Day 5 Notes – The Law of Cosines

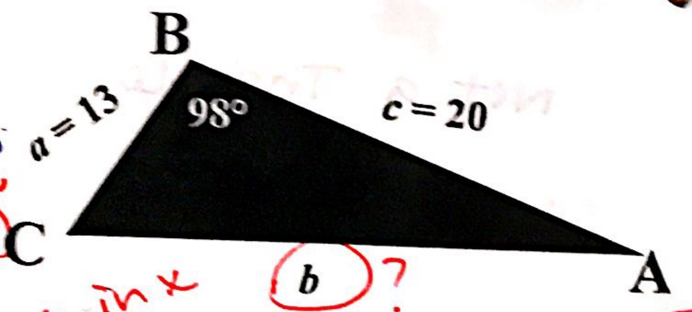
For any type of triangle (right, acute, or obtuse), you may use the following formula to solve for missing sides or angles:

$$\boxed{\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cdot\cos A \\ b^2 &= a^2 + c^2 - 2ac\cdot\cos B \\ c^2 &= a^2 + b^2 - 2ab\cdot\cos C \end{aligned}}$$

Use the Law of Cosines when… you have 3 dimensions of a triangle and you need to find the other 3 dimensions. They cannot be just ANY 3 dimensions though, or you won't have enough information to solve the Law of Cosines equations. Use the Law of Cosines if you are given:

- SAS
- SSS

Example 1
Find all the missing dimensions of triangle ABC, given that angle B = 98°, side $a = 13$ and side $c = 20$.



$b^2 = a^2 + c^2 - 2ac\cdot\cos B$

$b^2 = 13^2 + 20^2 - 2\cdot13\cdot20\cdot\cos 98°$

$b^2 = 169 + 400 - 520\cdot(-.139)$

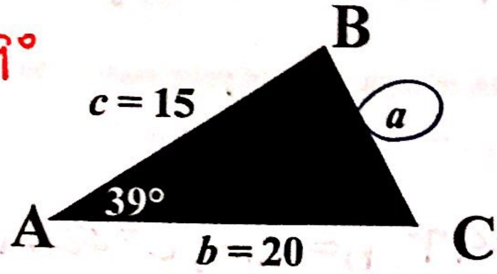
$b^2 = 569 - 520(-.139)$ ↑ store in x

$b^2 \approx 641.37\ldots$

$\boxed{b \approx 25.325}$

Law of Sines: $\dfrac{\sin 98°}{25.325} = \dfrac{\sin C}{20} \rightarrow$ $\boxed{\begin{aligned}\angle C &= 51.45° \\ \angle A &= 30.55°\end{aligned}}$

Example 2*
Find all the missing dimensions of triangle, ABC, given that angle A = 39°, side $b = 20$ and side $c = 15$.



$a^2 = b^2 + c^2 - 2bc\cdot\cos A$

$a^2 = 20^2 + 15^2 - 2\cdot20\cdot15\cdot\cos 39°$

$a^2 = 400 + 225 - 600\cdot\cos 39°$

$\sqrt{a^2} \approx \sqrt{158.71}$

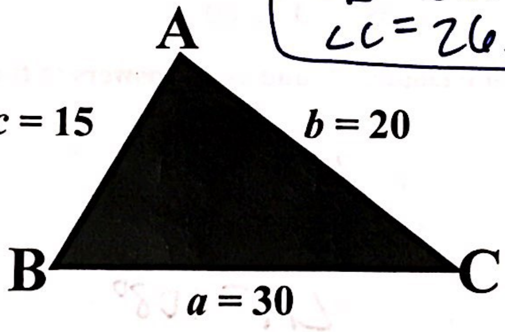
$\boxed{a \approx 12.598}$ $\boxed{\begin{aligned}\angle B &= 87.53° \\ \angle C &= 100.13°\end{aligned}}$

Scanned by CamScanner

Example 3*

Find all the missing dimensions of triangle, ABC, given that side $a = 30$, side $b = 20$ and side $c = 15$.

$$\boxed{\begin{array}{l} \angle B = 36.336^\circ \\ \angle C = 26.384^\circ \end{array}}$$



$a^2 = b^2 + c^2 - 2bc\cdot\cos A$

$30^2 = 20^2 + 15^2 - 2\cdot 20\cdot 15\cos A$

$900 = \underbrace{400 + 225}_{} - 600\cdot\cos A$

$275 = -600\cos A$

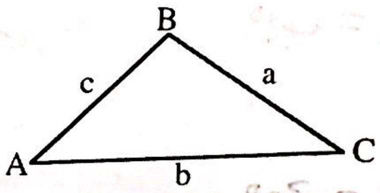
$-.458\overline{3} = \cos A$

$117.28^\circ = A$

***Important**: The Law of Sines will never produce an obtuse angle. If an angle *might* be obtuse, never use the Law of Sines to find it.

