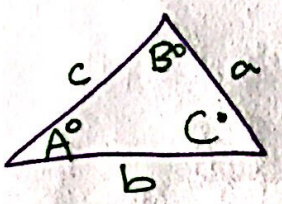
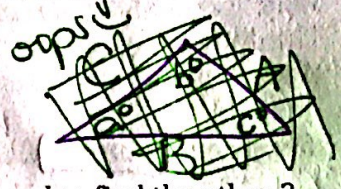
Day 4 Notes – The Law of Sines



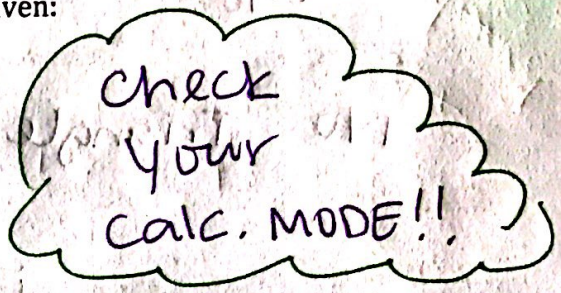
For any triangle (right, acute or obtuse), you may use the following formula to solve for missing sides or angles:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Use the Law of Sines when...**you have 3 dimensions** of a triangle and you need to find the other 3 dimensions - **they cannot be just ANY 3 dimensions** though, or you won't have enough info to solve the Law of Sines equation. Use the Law of Sines if you are given:

- AAS
- ASA
- SSA (Ambiguous Case)



Example 1

You are given a triangle, ABC, with angle A = 70°, angle B = 80° and side a = 12 cm. Find the measures of angle C and sides b and c.

$\angle A = 70°$ $a = 12$ cm

$\angle B = 80°$ $b = 12.58$ cm

$\angle C = 30°$ $c = 6.39$ cm

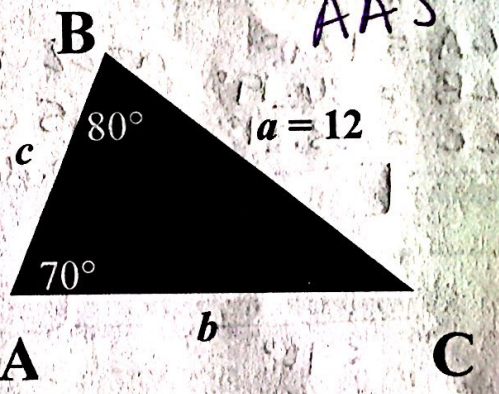
① $180 - 70 - 80$

② $\frac{\sin 70°}{12} = \frac{\sin 80°}{b}$

$b\sin 70° = 12\sin 80°$

$\frac{b\sin 70°}{\sin 70°} = \frac{12\sin 80°}{\sin 70°}$

③ $\frac{\sin 70°}{12} = \frac{\sin 30°}{c} \rightarrow c = \frac{12\sin 30°}{\sin 70°}$



Example 2

You are given a triangle, ABC, with angle C = 115°, angle B = 30° and side a = 30 cm. Find the measures of angle A and sides b and c.

$\angle A = 35°$ $a = 30$ cm

$\angle B = 30°$ $b = 26.15$ cm

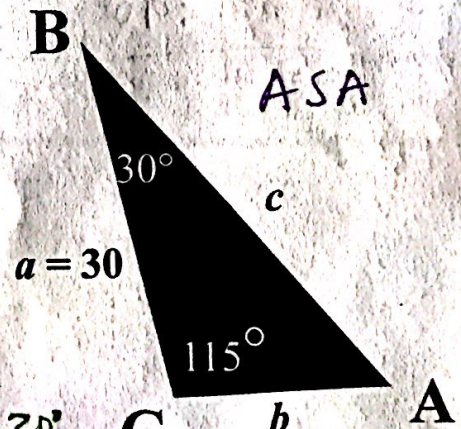
$\angle C = 115°$ $c = 41.4$ cm

① $180 - 115 - 30$

② $\frac{\sin 115°}{c} = \frac{\sin 35°}{30}$

$c = \frac{30\sin 115°}{\sin 35°}$

③ $\frac{\sin 35°}{30} = \frac{\sin 30°}{b} \rightarrow b = \frac{30\sin 30°}{\sin 35°}$



The Ambiguous Case

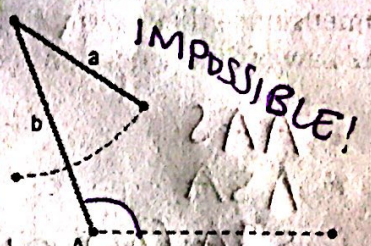
When given **SSA** (two sides and an angle that is NOT the included angle), the situation is ambiguous. The dimensions may not form a triangle, or there may be 1 or 2 triangles with the given dimensions. We first go through a series of tests to determine how many (if any) solutions exist.

If angle A is obtuse...

1. If angle A is obtuse, and a < b or a = b, no such triangle exists.

>90°

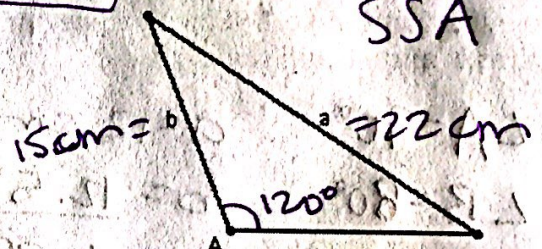
No Triangle



2. If angle A is obtuse, and a > b, one such triangle exists.

>90°

Given a triangle with angle A = 120°, side $a = 22$ cm and side $b = 15$ cm, find the other dimensions.



