p = 6$, $p = 1.5$

Want: $(y-k)^2 = 4p(x-h)$

$$\boxed{\begin{array}{l} \text{Vertex: } (2,-1) \\ \text{Focus: } (3.5,-1) \quad (+1.5) \\ \text{directrix: } x = 0.5 \end{array}}$$

2-1.5

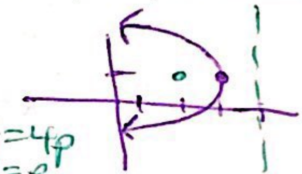
Example 4: Prove the graph of $y^2 - 2y + 4x - 11 = 0$ is a parabola and find its vertex, focus, and directrix.

$$y^2 - 2y + \underline{1} = -4x + 11 + \underline{1}$$

$$(y-1)^2 = -4x + 12$$

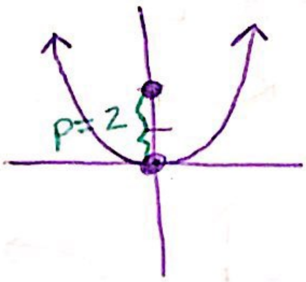
$$(y-1)^2 = -4(x-3)$$

$-4 = 4p$, $-1 = p$



$$\boxed{\text{vertex: } (3,1) \quad \text{focus: } (-2,1) \quad \text{directrix: } x = 4}$$

Example 5: Write the standard form of the parabola given: Vertex: (0, 0); Focus: (0, 2)



$$(x-h)^2 = 4p(y-k)$$

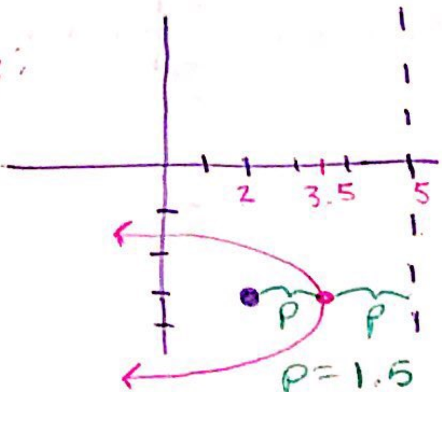
p = 2

$$\boxed{x^2 = 8y}$$

Example 6: Write the standard form of the parabola given: Focus: (2, -3); Directrix: x = 5

vertex:

$$x = \frac{2+5}{2} = 3.5$$



$$(y-k)^2 = 4p(x-h)$$

vertex: (3.5, -3)

$p = 1.5$, $4p = 6$

Left! 4p < 0

$$\boxed{(y+3)^2 = -6(x-3.5)}$$

Pre-Calc Unit 8
Homework Day 2

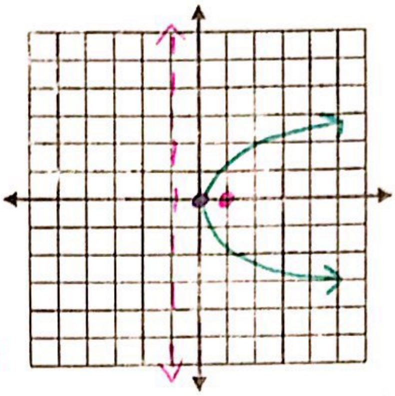
Parabolas

Name______________

Graph the parabola and identify the vertex, directrix, focus, and axis of symmetry.

