Unit 8 Test Review – Vectors and Polar
Pre-Calculus Honors

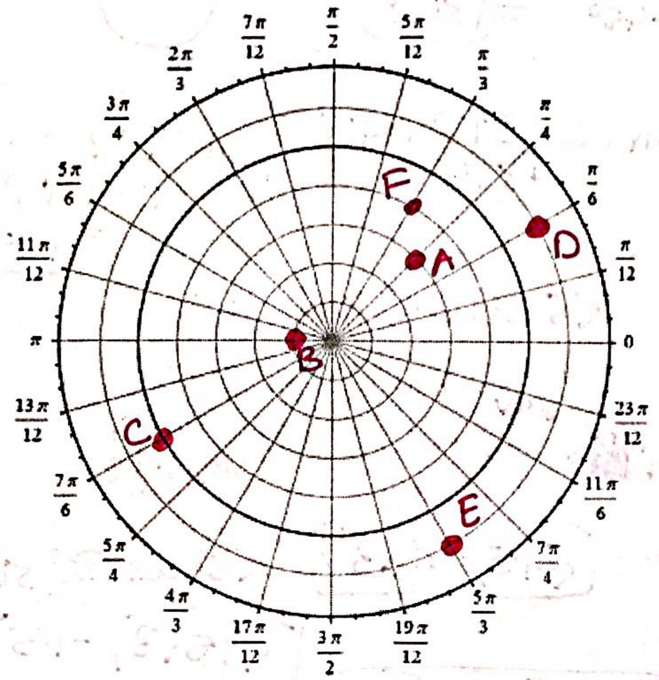
Name: Key

1. Plot the following points on the graph below. Then convert each point to a rectangular coordinate.

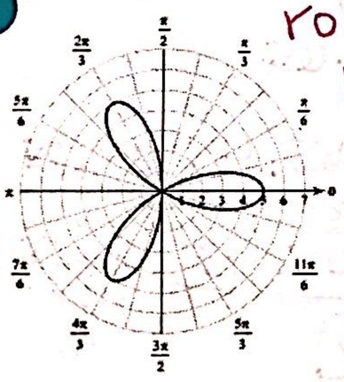
A. $(-3,-135°)$ B. $(1,180°)$

C. $\left(-5,-\frac{11\pi}{6}\right)$ D. $\left(-6,\frac{7\pi}{6}\right)$

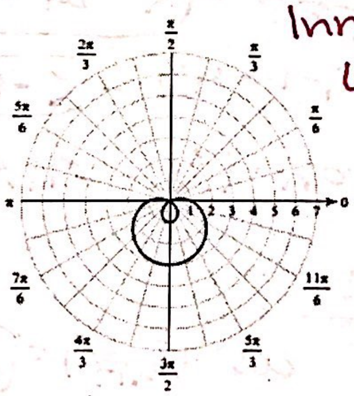
E. $\left(6,\frac{5\pi}{3}\right)$ F. $(-4,-120°)$



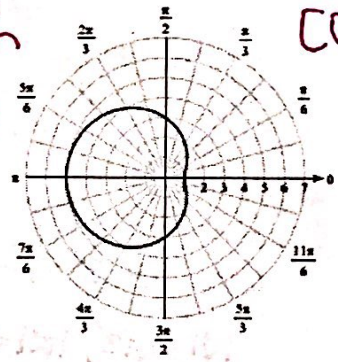
2. Classify each polar curve and state its domain.



