Unit 7 Packet – Identities Continued...
Pre-Calculus Honors

Name: Key!!

Unit ~~6~~ 7 Notes – **SOLVING TRIG EQUATIONS**

RECALL: We have solved basic trig equations before (they were on the last test).

Ex 1: Solve: sin Θ = ½. (ask yourself, "Self, where does sine = ½?") *Sine is ⊕ in I, II*

$\theta = \frac{\pi}{6} \pm 2\pi k$

and $\frac{5\pi}{6} \pm 2\pi k$

2πk takes into account k number of full rotations!

Ex 2: Solve: $2\sin^2\Theta - \sin\Theta - 1 = 0$ (this has a squared term and a linear term, so factor)

Let $y = \sin\theta$.

$2y^2 - y - 1 = 0$

$(y - \frac{2}{2})(y + \frac{1}{2}) = 0$

$(y-1)(2y+1) = 0$

$y - 1 = 0 \quad 2y + 1 = 0$

$y = 1 \quad y = -1/2$

$\sin\theta = 1 \quad \sin\theta = -1/2$ *Sine is ⊖ in III, IV*

$\theta = \frac{\pi}{2} + 2\pi k$ and $\frac{7\pi}{6} + 2\pi k$ and $\frac{11\pi}{6} + 2\pi k$

"BLOB" PROBLEMS:

These problems will be very similar to Example 1. The main difference is the angle.

Ex 3: Solve: $\sin\boxed{2\Theta} = 1$ Let ▭ $= 2\theta$

To do: Treat the problem like sin(▭) = 1. (▭) = blob (represents an unknown angle)

So, find the angle whose sine = 1 → π/2 ← where sine = 1

You know that the unknown angle must represent π/2 in order for the equation to be true...

consider possible full circle rotations

Therefore, (▭) = π/2 + 2πk In the original problem, we did not have a BLOB, we had 2Θ.

Therefore, $\frac{2\Theta}{2} = \frac{\pi/2}{2} + \frac{2\pi k}{2}$ Now, you can solve for theta by dividing *everything* by 2.

Answer: $\boxed{\Theta = \pi/4 + \pi k}$ $\frac{\pi}{2} \div 2 = \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$

Ex 4: Solve: cos Θ/4 = 1

▭ $= \frac{\theta}{4}$

Rewrite with a blob: cos ▭ = 1

Find out what angle has a cosine of 1: $0 + 2\pi k = 2\pi k$

Set blob contents = to that angle: $4 \cdot \frac{\theta}{4} = 2\pi k \cdot 4$

Solve for theta:

Answer: $\boxed{8\pi k} = \theta$

Ex 5: Solve: $\tan 3\Theta = \sqrt{3}$ * remember for tan, use "+ πk" in answer

jumps back and forth between QI and QIII

$\tan ▭ = \frac{\sqrt{3}}{1} \begin{matrix} \leftarrow y \\ \leftarrow x \end{matrix}$ $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ or $\left(-1/2, -\sqrt{3}/2\right)$

$\frac{\boxed{3\theta}}{3} = \frac{\pi/3}{3} + \frac{\pi k}{3}$ → $\boxed{\theta = \frac{\pi}{9} + \frac{\pi k}{3}}$

5.3 Solving Trig Equations Practice Worksheet #1
Pre-calculus

Name: ____________________
Date: ____________________ Block: ____

Solve for the unknown variable on the interval $\boxed{0 \le x < 2\pi}$.

1. $4\cos^2 x - 3 = 0$

$4x^2 - 3 = 0$

$\sqrt{x^2} = \pm\sqrt{\frac{3}{4}}$

$x = \frac{\pm\sqrt{3}}{2}$

$\cos\theta = \frac{\pm\sqrt{3}}{2}$

$\boxed{\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

2. $\sqrt{2}\sin(\boxed{2x}) = 1$

$\frac{\sqrt{2}}{\sqrt{2}} y\square = \frac{1}{\sqrt{2}}$

$y\square = \frac{1\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\frac{\boxed{2x}}{2} = \frac{\frac{\pi}{4}}{2}$ $\quad \frac{\boxed{2x}}{2} = \frac{3\pi}{4} \cdot \frac{1}{2}$

$\boxed{x = \frac{\pi}{8}, \frac{3\pi}{8}}$ & $\frac{9\pi}{8}, \frac{11\pi}{8}$

3. $3\cot^2 x - 1 = 0$

$\cot^2 x = 1/3$

$\cot x = \pm\sqrt{1/3} = \frac{\pm 1}{\sqrt{3}}$

$\hookrightarrow \tan x = \pm\sqrt{3}$

$\boxed{x = \frac{\pi}{3}, \frac{4\pi}{3}}$ & $\frac{2\pi}{3}, \frac{5\pi}{3}$

4. $\cos^3 x = \cos x$

$\cos^3 x - \cos x = 0$

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x+1)(x-1) = 0$

$x = 0 \quad x + 1 = 0 \quad x - 1 = 0$

$\cos x = 0 \quad \cos x = -1 \quad \cos x = 1$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$ $\boxed{x = \pi}$ $\boxed{x = 0}$

5. $\sin x - 2\sin x\cos x = 0$

$\sin x(1 - 2\cos x) = 0$

$\sin x = 0 \quad 1 - 2\cos x = 0$

$\boxed{x = 0, \pi}$ $\quad \cos x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

6. $2\sin^2 x - \sin x - 3 = 0$

$2y^2 - y - 3 = 0$

$(y - \frac{3}{2})(y + \frac{2}{2}) = 0$

$(2y - 3)(y + 1) = 0$

$2\sin x - 3 = 0 \quad \sin x + 1 = 0$

$\sin x = 3/2 \quad \sin x = -1$

$x =$ DNE $\quad \boxed{x = \frac{3\pi}{2}}$

7. $\csc^2 x - \csc x - 2 = 0$

$y^2 - y - 2 = 0$

$(y+1)(y-2) = 0$

$\csc x = -1 \quad \csc x = 2$

$\frac{1}{\sin x} = -1 \quad \frac{1}{\sin x} = 2$

$\sin x = -1 \quad \sin x = 1/2$

$\boxed{x = \frac{3\pi}{2}}$ $\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

8. $\cos^2 x = 1 - \sin x$

$1 - \sin^2 x = 1 - \sin x$

$-1 + \sin x \quad -1 + \sin x$

$-\sin^2 x + \sin x = 0$

$-\sin x(\sin x - 1) = 0$

$-\sin x = 0 \quad \sin x = 1$

$\boxed{x = 0, \pi}$ $\boxed{x = \frac{\pi}{2}}$

Solve for the unknown variable on the given interval.