$\angle A = 120°$ $\quad a = 22$ cm

$\angle B = 36.19°$ $\quad b = 15$ cm

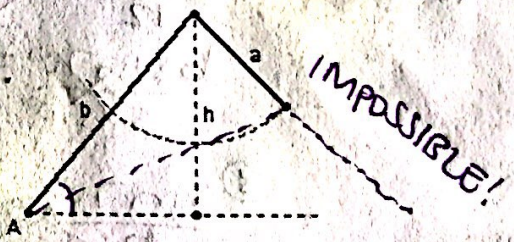
$\angle C = 23.81°$ $\quad c = 10.26$ cm

$\frac{\sin 120°}{22} \rightarrow \frac{\sin B°}{15}$

If angle A is acute...

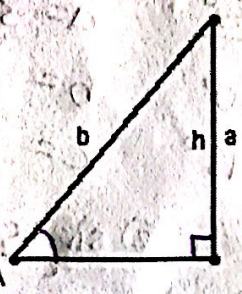
3. If angle A is acute, and a < h, no such triangle exists.

<90°



<90°

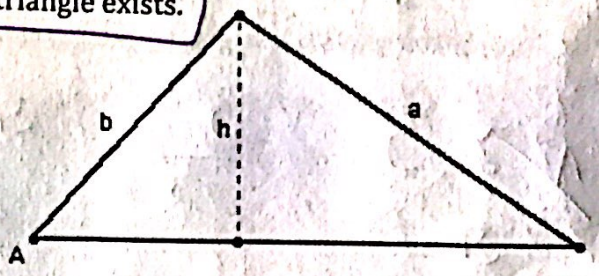
4. If angle A is acute, and a = h, one possible triangle exists. Angle B is a right angle.



and $a > h$

5. If angle A is acute, and $a > b$, one possible triangle exists.

   Given a triangle with angle A = 40°, side a = 12 cm and side b = 10 cm, find the other dimensions.



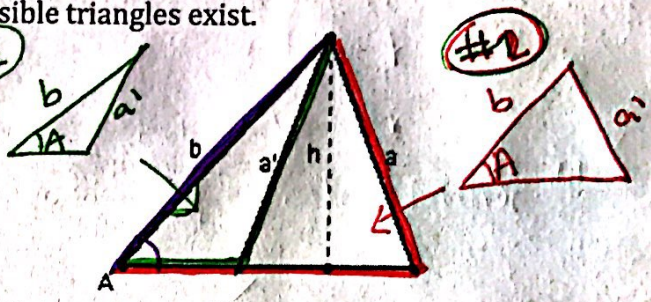
$\angle A = 40°$ $a = 12$ cm

$\angle B = 32.39°$ $b = 10$ cm

$\angle C = 107.61$ $c = 17.79$ cm

6. If angle A is acute, and $h < a < b$, two possible triangles exist.

   Given a triangle with angle A = 40°, side a = 12 cm and side b = 15 cm, find the other dimensions.



Triangle 1

$\angle A = 40°$ $a = 12$ cm

$\angle B = 126.54°$ $b = 15$ cm

$\angle C = 13.46°$ $c = 4.35$ cm

Triangle 2

$\angle A = 40°$ $a = 12$ cm

$\angle B = 53.46°$ $b = 15$ cm

$\angle C = 86.54°$ $c = 4.35$ cm

SSA Summary:

if angle A is **obtuse**	if $a < b$ → no solution
	if $a > b$ → one solution
if angle A is **acute**	if $a < h$ → no solution
find the height, $h = b * \sin A$	if $h < a < b$ → 2 solutions *one with angle B acute, one with angle B obtuse*
	if $a > b > h$ → 1 solution
	If $a = h$ → 1 solution *angle B is right*

Day 4 Homework

Law of Sines

State the number of possible triangles that can be formed using the given measurements.

1) $m\angle C = 24°$, $b = 29$ yd, $c = 14$ yd

2

2) $m\angle B = 104°$, $a = 8$ m, $b = 8$ m

0

3) $m\angle C = 70°$, $b = 34$ yd, $c = 5$ yd

0

4) $m\angle B = 40°$, $a = 14$ cm, $b = 24$ cm

1

Find each measurement indicated. Round your answers to the nearest tenth.

5) $m\angle B = 140°$, $m\angle A = 12°$, $c = 27$ m
Find b

37 m

6) $m\angle A = 30°$, $m\angle B = 36°$, $a = 23$ km
Find b

27 km

7) $m\angle C = 62°$, $b = 14$ mi, $c = 9$ mi
Find a

Not a Triangle

8) $m\angle A = 104°$, $m\angle B = 39°$, $a = 37$ yd
Find b

24 yd

9) $m\angle C = 27°$, $b = 23$ in, $c = 21$ in
Find $m\angle B$

29.8° or 150.2°

10) $m\angle A = 128°$, $c = 10$ ft, $a = 38$ ft
Find $m\angle C$

12°

11) $m\angle C = 43°$, $b = 33$ ft, $c = 17$ ft
Find $m\angle B$

Not a Triangle

12) $m\angle C = 96°$, $b = 5$ mi, $c = 26$ mi
Find $m\angle B$

11°

Solve each triangle. Round your answers to the nearest tenth.

13) $m\angle B = 59°$, $m\angle A = 24°$, $a = 9$ m

$\angle C = 97°$, $b = 19$m, $c = 22$m

14) $m\angle A = 37°$, $a = 35$ mi, $c = 17$ mi

$\angle B = 126°$, $\angle C = 17°$, $b = 41.1$mi

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