1. $y^2 = 4x$

$4p = 4$
$p = 1$



Vertex: $(0,0)$

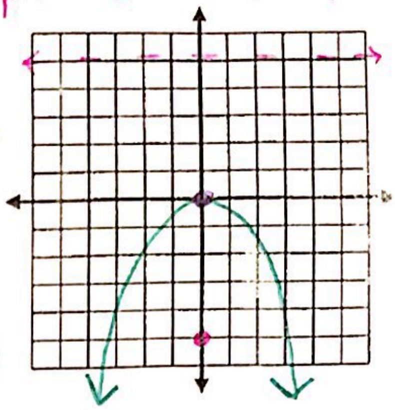
Focus: $(1,0)$

Directrix: $x = -1$

Axis of Symmetry: $y = 0$

2. $x^2 = -20y$

$4p = -20$
$p = -5$



Vertex: $(0,0)$

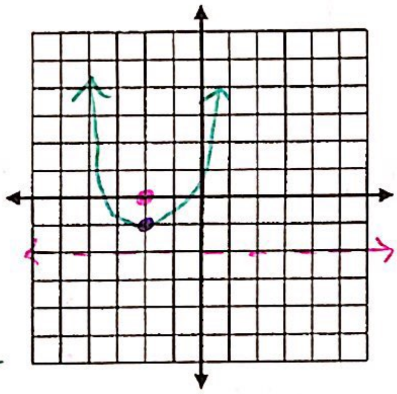
Focus: $(0,-5)$

Directrix: $y = 5$

Axis of Symmetry: $x = 0$

3. $(x+2)^2 = 4(y+1)$

$4p = 4$
$p = 1$



Vertex: $(-2,-1)$

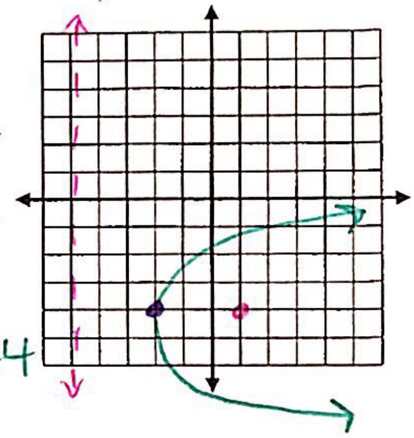
Focus: $(-2,0)$

Directrix: $y = -2$

Axis of Symmetry: $x = -2$

4. $(y+4)^2 = 12(x+2)$

$4p = 12$
$p = 3$



Vertex: $(-2,-4)$

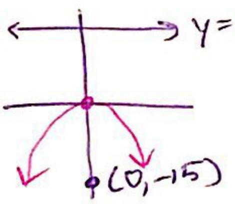
Focus: $(1,-4)$

Directrix: $x = -5$

Axis of Symmetry: $y = 4$

Write an equation in standard form for the parabola satisfying the given conditions.

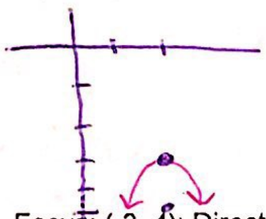
5. Focus: (0, -15); Directrix: y = 15



~~center~~: $(0,0)$ vertex $p = 15$ $4p = 60$

$\boxed{(x)^2 = -60y}$

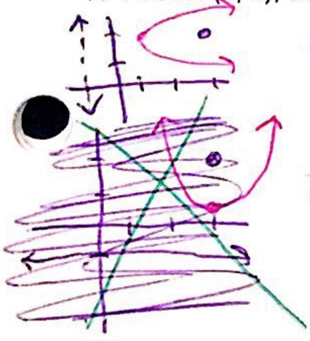
6. Vertex: (2, -3); Focus: (2, -5)



~~center~~: $(2,-3)$ vertex $p = -2$ $4p = -8$

$\boxed{(x-2)^2 = -8(y+3)}$

7. Focus: (3, 2); Directrix: x = -1



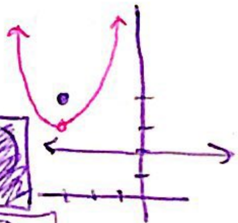
$(1,2)$

vertex:

$p = 2$ $4p = 8$

$\boxed{(y-2)^2 = 8(x-3)}$

8. Focus: (-3, 4); Directrix: y = 2



vertex: $(-3,3)$ $p = 1$ $4p = 4$

$\boxed{(x+3)^2 = 4(y-3)}$

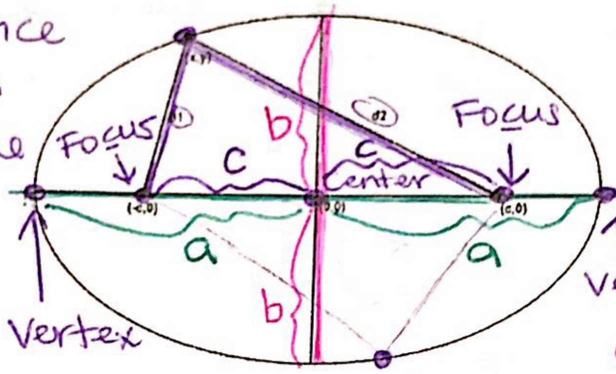
Unit 9 – Day 3: Ellipses
Pre-Calculus

Name: ______________________

Definition: An ellipse is the set of all points in a plane whose distances from two fixed points in the plane have a constant sum.

- The fixed points are the foci of the ellipse.
- The line through the foci is the Focal Axis.
- The point on the focal axis midway between the foci is the center.
- The points where the ellipse intersects its axis are the vertices of the ellipse.

The distance from each focus to the center is "c" units.