9. $\sqrt{3} + \tan(\boxed{2x}) = 0$ on $[0, 2\pi)$.

$\tan\square = -\sqrt{3}$

$\frac{\boxed{2x}}{2} = \frac{\frac{2\pi}{3}}{2} \quad \frac{\boxed{2x}}{2} = \frac{\frac{5\pi}{3}}{2}$

$\boxed{x = \frac{\pi}{3} \quad x = \frac{5\pi}{6}}$

10. $\cos(\boxed{\pi x}) = 0.5$ on $[0, 2)$.

$\cos\square = 1/2$

$\frac{\boxed{\pi x}}{\pi} = \frac{\frac{\pi}{3}}{\pi} \quad \frac{\boxed{\pi x}}{\pi} = \frac{\frac{5\pi}{3}}{\pi}$

$\boxed{x = \frac{1}{3} \quad x = \frac{5}{3}}$

11. $\sin\left(\boxed{\frac{x}{2}}\right) - 1 = 0$ on $[0, 8\pi)$.

= BLOB

$+1 \quad +1$

$y\square = 1$

$2\boxed{x/2} = \frac{\pi}{2} \cdot 2 + 2\pi k \cdot 2$

$x = \pi + 4\pi k$

$\boxed{x = \pi, 5\pi}$

5.3 Solving Trig Equations – Worksheet #2
Pre-calculus

Name: ____________________
Date: ____________________ Block: ___

Part 1: Solve for the unknown variable. Give all of the exact general solutions.

1. $\sin\theta = \frac{\sqrt{2}}{2}$

$\theta = \boxed{\frac{\pi}{4} \pm 2\pi k}$ and $\boxed{\frac{3\pi}{4} \pm 2\pi k}$

2. $\cos\theta = \sin\theta$

$\sin\theta \quad \sin\theta$

$\tan\theta = 1$

$\boxed{\theta = \frac{\pi}{4} \pm \pi k}$

3. $\tan\theta = 1$

$\boxed{\theta = \frac{\pi}{4} \pm \pi k}$

4. $1 + \sin\theta = 2\cos^2\theta$

$1 + \sin\theta = 2(1 - \sin^2 x)$

$1 + \sin\theta = 2 - 2\sin^2 x$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(\sin\theta + \frac{2}{2})(\sin\theta - \frac{1}{2}) = 0$

$\sin\theta = -1 \quad \sin\theta = 1/2$

$\boxed{\theta = \frac{3\pi}{2} \pm 2\pi k, \frac{\pi}{6} \pm 2\pi k, \frac{5\pi}{6} \pm 2\pi k}$

5. $2\cos^2\theta + \cos\theta = 0$

$\cos\theta(2\cos\theta + 1) = 0$

$\cos\theta = 0 \quad \cos\theta = -1/2$

$\boxed{\theta = \frac{\pi}{2} \pm 2\pi k, \frac{3\pi}{2} \pm 2\pi k}$ $\boxed{\theta = \frac{2\pi}{3} \pm 2\pi k, \frac{4\pi}{3} \pm 2\pi k}$

6. $\sin 3\theta = -1$

$\sin\square = -1$

$\frac{\boxed{3\theta}}{3} = \frac{\frac{3\pi}{2}}{3} \pm \frac{2\pi k}{3}$

$\boxed{\theta = \frac{\pi}{2} \pm \frac{2\pi k}{3}}$

7. $\sin^2\theta - 1 = 0$

$\sqrt{\sin^2\theta} = \pm\sqrt{1}$

$\sin\theta = \pm 1$

$\boxed{\theta = \frac{\pi}{2} \pm 2\pi k \text{ or } \frac{3\pi}{2} \pm 2\pi k}$

8. $\cos 2\theta = ½$

$\cos\square = 1/2$

$\frac{\boxed{2\theta}}{2} = \frac{\frac{\pi}{3}}{2} + \frac{2\pi k}{2}$

$\boxed{\theta = \frac{\pi}{6} \pm \pi k}$

$\frac{\boxed{2\theta}}{2} = \frac{\frac{5\pi}{3}}{2} + \frac{2\pi k}{2}$

$\boxed{\theta = \frac{5\pi}{6} \pm \pi k}$

9. $2\sin^2\theta - \sin\theta - 1 = 0$

$2x^2 - x - 1 = 0$

$(x - \frac{2}{2})(x + \frac{1}{2}) = 0$

$(\sin\theta - 1)(2\sin\theta + 1) = 0$

$\sin\theta = 1 \quad \sin\theta = -1/2$

$\boxed{\theta = \frac{\pi}{2} \pm 2\pi k, \frac{7\pi}{6} \pm 2\pi k, \frac{11\pi}{6} + 2\pi k}$

10. $\tan 4\theta = -1$ II, IV

$\frac{y}{x}\square = -1$

$\frac{\boxed{4\theta}}{4} = \frac{\frac{3\pi}{4}}{4} \pm \frac{\pi k}{4}$

$\boxed{\theta = \frac{3\pi}{16} \pm \frac{\pi k}{4}}$

11. $\tan^2 3x = 3$

$\pm\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\sqrt{\tan^2\square} = \pm\sqrt{3}$

$\tan\square = \pm\frac{\sqrt{3}}{1} = \frac{y}{x}$

$\frac{\boxed{3x}}{3} = \frac{\pi}{3}\cdot\frac{1}{3} \pm \frac{\pi k}{2}\cdot\frac{1}{3}$

$\boxed{x = \frac{\pi}{9} \pm \frac{\pi k}{6}}$

12. $\cos\frac{x}{2} = \frac{\sqrt{2}}{2}$ I, IV

$\cos\square = \frac{\sqrt{2}}{2}$

$2\cdot\boxed{x/2} = \frac{\pi}{4}\cdot 2 \pm 2\pi k\cdot 2$

$\boxed{x = \frac{\pi}{2} \pm 4\pi k}$

$2\cdot\boxed{x/2} = \frac{7\pi}{4}\cdot 2 \pm 2\pi k\cdot 2$

$\boxed{x = \frac{7\pi}{2} \pm 4\pi k}$ 3

Part 2: Solve by approximating the solutions on the interval $\boxed{[0,2\pi)}$.