rose
$[0,\pi]$



Inner Loop Limacon
$[0,2\pi]$



Limacon
$[0,2\pi]$

3. Convert $2x^2 + 2y^2 = 3$ from rectangular form to a polar equation, and solve for r.

$2(x^2+y^2)=3$

$2r^2 = 3$

$r^2 = 3/2$

$\boxed{r = \pm\sqrt{3/2} = \frac{\pm\sqrt{6}}{2}}$

4. Convert $2xy = 1$ from rectangular form to a polar equation, and solve for r.

$2r\sin\theta\cos\theta = 1$

$r^2(2\sin\theta\cos\theta)=1$

$r^2(\sin 2\theta) = 1$

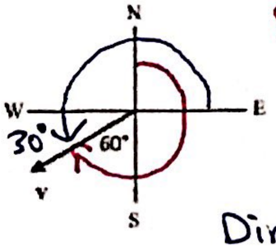
$r^2 = \csc 2\theta$

$\boxed{r = \pm\sqrt{\csc 2\theta}}$

5. Convert $r\sin\theta = 2$ and $r\cos\theta = -3$ to rectangular equations.

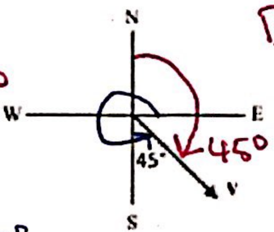
$\boxed{y=2}$ $\boxed{x=-3}$

6. State the bearing AND the direction angle for the vectors shown below.



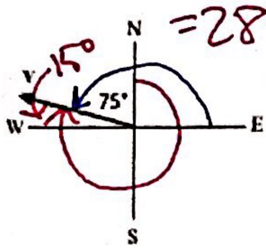
Bearing $= 180° + 60° = \boxed{240°}$

Dir. ∠ $= 180° + 30° = \boxed{210°}$



Bearing $= 90° + 45° = \boxed{135°}$

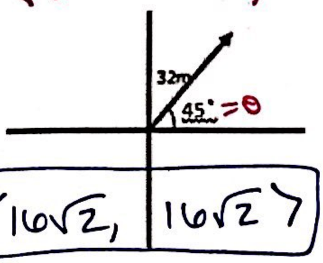
Dir. ∠ $= 270° + 45° = \boxed{315°}$



Bearing $= 270° + 15° = 285°$

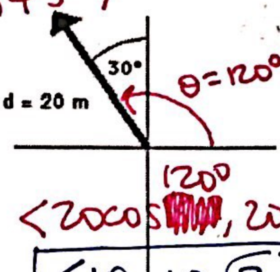
Dir. ∠ $= 90° + 75° = \boxed{165°}$

7. Write each of the following vectors in component form.



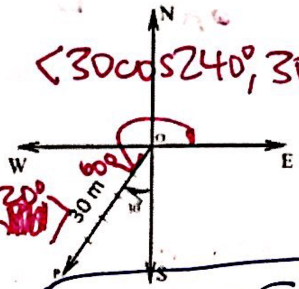
$\langle 32\cos 45°, 32\sin 45° \rangle$

$\boxed{\langle 16\sqrt{2}, 16\sqrt{2} \rangle}$



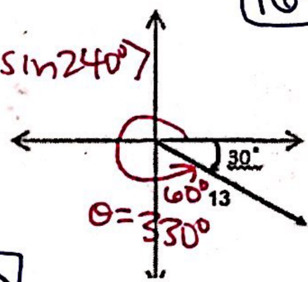
$\langle 20\cos 120°, 20\sin 120° \rangle$

$\boxed{\langle -10, 10\sqrt{3} \rangle}$



$\langle 30\cos 240°, 30\sin 240° \rangle$

$\boxed{\langle -15, -15\sqrt{3} \rangle}$



$\langle 30\cos 330°, 30\sin 330° \rangle$

$\boxed{\langle 15\sqrt{3}, -15 \rangle}$

8. Consider the vector **v** = 5**i** – 2**j**.

A) Find |**v**|. $\sqrt{5^2 + (-2)^2} = \boxed{\sqrt{29}}$

B) Find the unit vector in the same direction as **v**.

$\frac{\langle 5, -2 \rangle}{\sqrt{29}} = \boxed{\left\langle \frac{5\sqrt{29}}{29}, \frac{-2\sqrt{29}}{29} \right\rangle}$

9. Given that P(3, -4) and Q(2, -1), find:

A) The position vector $\overrightarrow{PQ}$. $\langle 2-3, -1--4 \rangle = \boxed{\langle -1, 3 \rangle}$ ← QII

B) The direction angle of the vector $\overrightarrow{PQ}$.

$\tan^{-1}\left(\frac{3}{-1}\right) = -71.57° = 288.43°$ ← QIV

-180

$\boxed{108.43°}$ ← QII ✓

10. Given **v** = ⟨3, −2⟩ and **w** = ⟨0, 5⟩, find:

A) V•W $= 3(0) - 2(5) = \boxed{-10}$

B) The angle between the two vectors.

$|\vec{v}| = \sqrt{13}$, $|\vec{w}| = \sqrt{25} = 5$

$\cos^{-1}\left(\frac{-10}{5\sqrt{13}}\right) = \boxed{123.69°}$

11. An airplane travels 250 mph due south. There is a steady 35 mph wind with a bearing of 90°.

A) Write the component form of the velocity vector of the airplane (without wind).

$\vec{v} = \langle 250\cos(180°), 250\sin(180°) \rangle = \boxed{\langle 0, -250 \rangle}$

B) Write the component form of the ~~velocity~~ wind vector of the airplane.

$\vec{w} = \langle 35\cos 0°, 35\sin 0° \rangle = \boxed{\langle 35, 0 \rangle}$

C) Write the component form of the actual velocity vector of the airplane.

$\vec{v} + \vec{w} = \boxed{\langle 35, -250 \rangle}$

D) What is the actual ground speed of the airplane?

$|\vec{v} + \vec{w}| = \boxed{252.44 \text{ mph}}$

E) What is the actual compass bearing of the airplane?

$\tan^{-1}\left(\frac{-250}{35}\right) = -82.03 \rightarrow$ bearing $= 90° + 82.03 = \boxed{172.03°}$