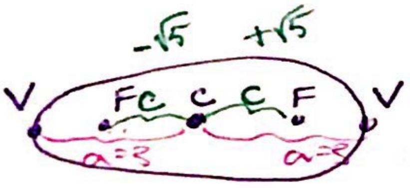
Focal Axis / "Major Axis" $= 2a$

"Minor Axis" $= 2b$

Standard Equation $a > b$	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
Sketch *Major Axis is whichever variable is over a*	horizontal	vertical
Center	(h, k)	(h, k)
Focal axis "Major Axis"	$y = k$	$x = h$
Foci $c^2 = a^2 - b^2$	$(h \pm c, k)$	$(h, k \pm c)$
Vertices	$(h \pm a, k)$	$(h, k \pm a)$
Length of major axis	$2a$	$2a$
Length of minor axis	$2b$	$2b$

Example 1: Find the vertices and the foci of the ellipse $4x^2 + 9y^2 = 36$

Center



$\boxed{\begin{array}{l} \text{Center: } (0, 0) \\ \text{Vertices: } (3, 0) \\ \qquad (-3, 0) \\ \text{Foci: } (\sqrt{5}, 0) \\ \qquad (-\sqrt{5}, 0) \end{array}}$

$\frac{4x^2}{36} + \frac{9y^2}{36} = \frac{36}{36}$

$\frac{x^2}{9} + \frac{y^2}{4} = 1$

$a^2 = 9 \quad b^2 = 4$

$a = 3 \quad b = 2$

For Finding c:

$c^2 = a^2 - b^2$

$c^2 = 9 - 4$

$c^2 = 5$

$c = \sqrt{5}$

Example 2: Find the center, vertices, foci, length of major and minor axes, and ~~sketch~~.

$h=1 \quad k=-3$

$\frac{(x-1)^2}{2} + \frac{(y+3)^2}{4} = 1$

$b^2 = 2 \quad a^2 = 4$

$b = \sqrt{2} \quad a = 2$

2b= Minor Axis: $2\sqrt{2}$

2a= Major Axis: 4

Finding c:

$c^2 = a^2 - b^2$

$c^2 = 4 - 2$

$c^2 = 2$

$c = \sqrt{2}$

Center: $(1, -3)$

$\pm a$ Vertices: $(1, \overset{+2}{-1}), (1, \overset{-2}{-5})$

$\pm c$ Foci: $(1, \overset{+\sqrt{2}}{-3+\sqrt{2}}), (1, \overset{-\sqrt{2}}{-3-\sqrt{2}})$

Example 3: Find the standard form of the equation for the ellipse whose major axis has endpoints (-2, -1) and (8, -1), and whose minor axis has length 8.

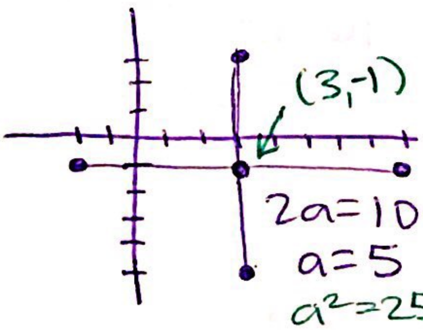
vertices

$2b = 8$

$b = 4$

$b^2 = 16$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$



$2a = 10$

$a = 5$

$a^2 = 25$

horizontal

Major: x-axis

h k

Need: Center: $(3, -1)$

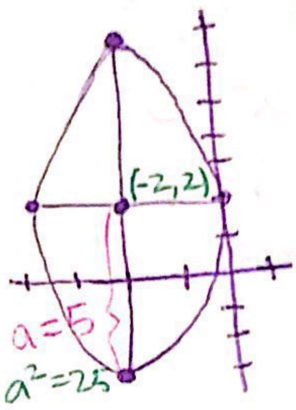
$a^2 = 25 \quad b^2 = 16$

$\boxed{\frac{(x-3)^2}{25} + \frac{(y+1)^2}{16} = 1}$

Example 4: Find the standard form of the equation for the ellipse whose major axis has endpoints (-2, -3) and (-2, 7); minor axis length 4.