13. $2\sin^2 x + 3\sin x + 1 = 0$

$2y^2+3y+1=0$

$(y+\frac{1}{2})(y+\frac{2}{2})=0$

$(2y+1)=0 \quad y+1=0$

III, IV $\quad \sin x = \frac{-1}{2} \quad \sin x = -1$

$\boxed{x=\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}}$

OMIT!!

* 14. $4\sin^2 x = 2\cos x + 1$ *

$4(1-\cos^2 x)=2\cos x+1$

$4(1-x^2)=2x+1$

$4x-4x^2-2x-1=0$

$+4x^2+2x+3=0$

Quad Formula $\quad x=\frac{1\pm\sqrt{13}}{-4}$

$\boxed{x=\cos^{-1}\left(\frac{1\pm\sqrt{13}}{-4}\right)}$

15. $\csc x + \cot x = 1$

$\frac{1}{\sin x}+\frac{\cos x}{\sin x}=1$

$\sin x \cdot \frac{1+\cos x}{\sin x} = 1(\sin x)$

$(1+\cos x)^2=(\sin x)^2$

$1+2\cos x+\cos^2 x=\sin^2 x$

$1+2\cos x+\cos^2 x = 1-\cos^2 x$

$-1 \quad\quad +\cos^2 x \quad -1 \; +\cos^2 x$

$2\cos x + 2\cos^2 x = 0$

$2x+2x^2=0$

$2x(1+x)=0$

$2\cos\theta=0 \qquad 1+\cos\theta=0$

$\cos\theta=-1$

$\boxed{\theta=\frac{\pi}{2}, \frac{3\pi}{2}} \qquad \boxed{\theta=\pi}$

16. $\frac{\cos x \cot x}{1-\sin x} = 3$

$\sin x \cdot \frac{\cos x \cos x}{\sin x} = 3(1-\sin x) \cdot \sin x$

$\cos^2 x = 3\sin x(1-\sin x)$

$1-\sin x^2 = 3\sin x - 3\sin^2 x$

$2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$\sin x = 1/2 \quad \sin x = 1 \quad \boxed{x=\pi/2, \pi/6, 5\pi/6}$

17. $\sec^2 x + 0.5\tan x = 1$

$\sec^2 x + \frac{1}{2}(\sec^2 x - 1) = 1$

$\sec^2 x + \frac{1}{2}\sec^2 x - \frac{1}{2} = 1$

$\frac{3}{2}\sec^2 x - \frac{3}{2} = 0$

$3/2(\sec^2 x - 1) = 0$

$\tan^2 x = 0$

$\boxed{x=0, \pi}$

Part 3: Use the calculator's inverse trig functions to approximate the solutions. Remember that you must also find the other solution by either adding π (tan), subtracting the value from π (sin), or subtracting the value from 2π (cos).

18. $\tan\theta = 4$

$+\pi$ QI QIII

$\tan^{-1} 4 = \theta$

$\boxed{\theta = 1.3258, 4.4674}$

$+\pi$

19. $\cos\theta = 0.84$

QI QIV $\quad 2\pi -$

$\cos^{-1}(0.84)=\theta$

$\boxed{\theta = 0.5735, 5.7097}$

$+ \quad = 2\pi$

20. $\sin\theta = 0.63$

$\pi -$ QII QI

$\sin^{-1}(0.63)=\theta$

$\boxed{\theta = 0.6816, 2.46}$

$+ \quad = \pi$

Day 2 Notes – Sum and Difference Identities

opp. signs

Sum and Difference Identities
$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

same

Example 1 Evaluate each trigonometric sum or difference.

a) $\sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \sin(\pi/3)\cos(\pi/4) + \cos(\pi/3)\sin(\pi/4)$

$= (\sqrt{3}/2)(\sqrt{2}/2) + (1/2)(\sqrt{2}/2) = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

b) $\cos(45 - 60) = \cos(45^\circ)\cos(60^\circ) + \sin(45^\circ)\sin(60^\circ)$

$= (\sqrt{2}/2)(1/2) + (\sqrt{2}/2)(\sqrt{3}/2)$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

c) $\tan\left(\frac{\pi}{6} - \frac{\pi}{3}\right) = \dfrac{\tan(\pi/6) - \tan(\pi/3)}{1 + \tan(\pi/6)\tan(\pi/3)} = \dfrac{\frac{\sqrt{3}}{3} - \frac{3\sqrt{3}}{1\cdot 3}}{1 + \frac{\sqrt{3}}{3}\left(\frac{\sqrt{3}}{1}\right) = \frac{3}{3}} = \dfrac{\frac{1\sqrt{3} - 3\sqrt{3}}{3}}{1+1} = \dfrac{\frac{-2\sqrt{3}}{3}}{2}$

$\frac{1\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{\sqrt{3}}{3}$

$= \frac{-2\sqrt{3}}{6} = \boxed{\frac{-\sqrt{3}}{3}}$

Example 2 Write each expression as a single trigonometric ratio.

a) $\sin\frac{\pi}{6}\cos\frac{\pi}{2} + \cos\frac{\pi}{6}\sin\frac{\pi}{2} = \cancel{\left(\frac{1}{2}\right)(0)}^{0} + \left(\frac{\sqrt{3}}{2}\right)(1) = \boxed{\frac{\sqrt{3}}{2}}$

$= \sin(\pi/6 + \pi/2) = \sin(4\pi/6) = \sin(2\pi/3)$

b) $\dfrac{\tan\frac{\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{4}\tan\frac{\pi}{6}} = \dfrac{\frac{3\cdot 1}{3} - \frac{\sqrt{3}}{3}}{\frac{3\cdot 1}{3} + 1\left(\frac{\sqrt{3}}{3}\right)} = \dfrac{\boxed{\frac{3-\sqrt{3}}{3}}}{\frac{3+\sqrt{3}}{3}} = \dfrac{(3-\sqrt{3})\,\cancel{3}}{\cancel{3}\,(3+\sqrt{3})}$

$= \boxed{\dfrac{3-\sqrt{3}}{3+\sqrt{3}}} \cdot \dfrac{(3-\sqrt{3})}{(3-\sqrt{3})}$

$= \boxed{\tan(\pi/12)}$

c) $\cos\frac{\pi}{3}\cos\frac{\pi}{6} + \sin\frac{\pi}{3}\sin\frac{\pi}{6}$

$= \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2}\left(\frac{1}{2}\right) = \frac{1\sqrt{3}}{4} + \frac{1\sqrt{3}}{4} = \frac{2\sqrt{3}}{4} = \boxed{\frac{\sqrt{3}}{2}}$

$0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ, \ldots$

Kuta Software - Infinite Algebra 2

Name______________________________

Angle Sum/Difference Identities

Date________________ Period____

Use the angle sum identity to find the exact value of each.

