Day 3 Notes – Double and Half Angle Identities

$R \to D$: $\theta \cdot \frac{180}{\pi}$ | $D \to R$: $\theta \cdot \frac{\pi}{180}$

Double Angle Identities (2θ)	$\sin(2A) = 2\sin A\cos A$; $\cos(2A) = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$; $\tan(2A) = \frac{2\tan A}{1-\tan^2 A}$
Half Angle Identities ($\frac{\theta}{2}$)	$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$; $\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$; $\tan\left(\frac{A}{2}\right) = \frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$

Evaluate the following expressions using double- and half-angle formulas.

1. $\sin 15°$

$= \sin(45° - 30°)$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

OR $= \sin\left(\frac{30°}{2}\right) \overset{A=30°}{=} \pm\sqrt{\frac{1-\cos(30°)}{2}} = \pm\sqrt{\frac{1-\sqrt{3}/2}{2}} = \pm\sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\pm\sqrt{2-\sqrt{3}}}{2}$

2. $\cos 22.5°$

$= \cos\left(\frac{45°}{2}\right) = \pm\sqrt{\frac{1+\cos(45°)}{2}} = \pm\sqrt{\frac{1+\sqrt{2}/2}{2}}$

$= \pm\sqrt{\frac{2/2+\sqrt{2}/2}{4/2}} = \pm\sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\pm\sqrt{2+\sqrt{2}}}{2}$

3. $\tan(-15°)$

$= \tan\left(\frac{-30°}{2}\right) = \frac{\sin(-30°)}{1+\cos(-30°)} = \frac{\sin(330°)}{1+\cos(330°)} = \frac{-1/2}{1+\sqrt{3}/2} = \frac{-1/2}{2/2+\sqrt{3}/2}$

$= -1 \cdot \frac{(2-\sqrt{3})}{(2+\sqrt{3})\cdot(2-\sqrt{3})} = \frac{-2+\sqrt{3}}{4-3} = -2+\sqrt{3}$

4. $\sin\left(-\frac{\pi}{12}\right)$

$\frac{\pi}{12}\cdot\frac{180}{\pi} = 15$, $\frac{-180}{12} = -15$

$= \sin(-15°) = \sin\left(\frac{-30°}{2}\right)$

$= \pm\sqrt{\frac{1-\cos(-30°)}{2}} = \pm\sqrt{\frac{1-\sqrt{3}/2}{2}} = \pm\sqrt{\frac{2/2-\sqrt{3}/2}{4/2}} = \frac{\pm\sqrt{2-\sqrt{3}}}{2}$

5. $\cos\frac{5\pi}{8}$

$\frac{5\pi}{2}\cdot\frac{\pi}{180} = \frac{90}{2} = 225$

$= \cos\left(\frac{225°}{2}\right) = \pm\sqrt{\frac{1+\cos(225°)}{2}} = \pm\sqrt{\frac{1+(-\sqrt{2}/2)}{2}}$

$= \pm\sqrt{\frac{2/2-\sqrt{2}/2}{4/2}} = \pm\sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\pm\sqrt{2-\sqrt{2}}}{2}$

6. $\tan\frac{\pi}{8}$

$\frac{\pi}{8}\cdot\frac{180}{\pi} = \frac{45}{2}$

$= \tan\left(\frac{45°}{2}\right) = \frac{1-\cos(45°)}{\sin(45°)} = \frac{1-\sqrt{2}/2}{\sqrt{2}/2} = \frac{2/2-\sqrt{2}/2}{\sqrt{2}/2}$

$= \frac{2-\sqrt{2}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{2\sqrt{2}-2}{2} = \sqrt{2}-1$

Find the exact value of the function.