Vertices

vertical



$2b = 4$

$b = 2$

$b^2 = 4$

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

$$\boxed{\frac{(y-2)^2}{25} + \frac{(x+2)^2}{4} = 1}$$

Example 5: Prove that the following equation is an ellipse.

$$3x^2 + 5y^2 - 12x + 30y + 42 = 0$$

$$3x^2 - 12x + 5y^2 + 30y = -42$$

$$3(x^2 - 4x + \underline{4}) + 5(y^2 + 6y + \underline{9}) = -42 + \underline{12} + \underline{45}$$

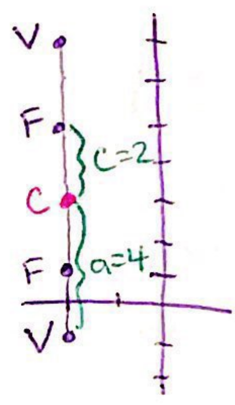
$$\frac{3(x-2)^2}{15} + \frac{5(y+3)^2}{15} = \frac{15}{15}$$

$$\boxed{\frac{(x-2)^2}{5} + \frac{(y+3)^2}{3} = 1}$$

Example 6: Find an equation in standard form for the ellipse with foci (-2, 1) and (-2, 5) and major axis endpoints (-2, -1) and (-2, 7).

vertices

Center: $(-2, 3)$ $a = 4$ $b = \sqrt{12}$ $c = 2$



$$\boxed{\frac{(x+2)^2}{16} + \frac{(y-3)^2}{12} = 1}$$

$c^2 = a^2 - b^2$

$4 = 16 - b^2$

$12 = b^2 \rightarrow b = \sqrt{12}$

vertical

$x \rightarrow a^2$

Day 3 Homework

ELLIPSE

1. For each of the following, determine the center, vertices, and foci.

a. $\frac{x^2}{64}+\frac{y^2}{36}=1$ Center: $(0,0)$ Vertices: $(-8, 0)$ $(8, 0)$ (0 ± 8) Foci: $(-\sqrt{28}, 0)$ $(\sqrt{28}, 0)$ $(0\pm\sqrt{28})$

$=64$ $=36$ $c^2=28$

b. $\frac{x^2}{16}+\frac{y^2}{49}=1$ center: $(0,0)$ Vertices: $(0, -7)$ $(0, 7)$ (0 ± 7) Foci: $(0, -\sqrt{33})$ $(0, \sqrt{33})$ $(0\pm\sqrt{33})$

$=49$ $=16$ $c^2=33$

c. $\frac{(x+7)^2}{4}+\frac{(y-5)^2}{25}=1$ Center: $(-7,5)$ Vertices: $(-7, 0)$ $(-7, 10)$ (5 ± 5) Foci: $(-7, 5-\sqrt{21})$ $(-7, 5+\sqrt{21})$ $(5\pm\sqrt{21})$

$=25$ $c^2=21$ $=4$

d. $\frac{(x-3)^2}{9}+\frac{(y-8)^2}{100}=1$ Center: $(3,8)$ Vertices: $(3,-2)$ $(3, 18)$ (8 ± 10) Foci: $(3, 8-\sqrt{91})$ $(3, 8+\sqrt{91})$ $(8\pm\sqrt{91})$

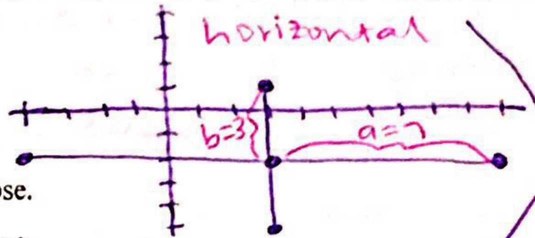
$a^2=100$ $b^2=9$ $c^2=91$

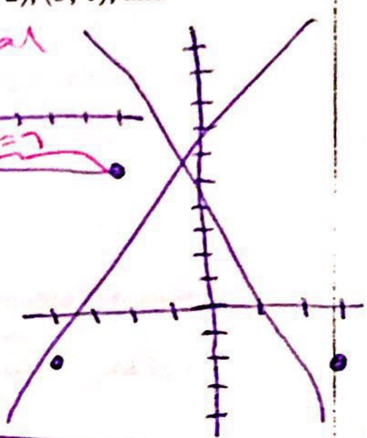
2. Write the equation of an ellipse with a center (3, –2), passing through (–4, –2), (10, –2), (3, 1), and (3, –5).

h k

$$\boxed{\frac{(x-3)^2}{49}+\frac{(y+2)^2}{9}=1}$$

horizontal





3. Change the following to standard form for each ellipse.

a. $4x^2+y^2+8x-4y=0$

$4(x^2+2x+1)+(y^2-4y+4)=0+4+4$

$4(x+1)^2+(y-2)^2=8 \longrightarrow \boxed{\frac{(x+1)^2}{2}+\frac{(y-2)^2}{8}=1}$

b. $9x^2+25y^2-36x+50y-164=0$

$9(x^2-4x+4)+25(y^2+2y+1)=164+36+25$

$\frac{9(x-2)^2}{225}+\frac{25(y+1)^2}{225}=\frac{225}{225} \rightarrow \boxed{\frac{(x-2)^2}{25}+\frac{(y+1)^2}{9}=1}$