1) cos 105° $= \cos(\overset{A}{60^\circ} + \overset{B}{45^\circ})$

$= \cos(60^\circ)\cos(45^\circ) - \sin(60^\circ)\sin(45^\circ)$

$= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

2) sin 195°

$\frac{\sqrt{2}-\sqrt{6}}{4}$

3) cos 195° $= \cos(150^\circ + 45^\circ)$

$\boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$

4) cos 165°

$\frac{-\sqrt{6}-\sqrt{2}}{4}$

5) cos 285° $= \cos(225^\circ + 60^\circ)$

$\boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

6) cos 255°

$\frac{\sqrt{2}-\sqrt{6}}{4}$

7) sin 105° $= \sin(60^\circ + 45^\circ)$

$\boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

8) sin 285°

$\frac{-\sqrt{6}-\sqrt{2}}{4}$

9) cos 75° $= \cos(30^\circ + 45^\circ)$

$= \cos(30^\circ)\cos(45^\circ) - \sin(30^\circ)\sin(45^\circ)$

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

10) sin 255°

$\frac{-\sqrt{6}\mp\sqrt{2}}{4}$

Use the angle difference identity to find the exact value of each.

11) cos 75° $= \cos(225^\circ - 150^\circ)$

$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

12) cos −15°

$\frac{\sqrt{6}+\sqrt{2}}{4}$

13) tan 75°

$2+\sqrt{3}$

14) cos 15°

$\frac{\sqrt{6}+\sqrt{2}}{4}$

15) tan −105°

$2+\sqrt{3}$

16) sin 105°

$\frac{\sqrt{6}+\sqrt{2}}{4}$

17) tan 15°

$2-\sqrt{3}$

18) sin 15°

$\frac{\sqrt{6}-\sqrt{2}}{4}$

19) tan −15°

$\sqrt{3}-2$

20) sin −75°

$\frac{-\sqrt{6}-\sqrt{2}}{4}$

Use the angle sum or difference identity to find the exact value of each.

21) sin −105°

$\frac{-\sqrt{6}-\sqrt{2}}{4}$

22) cos 195°

$\frac{-\sqrt{6}-\sqrt{2}}{4}$

23) $\cos \frac{7\pi}{12}$

$\frac{\sqrt{2}-\sqrt{6}}{4}$

24) $\tan \frac{13\pi}{12} = \tan(5\pi/6 + \pi/4)$

$= \frac{\tan(5\pi/6)+\tan(\pi/4)}{1-\tan(5\pi/6)(\pi/4)} = \frac{\frac{-\sqrt{3}}{\cancel{3}} + 1\cdot\frac{3}{\cancel{3}}}{\frac{3}{\cancel{3}}\cdot 1 + (+\frac{\sqrt{3}}{\cancel{3}})\cdot 1}$

$= \frac{(-\sqrt{3}+3)}{(3+\sqrt{3})}\cdot\frac{(3-\sqrt{3})}{(3-\sqrt{3})}$ FOIL $= \frac{\boxed{-3\sqrt{3}}+3+9\boxed{-3\sqrt{3}}}{9-3}$

$= \frac{-6\sqrt{3}+12}{6}$

$= \boxed{-\sqrt{3}+2}$

25) $\sin \frac{\pi}{12}$

$\frac{\sqrt{6}-\sqrt{2}}{4}$

26) $\cos -\frac{7\pi}{12}$

$\frac{\sqrt{2}-\sqrt{6}}{4}$

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Day 3 Notes – Double and Half Angle Identities

R → D: $\theta \cdot \frac{180}{\pi}$ | D → R: $\theta \cdot \frac{\pi}{180}$

2θ **Double Angle Identities**	$\sin(2A) = 2\sin A\cos A$	$\cos(2A) = \cos^2 A - \sin^2 A$ $= 1 - 2\sin^2 A$ $= 2\cos^2 A - 1$	$\tan(2A) = \frac{2\tan A}{1-\tan^2 A}$
$\frac{\theta}{2}$ **Half Angle Identities**	$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$	$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$	$\tan\left(\frac{A}{2}\right) = \frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$

Evaluate the following expressions using double- and half-angle formulas.

1. $\sin 15°$
$= \sin(45° - 30°)$
$= \frac{\sqrt{6}-\sqrt{2}}{4}$

OR $= \sin\left(\frac{30°}{2}\right)$ $A = 30°$ $\rightarrow \pm\sqrt{\frac{1-\cos(30°)}{2}} = \pm\sqrt{\frac{1-\sqrt{3}/2}{2 \cdot 2}}$ $= \pm\sqrt{\frac{2/2 - \sqrt{3}/2}{4/2}} = \pm\sqrt{\frac{2-\sqrt{3}}{4}} = \boxed{\frac{\pm\sqrt{2-\sqrt{3}}}{2}}$

2. $\cos 22.5°$
$= \cos\left(\frac{45°}{2}\right) = \pm\sqrt{\frac{1+\cos(45°)}{2}} = \pm\sqrt{\frac{1+\sqrt{2}/2}{2}}$
$= \pm\sqrt{\frac{2/2 + \sqrt{2}/2}{4/2}} = \pm\sqrt{\frac{2+\sqrt{2}}{4}} = \boxed{\frac{\pm\sqrt{2+\sqrt{2}}}{2}}$

3. $\tan(-15°)$
$= \tan\left(\frac{-30°}{2}\right) = \frac{\sin(-30°)}{1+\cos(-30°)} = \frac{\sin(330°)}{1+\cos(330°)} = \frac{-1/2}{1+\sqrt{3}/2} = \frac{-1/2}{2/2+\sqrt{3}/2}$
$= \frac{-1}{(2+\sqrt{3})} \cdot \frac{(2-\sqrt{3})}{(2-\sqrt{3})} = \frac{-2+\sqrt{3}}{4-3} = \boxed{-2+\sqrt{3}}$

4. $\sin\left(-\frac{\pi}{12}\right) = \sin(-15°) = \sin\left(\frac{-30°}{2}\right)$
$= \pm\sqrt{\frac{1-\cos(-30°)}{2}} = \pm\sqrt{\frac{1-\sqrt{3}/2}{2}} = \pm\sqrt{\frac{2/2 - \sqrt{3}/2}{4/2}} = \boxed{\frac{\pm\sqrt{2-\sqrt{3}}}{2}}$