7. $\cos\frac{u}{2}$ if $\cos u = \frac{4}{5}, \boxed{0 \le u < \frac{\pi}{2}}$ QI

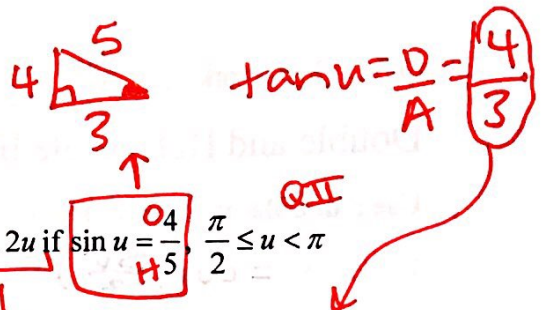
$\frac{u}{2} = A$

$\pm\sqrt{\frac{1+\cos u}{2}} = \pm\sqrt{\frac{1+(4/5)}{2/1}}$

$= \pm\sqrt{\frac{5/5+4/5}{10/5}} = \pm\sqrt{\frac{5+4}{10}}$

$= \frac{\pm\sqrt{9}}{\sqrt{10}} = \frac{\pm 3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \boxed{\frac{3\sqrt{10}}{10}}$

8. $\tan 2u$ if $\sin u = \frac{4}{5}, \frac{\pi}{2} \le u < \pi$ QII



$\frac{2\tan u}{1-\tan^2 u} = \frac{2(4/3)}{1-(4/3)^2}$

$= \frac{8/3}{1-16/9} \cdot \frac{3/3}{} = \frac{24/9}{9/9-16/9}$

$= \frac{24}{9-16} = \boxed{\frac{24}{-7}}$

Verify the following identities.

9. $2\sin^2 2x + \cos 4x = 1$

$2\sin^2 2x + \cos(2\cdot 2x) = 1$

$2\sin^2 2x + 1 - 2\sin^2 2x = 1$

$1 = 1$ ✓

10. $(\sin x + \cos x)^2 = 1 + \sin 2x$

$\sin^2 x + 2\sin x\cos x + \cos^2 x$

$+1 + 2\sin x\cos x$

$1 + \sin 2x$ ✓

Solve the equations for $0 \le x < 2\pi$.

11. $\sin x = \cos 2x$

$\sin x = 1 - 2\sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0 \quad \sin x + 1 = 0$

$\sin x = 1/2 \quad \sin x = -1$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}$

12. $\cos 2x - \cos x - 2 = 0$

$2\cos^2 x - 1 - \cos x - 2 = 0$

$2\cos^2 x - \cos x - 3 = 0$

$(2\cos x - 3)(\cos x + 1) = 0$

$2\cos x - 3 = 0 \quad \cos x + 1 = 0$

$\cos x = \frac{3}{2} \quad \cos x = -1$

$\boxed{x = \pi}$

Day 3 Homework

Double and Half Angle ID's

Use a double-angle or half-angle identity to find the exact value of each expression.

1) $\cos 75° = \cos\left(\frac{150°}{2}\right) = \pm\sqrt{\frac{1+\cos(150°)}{2}}$

$= \pm\sqrt{\frac{1+-\sqrt{3}/2}{2}} = \pm\sqrt{\frac{2-\sqrt{3}}{4}}$

$= \frac{\pm\sqrt{2-\sqrt{3}}}{2}$

2) $\sin\frac{5\pi}{3}$

$-\frac{\sqrt{3}}{2}$

3) $\cos\frac{\pi}{8}$

$\frac{\sqrt{2+\sqrt{2}}}{2}$

4) $\tan\frac{4\pi}{3}$

$\sqrt{3}$

5) $\sin\frac{5\pi}{8}$

$\frac{\sqrt{2+\sqrt{2}}}{2}$

6) $\tan 60°$

$\sqrt{3}$

7) $\sec\theta = \frac{17}{15}$ and $0 < \theta < \frac{\pi}{2}$

Find $\cos 2\theta$

$\frac{161}{289}$

8) $\cot\theta = -\frac{4}{3}$ and $90° < \theta < 180°$

Find $\sin 2\theta$

$-\frac{24}{25}$

9) $\tan\theta = \frac{3}{4}$ and $180° < \theta < 270°$

Find $\tan 2\theta$

$\frac{24}{7}$

10) $\cos\theta = \frac{4}{5}$ and $0° < \theta < 90°$

Find $\sin\frac{\theta}{2}$

$\frac{\sqrt{10}}{10}$

11) $\csc\theta = \frac{\sqrt{34}}{5}$ and $0° < \theta < 90°$

Find $\cos\frac{\theta}{2}$

$\frac{\sqrt{578+51\sqrt{34}}}{34}$

12) $\csc\theta = \sqrt{17}$ and $90° < \theta < 180°$

Find $\tan\frac{\theta}{2}$

$\sqrt{17}+4$