4. Find the equation of an ellipse satisfying the given conditions:

a. center at (2, 5) with the longer axis of length 12 and parallel to the x–axis, shorter axis of length 10

h k $2a=12$ $a=6$ horizontal $2b=10$ $b=5$

$$\boxed{\frac{(x-2)^2}{36}+\frac{(y-5)^2}{25}=1}$$

b. center at (–3, 4) with the longer axis of length 8 and parallel to the y–axis, shorter axis of length 2

h k $2a=8$ $a=4$ vertical $2b=2$ $b=1$

$$\boxed{\frac{(y-4)^2}{16}+\frac{(x+3)^2}{1}=1}$$

c. center at (2, -2), one vertex at (7, -2) and one focus at (4, -2)

h k $a=5$ $c=2$

$c^2=a^2-b^2$

$4=25-b^2$

$21=b^2$

Horizontal

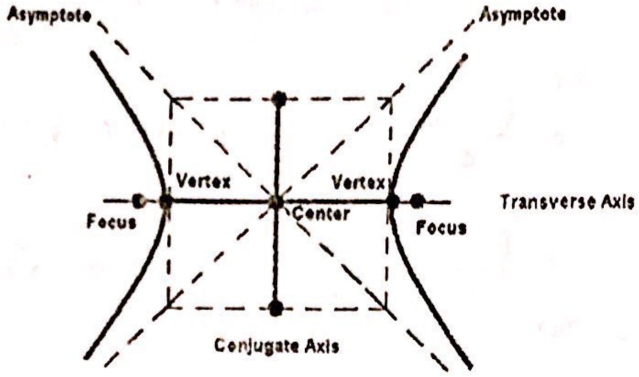
$$\boxed{\frac{(x-2)^2}{25}+\frac{(y+2)^2}{21}=1}$$

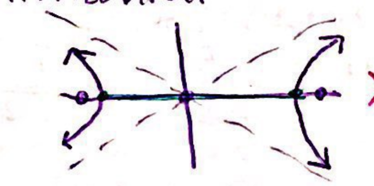
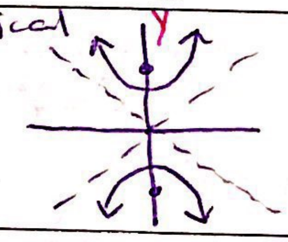
Unit 9 – Day 4: Hyperbolas
Pre-Calculus

Name: ______________________

Definition: A hyperbola is the set of all points in a plane whose distances from two fixed points in the plane have a constant difference

- The fixed points are the foci of the hyperbola.
- The line through the foci is the focal axis.
- The point on the focal axis midway between the foci is the center.
- The points where the hyperbola intersects its axis are the vertices.
- The chord lying on the focal axis connecting the vertices is the transverse axis.
- The line segment that is perpendicular to the focal axis and that has the center of the hyperbola as its midpoint is the conjugate axis.



Standard Equation *a is not always bigger, so check signs!!	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Sketch	Horizontal  x	Vertical y 
Center	(h, k)	(h, k)
Focal axis "Transverse Axis"	$y = k$	$x = h$
Foci $c^2 = a^2 + b^2$	$(h \pm c, k)$	$(h, k \pm c)$
Vertices	$(h \pm a, k)$	$(h, k \pm a)$
Length of transverse axis Major	$2a$	$2a$
Length of conjugate axis Minor	$2b$	$2b$
Asymptotes	$y = \pm\frac{b}{a}(x-h) + k$	$y = \pm\frac{a}{b}(x-h) + k$

$\pm a$ $\pm c$

Example 1: Find the vertices and the foci of the hyperbola $4x^2 - 9y^2 = 36$

$\frac{4x^2}{36} - \frac{9y^2}{36} = \frac{36}{36}$

h k

Center $(0, 0)$

Vertices $(3, 0)$, $(-3, 0)$

(0 ± 3)

$\uparrow$ h

Foci $(\sqrt{13}, 0)$, $(-\sqrt{13}, 0)$

$(0 \pm \sqrt{13})$

$\uparrow$ h

$\frac{x^2}{9} - \frac{y^2}{4} = 1$

$a^2 = 9 \quad b^2 = 4$

$a = 3 \quad b = 2$

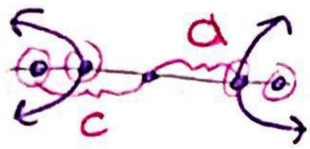
To find c:

$c^2 = a^2 + b^2$

$c^2 = 9 + 4 = 13$

$c = \sqrt{13}$

* Since x>0, horizontal!