$\frac{-\pi}{12} \cdot \frac{180}{\pi} = \frac{-180}{12} = -15$

5. $\cos\frac{5\pi}{8} = \cos\left(\frac{225°}{2}\right) = \pm\sqrt{\frac{1+\cos(225°)}{2}} = \pm\sqrt{\frac{1+(-\sqrt{2}/2)}{2}}$
$= \pm\sqrt{\frac{2/2 + -\sqrt{2}/2}{4/2}} = \pm\sqrt{\frac{2-\sqrt{2}}{4}} = \boxed{\frac{\pm\sqrt{2-\sqrt{2}}}{2}}$

$\frac{5\pi}{8} \cdot \frac{180}{\pi}$ $= \frac{90}{2} = \frac{225}{2}$

6. $\tan\frac{\pi}{8} = \tan\left(\frac{45°}{2}\right) = \frac{1-\cos(45°)}{\sin(45°)} = \frac{1-\sqrt{2}/2}{\sqrt{2}/2} = \frac{2/2 - \sqrt{2}/2}{\sqrt{2}/2}$
$= \frac{2-\sqrt{2}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{2\sqrt{2}-2}{2} = \boxed{\sqrt{2}-1}$

$\frac{\pi}{8} \cdot \frac{180}{\pi}$ $= \frac{45}{2}$

Find the exact value of the function.

7. $\cos\frac{u}{2}$ if $\cos u = \frac{4}{5}$, $\boxed{0 \le u < \frac{\pi}{2}}$ QI

$\frac{u}{2} = A$

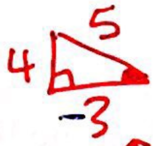
$$\pm\sqrt{\frac{1+\cos u}{2}} = \pm\sqrt{\frac{1+(4/5)}{2/1}}$$

$$= \pm\sqrt{\frac{5/5+4/5}{10/5}} = \pm\sqrt{\frac{5+4}{10}}$$

$$= \frac{\pm\sqrt{9}}{\sqrt{10}} = \frac{\pm 3}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = \boxed{\frac{3\sqrt{10}}{10}}$$

8. $\tan 2u$ if $\boxed{\sin u = \frac{4}{5}}$, $\frac{\pi}{2} \le u < \pi$ QII

O 4, H 5



$\tan u = \frac{O}{A} = \frac{-4}{3}$

$$\frac{2\tan u}{1-\tan^2 u} = \frac{2(-4/3)}{1-(4/3)^2}$$

$$= \frac{-8/3}{1-16/9}\cdot\frac{3}{3} = \frac{24/9}{9/9-16/9}$$

$$= \frac{24}{9-16} = \boxed{\frac{+24}{+7}}$$

Verify the following identities.

9. $2\sin^2 2x + \cos 4x = 1$

$2\sin^2 2x + \cos(2\cdot 2x) = 1$

$\cancel{2\sin^2 2x} + 1 - \cancel{2\sin^2 2x} = 1$

$1 = 1$ ✓

10. $(\sin x + \cos x)^2 = 1 + \sin 2x$

$\sin^2 x + 2\sin x\cos x + \cos^2 x$

$+1 + 2\sin x\cos x$

$1 + \sin 2x$ ✓

Solve the equations for $0 \le x < 2$p. 2π

11. $\sin x = \cos 2x$

$\sin x = 1 - 2\sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0 \qquad \sin x + 1 = 0$

$\sin x = 1/2 \qquad \sin x = -1$

$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}$$

12. $\cos 2x - \cos x - 2 = 0$

$2\cos^2 x - 1 - \cos x - 2 = 0$

$2\cos^2 x - \cos x - 3 = 0$

$(2\cos x - 3)(\cos x + 1) = 0$

$2\cos x - 3 = 0 \qquad \cos x + 1 = 0$

$\cancel{\cos x = \frac{3}{2}} \qquad \cos x = -1$

$$\boxed{x = \pi}$$

Day 3 Homework

Double and Half Angle ID's

Use a double-angle or half-angle identity to find the exact value of each expression.

1) $\cos 75° = \cos\left(\frac{150°}{2}\right) = \pm\sqrt{\frac{1+\cos(150°)}{2}}$

$= \pm\sqrt{\frac{1+-\sqrt{3}/2}{2}} = \pm\sqrt{\frac{2-\sqrt{3}}{4}}$

$= \boxed{\frac{\pm\sqrt{2-\sqrt{3}}}{2}}$

2) $\sin \frac{5\pi}{3}$

$-\frac{\sqrt{3}}{2}$

3) $\cos \frac{\pi}{8}$

$\frac{\sqrt{2+\sqrt{2}}}{2}$

4) $\tan \frac{4\pi}{3}$

$\sqrt{3}$

5) $\sin \frac{5\pi}{8}$

$\frac{\sqrt{2+\sqrt{2}}}{2}$

6) $\tan 60°$

$\sqrt{3}$

7) $\sec \theta = \frac{17}{15}$ and $0 < \theta < \frac{\pi}{2}$

Find $\cos 2\theta$

$\frac{161}{289}$

8) $\cot \theta = -\frac{4}{3}$ and $90° < \theta < 180°$

Find $\sin 2\theta$

$-\frac{24}{25}$

9) $\tan \theta = \frac{3}{4}$ and $180° < \theta < 270°$

Find $\tan 2\theta$

$\frac{24}{7}$

10) $\cos \theta = \frac{4}{5}$ and $0° < \theta < 90°$

Find $\sin \frac{\theta}{2}$

$\frac{\sqrt{10}}{10}$

11) $\csc \theta = \frac{\sqrt{34}}{5}$ and $0° < \theta < 90°$

Find $\cos \frac{\theta}{2}$

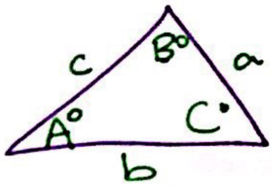
$\frac{\sqrt{578+51\sqrt{34}}}{34}$

12) $\csc \theta = \sqrt{17}$ and $90° < \theta < 180°$

Find $\tan \frac{\theta}{2}$

$\sqrt{17}+4$

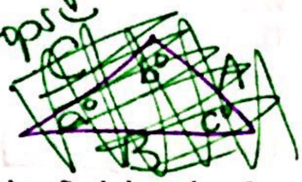
Day 4 Notes – The Law of Sines



For any triangle (right, acute or obtuse), you may use the following formula to solve for missing sides or angles:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

oops



Use the Law of Sines when...**you have 3 dimensions** of a triangle and you need to find the other 3 dimensions - **they cannot be just ANY 3 dimensions** though, or you won't have enough info to solve the Law of Sines equation. Use the Law of Sines if you are given:

- AAS
- ASA
- SSA (Ambiguous Case)

Check Your Calc. MODE!!