Area of a Triangle

You can find the area of any triangle given at least three pieces of information...



1. **SAS**

Δ Area $= \underbrace{\frac{1}{2}bc\sin A}_{} = \underbrace{\frac{1}{2}ac\sin B}_{} = \underbrace{\frac{1}{2}ab\sin C}_{}$

Ex) Given: B = 75°, a = 20, c = 18... find the area of the Δ

$A = \frac{1}{2}a\cdot c\cdot\sin B$

$= \frac{1}{2}\cdot 20\cdot 18\cdot\sin 75^\circ \approx \boxed{173.867}$

2. **SSS** "Heron's Formula"

Δ Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$s =$ Half of the Perimeter $= \dfrac{a+b+c}{2}$

Ex) Given a = 6, b = 8, c = 12... find the area of the Δ

$A = \sqrt{13(13-6)(13-8)(13-12)}$

$= \sqrt{13(7)(5)(1)}$

≈ 21.33

$s = \dfrac{6+8+12}{2} = \dfrac{26}{2} = 13$

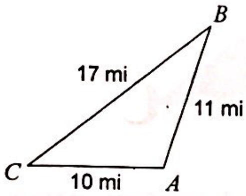
Scanned by CamScanner

Day 5 Homework

Law of Cosines and Area

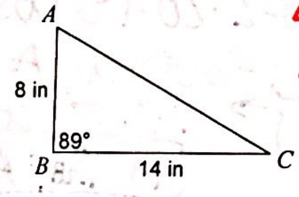
Solve each triangle. Round your answers to the nearest tenth.

1)



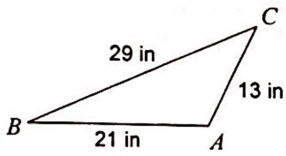
$\angle B = 34^\circ$
$\angle C = 38^\circ$
$\angle A = 108^\circ$

2)



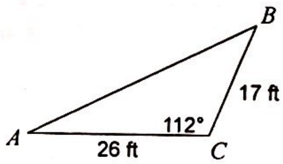
$\angle A = 61^\circ$
$\angle C = 30^\circ$
$b = 16$ in

3)



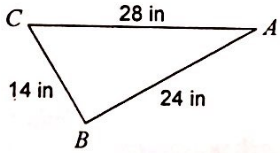
$\angle A = 115^\circ$
$\angle B = 24^\circ$
$\angle C = 41^\circ$

4)



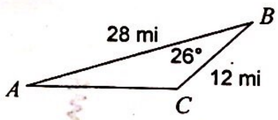
$\angle A = 26^\circ$
$\angle B = 42^\circ$
$c = 36$ ft

5)



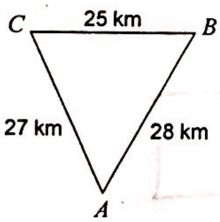
$\angle A = 30^\circ$
$\angle B = 91^\circ$
$\angle C = 59^\circ$

6)



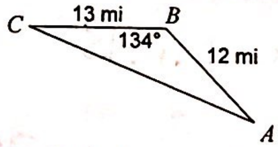
$\angle A = 17^\circ$
$\angle C = 137^\circ$
$b = 18$ mi

7)



$\angle A = 54^\circ$
$\angle B = 61^\circ$
$\angle C = 65^\circ$

8)



$\angle A = 24^\circ$
$\angle C = 22^\circ$
$b = 23$ mi

9) $b = 28$ km, $a = 18$ km, $c = 21$ km

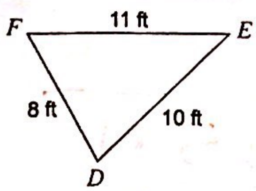
$\angle A = 40^\circ$ $\angle B = 91.4^\circ$ $\angle C = 48.6^\circ$

10) $c = 30$ cm, $b = 17$ cm, $a = 18$ cm

$\angle A = 32^\circ$ $\angle B = 30^\circ$ $\angle C = 118^\circ$

Find the area of each triangle to the nearest tenth.

11)



38.5 ft^2

12)

36.5 m^2

18

Scanned by CamScanner