Example 2: Find an equation in standard form for the hyperbola with foci (0, -3) and (0, 3) whose conjugate axis has length 4.

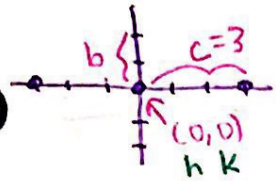
minor

$2b = 4$

$b = 2$

$b^2 = 4$

Horizontal: x>0



$c^2 = a^2 + b^2$

$9 = a^2 + 4$

$5 = a^2$

$\sqrt{5} = a$

$$\boxed{\frac{(x-0)^2}{5} - \frac{(y-0)^2}{4} = 1}$$

$\uparrow$

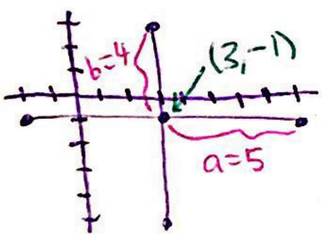
$$\frac{x^2}{5} - \frac{y^2}{4} = 1$$

Example 3: Find the standard form of the equation for the hyperbola with transverse axis endpoints (-2, -1) and (8, -1), and whose conjugate axis has length 8.

Vertices

minor $\quad 2b = 8$

$b = 4$



Horizontal: x>0

$$\boxed{\frac{(x-3)^2}{25} - \frac{(y+1)^2}{16} = 1}$$

Example 4: Find the center, vertices, and foci of the hyperbola.

Horizontal ← $\frac{(x+2)^2}{9} - \frac{(y-5)^2}{49} = 1$

$a^2 = 9$ $\quad b^2 = 49$

$a = 3$ $\quad b = 7$

Center: $(-2, 5)$

$c^2 = a^2 + b^2$

-2 ± 3 Vertices: $(1, 5), (-5, 5)$

$c^2 = 9 + 49$

$c^2 = 58$

$-2 \pm \sqrt{58}$ Foci: $(-2+\sqrt{58}, 5), (-2-\sqrt{58}, 5)$

$c = \sqrt{58}$

Example 5: Prove that the following equation is a hyperbola.

$25y^2 - 9x^2 - 54x + 50y - 281 = 0$

$25y^2 - 50y - 9x^2 - 54x = 281$

$25(y^2 - 2y + 1) - 9(x^2 + 6x + 9) = 281 + 25 + -81$

$\left(\frac{}{2}\right)^2$ $\quad \left(\frac{}{2}\right)^2$

$\frac{25(y-1)^2}{225} - \frac{9(x+3)^2}{225} = \frac{225}{225}$

$\boxed{\frac{(y-1)^2}{9} - \frac{(x+3)^2}{25} = 1}$

Example 6: Identify the conic without completing the square.

1) $2x^2 - y - 8x = 0$

$B = C = 0$ parabola

2) $x^2 - 8y^2 - x - 2y = 0$

$C < 0$ hyperbola

3) $x^2 + 2y^2 + 4x - 8y + 2 = 0$

$A \neq C$ $\quad A = 1 > 0$ $\quad C = 2 > 0$ ellipse

4) $3x - 9y^2 + 4y - 9x^2 = 4$

~~[illegible]~~ none

Identifying Conic Sections

General Form:

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

Parabola: A = 0 or C = 0, B = 0

Circle: A = C = 1, B = 0

Ellipse: A ≠ C , A > 0, C > 0

Hyperbola: A < 0 or C < 0

$$\frac{(\quad)^2}{a^2} - \frac{(\quad)^2}{b^2} = 1$$

Precalculus
Worksheet #5

Chapter 9 - Conic Sections
"The hyperbola"

Name________________________
Date________________ Period__________

Directions: Find the center, vertices, foci, & asymptotes for each below

(h,k) $\pm a$ $\pm c$ $y = \pm \square (x-h) + k$

1. $\frac{x^2}{8} - \frac{y^2}{12} = 1$

Center: $\boxed{(0,0)}$ a: $\sqrt{8}$ b: $\sqrt{12}$ $c^2 = 8+12$ $c = \sqrt{20}$