Example 1

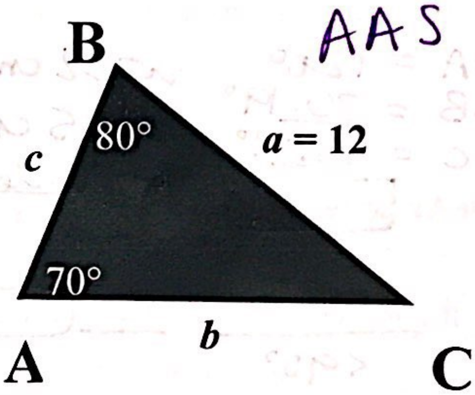
You are given a triangle, ABC, with angle A = 70°, angle B = 80° and side *a* = 12 cm. Find the measures of angle C and sides *b* and *c*.

$\angle A = 70°$ $\quad a = 12$ cm

$\angle B = 80°$ $\quad b = \underline{12.58}$ cm

$\angle C = \underline{30°}$ $\quad c = \underline{6.39}$ cm

AAS



① 180 − 70 − 80

② $\frac{\sin 70°}{12} = \frac{\sin 80°}{b}$

$\frac{b \sin 70°}{\sin 70°} = \frac{12 \sin 80°}{\sin 70°}$

③ $\frac{\sin 70°}{12} = \frac{\sin 30°}{c} \rightarrow c = \frac{12 \sin 30°}{\sin 70°}$

Example 2

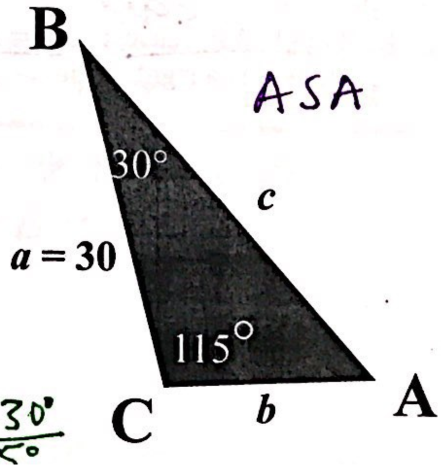
You are given a triangle, ABC, with angle C = 115°, angle B = 30° and side *a* = 30 cm. Find the measures of angle A and sides *b* and *c*.

$\angle A = \underline{35°}$ $\quad a = 30$ cm

$\angle B = 30°$ $\quad b = \underline{26.15}$ cm

$\angle C = 115°$ $\quad c = \underline{41.4}$ cm

ASA



① 180 − 115 − 30

② $\frac{\sin 115°}{c} = \frac{\sin 35°}{30}$

$c = \frac{30 \sin 115°}{\sin 35°}$

③ $\frac{\sin 35°}{30} = \frac{\sin 30°}{b} \rightarrow b = \frac{30 \sin 30°}{\sin 35°}$

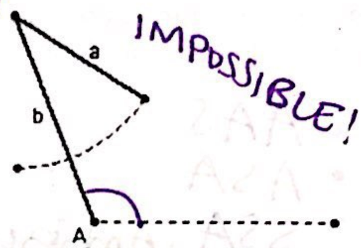
The Ambiguous Case

When given **SSA** (two sides and an angle that is NOT the included angle), the situation is ambiguous. The dimensions may not form a triangle, or there may be 1 or 2 triangles with the given dimensions. We first go through a series of tests to determine how many (if any) solutions exist.

If angle A is obtuse...

1. If angle A is obtuse, and $a < b$ or $a = b$, no such triangle exists.

$>90°$



IMPOSSIBLE!

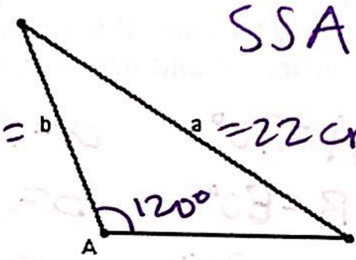
No Triangle

2. If angle A is obtuse, and $a > b$, one such triangle exists.

$>90°$

Given a triangle with angle $A = 120°$, side $a = 22$ cm and side $b = 15$ cm, find the other dimensions.

SSA



15cm = b

a = 22 cm

120°

$\angle A = 120°$ $\quad a = 22$ cm

$\angle B = 36.19°$ $\quad b = 15$ cm

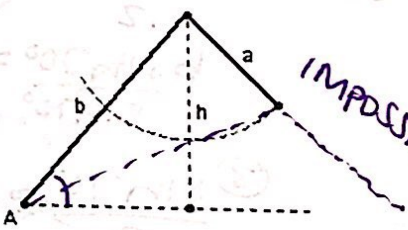
$\angle C = 23.81°$ $\quad c = 10.26$ cm

$\frac{\sin 120°}{22} = \frac{\sin B°}{15}$

If angle A is acute...

3. If angle A is acute, and $a < h$, no such triangle exists.

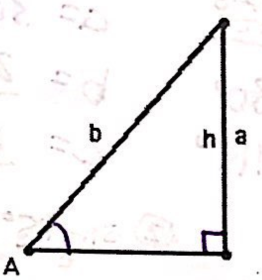
$<90°$



IMPOSSIBLE!

4. If angle A is acute, and $a = h$, one possible triangle exists. Angle B is a right angle.

$<90°$

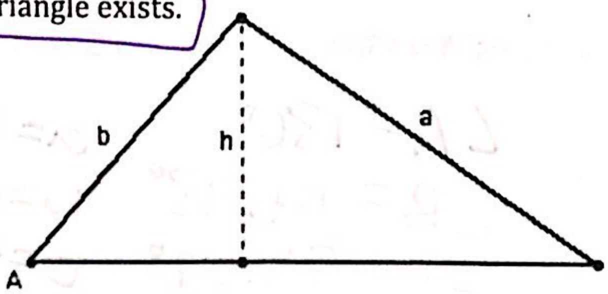


Scanned with CamScanner

and a>h

5. If angle A is acute, and a > b, one possible triangle exists.
<90°

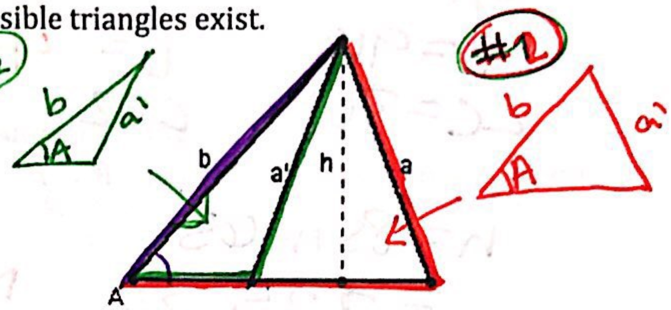
Given a triangle with angle A = 40°,
side a = 12 cm and side b = 10 cm,
find the other dimensions.



∠A = 40° a = 12 cm
∠B = 32.39° b = 10 cm
∠C = 107.61 c = 17.79 cm

6. If angle A is acute, and h < a < b, two possible triangles exist.

Given a triangle with angle A = 40°,
side a = 12 cm and side b = 15 cm,
find the other dimensions.



Triangle 1

∠A = 40° a = 12 cm
∠B = 126.54° b = 15 cm
∠C = 13.46° c = 4.35 cm

Triangle 2

∠A = 40° a = 12 cm
∠B = 53.46° b = 15 cm
∠C = 86.54° c = 4.35 cm

SSA Summary:

if angle A is **obtuse**	if $a < b$ → no solution
	if $a > b$ → one solution
if angle A is **acute** *find the height,* $h = b*\sin A$	if $a < h$ → no solution
	if $h < a < b$ → 2 solutions *one with angle B acute,* *one with angle B obtuse*
	if $a > b > h$ → 1 solution
	If $a = h$ → 1 solution *angle B is right*

Ambiguous Examples:

a. Given: A = 130°, c = 9, a = 12... Find: B, C, b

$\angle A = 130°$ $a = 12$
$\angle B = 14.93°$ $b = 4.04$
$\angle C = 35.07°$ $c = 9$

b. Given: B = 90°, C = 30°, c = 2... Find: A, a, b

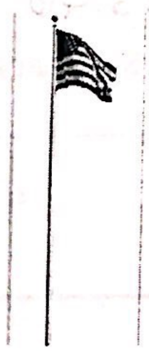
$\angle A = 60°$ $a = 3.46$
$\angle B = 90°$ $b = 4$
$\angle C = 30°$ $c = 2$

NOT Ambiguous

c. Given: A = 65°, a = 6, c = 8... Find: B, C, b

$h = 8 \sin 65$
$= 7.25 > a$ NOT a triangle

Example 3
Two observers are 600 ft apart on opposite sides of a flagpole. The angles of elevation from the observers to the top of the pole are 19° and 21°. Find the height of the flagpole.



Example 4
Officer Chamblee at checkpoint A notices 2 wrecked cars in the direction 48° east of north. Officer Thorne at checkpoint B, 12 miles due east of A, spots the same accident 30° west of north. Find the distance from each checkpoint to the accident.

Day 4 Homework

Law of Sines

State the number of possible triangles that can be formed using the given measurements.

1) $m\angle C = 24°$, $b = 29$ yd, $c = 14$ yd

2

2) $m\angle B = 104°$, $a = 8$ m, $b = 8$ m

0

3) $m\angle C = 70°$, $b = 34$ yd, $c = 5$ yd

0

4) $m\angle B = 40°$, $a = 14$ cm, $b = 24$ cm

1

Find each measurement indicated. Round your answers to the nearest tenth.

5) $m\angle B = 140°$, $m\angle A = 12°$, $c = 27$ m
Find b

37 m

6) $m\angle A = 30°$, $m\angle B = 36°$, $a = 23$ km
Find b

27 km

7) $m\angle C = 62°$, $b = 14$ mi, $c = 9$ mi
Find a

Not a Triangle

8) $m\angle A = 104°$, $m\angle B = 39°$, $a = 37$ yd
Find b

24 yd

9) $m\angle C = 27°$, $b = 23$ in, $c = 21$ in
Find $m\angle B$

29.8° or 150.2°

10) $m\angle A = 128°$, $c = 10$ ft, $a = 38$ ft
Find $m\angle C$

12°

11) $m\angle C = 43°$, $b = 33$ ft, $c = 17$ ft
Find $m\angle B$

Not a Triangle

12) $m\angle C = 96°$, $b = 5$ mi, $c = 26$ mi
Find $m\angle B$

11°

Solve each triangle. Round your answers to the nearest tenth.

13) $m\angle B = 59°$, $m\angle A = 24°$, $a = 9$ m

$\angle C = 97°$, $b = 19$m, $c = 22$m

14) $m\angle A = 37°$, $a = 35$ mi, $c = 17$ mi

$\angle B = 126°$, $\angle C = 17°$, $b = 47.1$mi

Day 5 Notes – The Law of Cosines

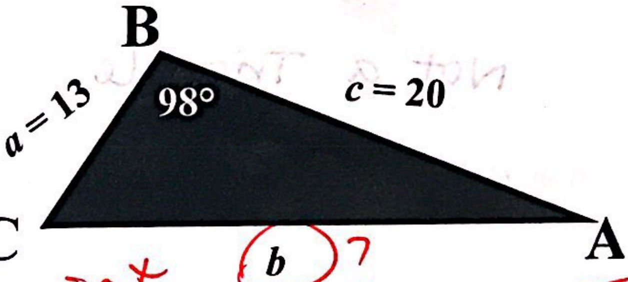
For any type of triangle (right, acute, or obtuse), you may use the following formula to solve for missing sides or angles:

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

Use the Law of Cosines when… you have 3 dimensions of a triangle and you need to find the other 3 dimensions. They cannot be just ANY 3 dimensions though, or you won't have enough information to solve the Law of Cosines equations. Use the Law of Cosines if you are given:

- SAS
- SSS

Example 1
Find all the missing dimensions of triangle ABC, given that angle B = 98°, side $a = 13$ and side $c = 20$.