$\pm\sqrt{8}$ Vertices: $(-\sqrt{8}, 0), (\sqrt{8}, 0)$

$\pm\sqrt{20}$ Foci: $(-\sqrt{20}, 0), (\sqrt{20}, 0)$

$y = \pm\sqrt{\frac{12}{8}}\,x \rightarrow \boxed{y = \pm\sqrt{\frac{3}{2}}\,x}$

2. $x^2 = 9 + y^2$

$x^2 - y^2 = 9$ Center: $\boxed{(0,0)}$

$\frac{x^2}{9} - \frac{y^2}{9} = 1$ a: 3 b: 3 $c^2 = 9+9$ $c = \sqrt{18}$

0 ± 3 Vertices: $(-3, 0), (3, 0)$

$0 \pm \sqrt{18}$ Foci: $(-\sqrt{18}, 0), (\sqrt{18}, 0)$

$\boxed{y = \pm(x)}$

3. $25x^2 - 4y^2 = -100$ Center: $\boxed{(0,0)}$

$\frac{y^2}{25} - \frac{x^2}{4} = 1$ a: 5 b = 2 $c^2 = 25+4$ $c = \sqrt{29}$

± 5 Vertices: $(0, -5)$, $(0, 5)$

$\pm\sqrt{29}$ Foci: $(0, -\sqrt{29}), (0, \sqrt{29})$

$\boxed{y = \pm 5/2\, x}$

4. $\frac{(x-3)^2}{16} - \frac{(y+2)^2}{49} = 1$

Center: $\boxed{(3,-2)}$ a: 4 b: 7 $c^2 = 16+49$ $c = \sqrt{65}$

Vertices: $(-1, -2), (7, -2)$

3 ± 4

$3 \pm \sqrt{65}$ Foci: $(3-\sqrt{65}, -2), (3+\sqrt{65}, -2)$

$\boxed{y = \pm\frac{7}{4}(x-3) - 2}$

5. $4x^2 - y^2 = -16$ Center: $\boxed{(0,0)}$

$\frac{y^2}{16} - \frac{x^2}{4} = 1$ a = 4 b = 2 $c^2 = 16+4$ $c = \sqrt{20}$

± 4 Vertices: $(0, -4)$, $(0, 4)$

$\pm\sqrt{20}$ Foci: $(0, -\sqrt{20})$, $(0, \sqrt{20})$

$\boxed{y = \pm 2x}$

6. $\frac{(y+1)^2}{25} - \frac{(x-3)^2}{36} = 1$

$c^2 = 25+36$ $c = \sqrt{61}$

Center: $\boxed{(3,-1)}$ a = 5 b = 6

-1 ± 5 Vertices: $(3, -6), (3, +4)$

$-1 \pm \sqrt{61}$ Foci: $(3, -1-\sqrt{61}), (3, -1+\sqrt{61})$

$\boxed{y = \pm\frac{5}{6}(x-3) - 1}$

Directions: Write an equation for each of the following hyperbolas.

$h=0 \quad k=0 \quad a=2 \quad b=1$

7. Hyperbola, center at the origin, vertex (0, 2), a = 2b

$$\boxed{\frac{(y)^2}{4} - \frac{(x)^2}{1} = 1}$$

$h=0 \quad k=0 \quad 2a=8 \quad a=4 \quad b=12$

8. Hyperbola, center at the origin, length of transverse axis is 8, b = 3a, parallel to the x-axis

$$\boxed{\frac{x^2}{16} - \frac{y^2}{144} = 1}$$

$h \; k$

9. Hyperbola, center at (-3, 1), a = 4, b = 2, transverse axis parallel to the y-axis

$$\boxed{\frac{(y-1)^2}{16} - \frac{(x+3)^2}{4} = 1}$$

$h \; k \quad a=3$

10. Hyperbola, center at (4, 3), vertex (1, 3), b = 2

$$\boxed{\frac{(x-4)^2}{9} - \frac{(y-3)^2}{4} = 1}$$

POLAR FORM OF CONICS

Name: ______________

I. Definition of Conics:

A conic is the set of all points such that the distance between a point on the conic and a fixed point is related to the distance from that point to a fixed line by a constant ratio.

- The fixed point is the focus.
- The fixed line is the directrix.
- The constant ratio is the eccentricity of the conic and is denoted by *e*.

The conic is.......	an ellipse if $e < 1$,	a parabola if $e = 1$,	and a hyperbola if $e > 1$.

"less" → ellipse

"hyper" = more

II. Polar Equations of Conics:

$$\boxed{r = \frac{ep}{1 \pm e\cos\theta}} \quad \text{or} \quad \boxed{r = \frac{ep}{1 \pm e\sin\theta}}$$

$e > 0$ is the eccentricity and $|p|$ is the distance between the pole (focus) and the directrix.

Nice Things to Know:

If the denominator is $1 + e\sin\theta$, it has a horizontal directrix **above** the pole.
If the denominator is $1 - e\sin\theta$, it has a horizontal directrix **below** the pole.
If the denominator is $1 + e\cos\theta$, it has a vertical directrix to the **right** of the pole.
If the denominator is $1 - e\cos\theta$, it has a vertical directrix to the **left** of the pole.

III. Examples:

Guided Example 1: Using the given equation, determine the type of conic and state the location of the directrix:

$$r = \frac{6}{1 - 2\cos\theta}$$

- The first # in the denominator is a 1, so this problem is already in the correct format. ✓ ☺
- The number in front of cosine is a 2. This means e = 2. e > 1, so this is a HYPERBOLA.
- The numerator represents "ep", so since e = 2, p must = 3. This means the directrix is 3 units away from the pole. We need to decide if it is left/right/above/below.
- Since the denominator has " – cosine", we know the directrix is to the LEFT of the pole (a focal point).

ANSWER: This is a hyperbola with the directrix located 3 units to the left of the pole.

$r = \dfrac{ep}{1 \pm e\_\_\_\theta}$

Example 2: $r = \dfrac{3/4}{4 + 2\sin\theta}$ $\begin{matrix}\div 4 & \div 4\end{matrix}$ $= \dfrac{3/4}{1 + 1/2\sin\theta}$ $e = 1/2 < 1$

$ep = 3/4 \rightarrow 1/2\, p = 3/4 \rightarrow p = 3/2$

above

$\boxed{\text{This is an \underline{ellipse} with a directrix located \underline{3/2} units \underline{ABOVE} the pole.}}$

Example 3: Find the equation with $e = 1$; directrix $x = -1$ 1 unit left $\Rightarrow \ominus\cos\theta$, $p=1$

$\boxed{r = \dfrac{1}{1-\cos\theta}}$

This is a <u>parabola</u> with a directrix located <u>1</u> units <u>left of</u> the pole.

Guided Example 4: Find the equation with $e = 3/4$; directrix $y = -2$ down 2 $\Rightarrow \ominus\sin\theta$, $p=2$

$\hookrightarrow < 1$

$ep = 3/4(2) = 6/4 = 3/2$

$r = \dfrac{3/2}{1 - 3/4\sin\theta}$ $\cdot 4$

$\boxed{r = \dfrac{6}{4-3\sin\theta}}$

This is an <u>ellipse</u> with a directrix located <u>2</u> units <u>below</u> the pole.

You try:

For the following, state the value of e, identify the type of conic, and describe the location of the directrix.

1. $r = \dfrac{8}{1 + 2\cos\theta}$ $2p = 8$, $p = 4$ $\rightarrow 4$

$\boxed{e = 2 \quad 2 > 1 \quad \text{directrix: } x = 4 \quad \text{conic} = \text{hyperbola}}$

2. $r = \dfrac{8/2}{2/2 - 2/2\sin\theta} = \dfrac{4 = ep}{1 - 1\sin\theta}$ $\downarrow 4$

$\boxed{e = 1 \quad e = 1 \quad \text{directrix: } y = -4 \quad \text{conic} = \text{parabola}}$

3. $r = \dfrac{4/3}{3/3 + 1/3\sin\theta} = \dfrac{4/3}{1 + 1/3\sin\theta}$ $\uparrow 4$

$1/3\, p = 4/3 \rightarrow p = 4$

$\boxed{e = 1/3 \quad 1/3 < 1 \quad \text{directrix: } y = 4 \quad \text{conic} = \text{ellipse}}$

4. $r = \dfrac{5}{4 - 6\cos\theta} = \dfrac{5/4}{1 - 3/2\cos\theta}$ $\leftarrow 5/6$

$\frac{3}{2}p = \frac{5}{4}$

$p = \frac{5}{4}\cdot\frac{2}{3} = \frac{5}{6}$

$\boxed{e = 3/2 \quad 3/2 > 1 \quad \text{directrix: } x = -5/6 \quad \text{conic} = \text{hyperbola}}$

Find the polar equation of the conic described:

5. Focus is at the pole, eccentricity is 2/5 and the equation of the directrix is x = - 3.

$|p| = 3$ $e = 2/5$ Left 3 $\Rightarrow \ominus\cos\theta$ $r = \dfrac{2/5(3)\cdot 5}{5\cdot 1 - 2/5\cos\theta} \Rightarrow \boxed{r = \dfrac{6}{5 - 2\cos\theta}}$

6. Eccentricity, e = 3 and the equation of directrix is y = - 2.

$p = 2$ $\downarrow 2 \Rightarrow \ominus\sin\theta$

$r = \dfrac{3(2)}{1 - 3\sin\theta} \rightarrow \boxed{r = \dfrac{6}{1 - 3\sin\theta}}$