$b^2 = a^2 + c^2 - 2ac \cdot \cos B$

$b^2 = 13^2 + 20^2 - 2 \cdot 13 \cdot 20 \cos 98°$

$b^2 = 169 + 400 - 520 \cdot (-.139)$

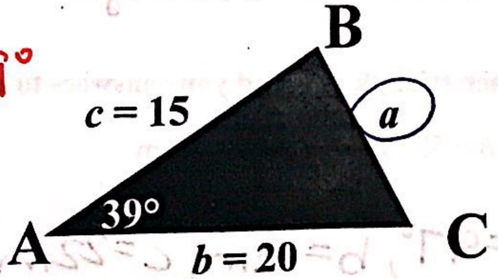
$b^2 = 569 - 520(-.139)$ ↑ store in x

$b^2 \approx 641.37...$

$\boxed{b \approx 25.325}$

Law of Sines:
$\frac{\sin 98°}{25.325} = \frac{\sin C}{20} \rightarrow$ $\boxed{\angle C = 51.45° \quad \angle A = 30.55°}$

Example 2*
Find all the missing dimensions of triangle, ABC, given that angle A = 39°, side $b = 20$ and side $c = 15$.



$a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$a^2 = 20^2 + 15^2 - 2 \cdot 20 \cdot 15 \cdot \cos 39°$

$a^2 = 400 + 225 - 600 \cdot \cos 39°$

$\sqrt{a^2} \approx \sqrt{158.71}$

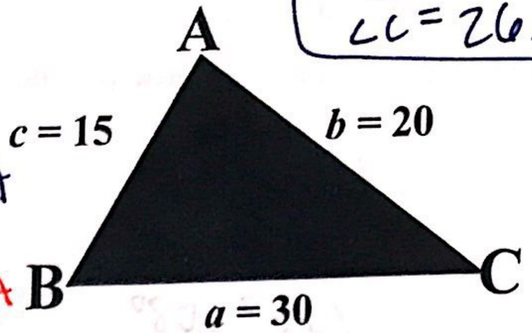
$\boxed{a \approx 12.598}$

$\boxed{\angle B = 87.53° \quad \angle C = 100.13°}$

Example 3*

Find all the missing dimensions of triangle, ABC, given that side $a = 30$, side $b = 20$ and side $c = 15$.

$$\boxed{\begin{array}{l}\angle B = 36.336^\circ \\ \angle C = 26.384^\circ\end{array}}$$



$a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$30^2 = 20^2 + 15^2 - 2 \cdot 20 \cdot 15 \cos A$

$900 = \underbrace{400 + 225}_{} - 600 \cdot \cos A$

$275 = -600 \cos A$

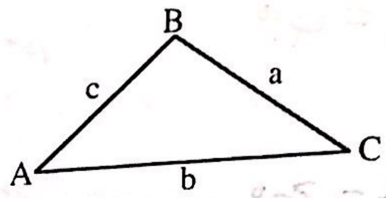
$-.458\overline{3} = \cos A$

$\boxed{117.28^\circ = A}$

***Important**: The Law of Sines will never produce an obtuse angle. If an angle *might* be obtuse, never use the Law of Sines to find it.

Area of a Triangle

You can find the area of any triangle given at least three pieces of information…



1. SAS

Δ Area $= \underbrace{\frac{1}{2} bc \sin A}_{} = \underbrace{\frac{1}{2} ac \sin B}_{} = \underbrace{\frac{1}{2} ab \sin C}_{}$

Ex) Given: B = 75°, a = 20, c = 18… find the area of the Δ

$A = \frac{1}{2} a \cdot c \cdot \sin B$

$= \frac{1}{2} \cdot 20 \cdot 18 \cdot \sin 75^\circ \approx \boxed{173.867}$

2. SSS "Heron's Formula"

Δ Area $= \sqrt{s(s - a)(s - b)(s - c)}$

$s =$ Half of the Perimeter $= \frac{a + b + c}{2}$

Ex) Given a = 6, b = 8, c = 12… find the area of the Δ

$A = \sqrt{13(13 - 6)(13 - 8)(13 - 12)}$

$= \sqrt{13(7)(5)(1)}$

≈ 21.33

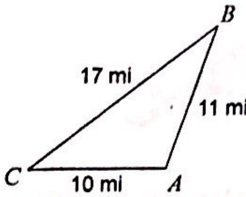
$s = \frac{6 + 8 + 12}{2} = \frac{26}{2} = 13$

Day 5 Homework

Law of Cosines and Area

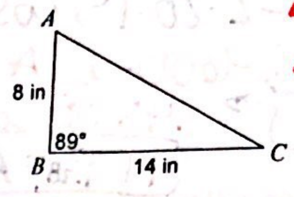
Solve each triangle. Round your answers to the nearest tenth.

1)



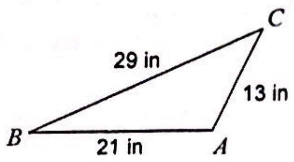
∠B = 34°
∠C = 38°
∠A = 108°

2)



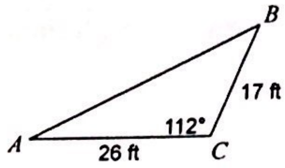
∠A = 61°
∠C = 30°
b = 16 in

3)



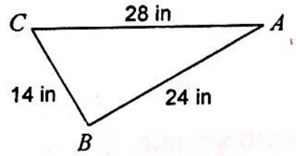
∠A = 115°
∠B = 24°
∠C = 41°

4)



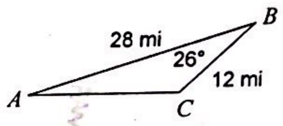
∠A = 26°
∠B = 42°
c = 36 ft

5)



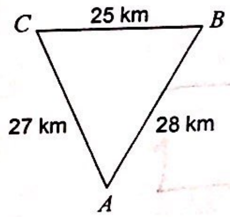
∠A = 30°
∠B = 91°
∠C = 59°

6)



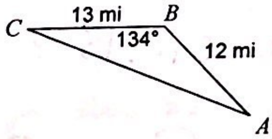
∠A = 17°
∠C = 137°
b = 18 mi

7)



∠A = 54°
∠B = 61°
∠C = 65°

8)



∠A = 24°
∠C = 22°
b = 23 mi

9) $b = 28$ km, $a = 18$ km, $c = 21$ km

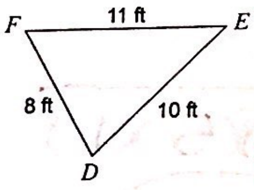
∠A = 40° ∠B = 91.4° ∠C = 48.6°

10) $c = 30$ cm, $b = 17$ cm, $a = 18$ cm

∠A = 32° ∠B = 30° ∠C = 118°

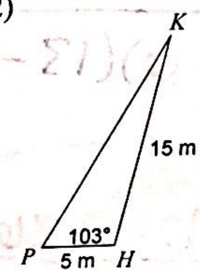
Find the area of each triangle to the nearest tenth.

11)



38.5 ft²

12)



36.5 m²