opp. signs

Example 1   Evaluate each trigonometric sum or difference.

Sum and Difference Identities
same
$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

a) $\sin\left(\dfrac{\pi}{3} \underset{A}{\oplus} \dfrac{\pi}{4}\right) = \sin(\pi/3)\cos(\pi/4) +$

$\cos(\pi/3)\sin(\pi/4)$

$= \left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{\sqrt{2}}{2}\right) + \left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\sqrt{6}}{4} + \dfrac{\sqrt{2}}{4} = \boxed{\dfrac{\sqrt{6}+\sqrt{2}}{4}}$

b) $\cos(\underset{A}{45} \ominus \underset{B}{60}) = \cos(45°)\cos(60°) + \sin(45°)\sin(60°)$

$= \left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{1}{2}\right) \oplus \left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right)$

$= \dfrac{\sqrt{2}}{4} + \dfrac{\sqrt{6}}{4} = \boxed{\dfrac{\sqrt{2}+\sqrt{6}}{4}}$

c) $\tan\left(\dfrac{\pi}{6} \ominus \dfrac{\pi}{3}\right) = \dfrac{\tan(\pi/6) \ominus \tan(\pi/3)}{1 + \tan(\pi/6)\tan(\pi/3)} = \dfrac{\dfrac{\sqrt{3}}{3} - \dfrac{3\sqrt{3}}{1\cdot3}}{1 + \dfrac{\sqrt{3}}{3}\left(\dfrac{\sqrt{3}}{1}\right) = \dfrac{3}{3}}$

$\underset{A \qquad B}{}$

$\dfrac{1\sqrt{3}}{\sqrt{3}\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

$= \dfrac{\dfrac{\sqrt{3} - 3\sqrt{3}}{3}}{1 + 1} = \dfrac{-2\sqrt{3}}{\dfrac{3}{2}}$

$= \dfrac{-2\sqrt{3}}{6} = \boxed{\dfrac{-\sqrt{3}}{3}}$

Example 2   Write each expression as a single trigonometric ratio.

a) $\sin\dfrac{\pi}{6}\cos\dfrac{\pi}{2} + \cos\dfrac{\pi}{6}\sin\dfrac{\pi}{2} = \left(\dfrac{1}{2}\right)(0) + \left(\dfrac{\sqrt{3}}{2}\right)(1) = \boxed{\dfrac{\sqrt{3}}{2}}$

b) $\dfrac{\tan\dfrac{\pi}{4} \ominus \tan\dfrac{\pi}{6}}{1 + \tan\dfrac{\pi}{4}\tan\dfrac{\pi}{6}} = \dfrac{\dfrac{3}{3} - \dfrac{\sqrt{3}}{3}}{\dfrac{3\cdot1 \oplus 1\left(\dfrac{\sqrt{3}}{3}\right)}{3}} = \dfrac{\dfrac{3-\sqrt{3}}{3}}{\dfrac{3+\sqrt{3}}{3}} = \dfrac{(3-\sqrt{3})}{(3+\sqrt{3})} \cdot \dfrac{3}{3}$

$= \dfrac{3\ominus\sqrt{3}}{3\oplus\sqrt{3}} \cdot \dfrac{(3-\sqrt{3})}{(3-\sqrt{3})}$

c) $\cos\dfrac{\pi}{3}\cos\dfrac{\pi}{6} \ominus \sin\dfrac{\pi}{3}\sin\dfrac{\pi}{6}$

$= \dfrac{1}{2}\left(\dfrac{\sqrt{3}}{2}\right) \oplus \dfrac{\sqrt{3}}{2}\left(\dfrac{1}{2}\right) = \dfrac{\sqrt{3}}{4} \oplus \dfrac{\sqrt{3}}{4} = \dfrac{2\sqrt{3}}{4 \div 2} = \boxed{\dfrac{\sqrt{3}}{2}}$

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$0°, 30°, 45°, 60°, 90°, \cdots$

Angle Sum/Difference Identities

Date_____ Period_____

Use the angle sum identity to find the exact value of each.

1) $\cos 105° = \cos(60° + 45°)$
   $A \quad B$
   $60°$
   $= \cos(\#)\cos(45°) \boxed{-} \sin(60°) \sin(45°)$
   $= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$
   $= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$

2) $\sin 195°$

   $\dfrac{\sqrt{2} - \sqrt{6}}{4}$

3) $\cos 195°$
   $= \cos(150° + 45°)$

   $\boxed{\dfrac{-\sqrt{6} - \sqrt{2}}{4}}$

4) $\cos 165°$

   $\dfrac{-\sqrt{6} - \sqrt{2}}{4}$

5) $\cos 285° = \cos(225° + 60°)$

   $\boxed{\dfrac{\sqrt{6} - \sqrt{2}}{4}}$

6) $\cos 255°$

   $\dfrac{\sqrt{2} - \sqrt{6}}{4}$

7) $\sin 105° = \sin(60° + 45°)$

   $\boxed{\dfrac{\sqrt{6} + \sqrt{2}}{4}}$

8) $\sin 285°$

   $\dfrac{-\sqrt{6} - \sqrt{2}}{4}$

9) $\cos 75° = \cos(30° + 45°)$
   $= \cos(30°)\cos(45°) - \sin(30°)\sin(45°)$
   $= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{\dfrac{\sqrt{6} - \sqrt{2}}{4}}$

10) $\sin 255°$

   $\dfrac{-\sqrt{6} + \sqrt{2}}{4}$

Use the angle difference identity to find the exact value of each.

11) $\cos 75° = \cos(225° - 150°)$

   $= \boxed{\dfrac{\sqrt{6} - \sqrt{2}}{4}}$

12) $\cos -15°$

   $\dfrac{\sqrt{6} + \sqrt{2}}{4}$

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13) tan 75°

$$2 + \sqrt{3}$$

14) cos 15°

$$\frac{\sqrt{6} + \sqrt{2}}{4}$$

15) tan −105°

$$2 + \sqrt{3}$$

16) sin 105°

$$\frac{\sqrt{6} + \sqrt{2}}{4}$$

17) tan 15°

$$2 - \sqrt{3}$$

18) sin 15°

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$

19) tan −15°

$$\sqrt{3} - 2$$

20) sin −75°

$$\frac{-\sqrt{6} - \sqrt{2}}{4}$$

Use the angle sum or difference identity to find the exact value of each.

21) sin −105°

$$\frac{-\sqrt{6} - \sqrt{2}}{4}$$

22) cos 195°

$$\frac{-\sqrt{6} - \sqrt{2}}{4}$$

23) $\cos \frac{7\pi}{12}$

$$\frac{\sqrt{2} - \sqrt{6}}{4}$$

24) $\tan \frac{13\pi}{12} = \tan(5\pi/6 + \pi/4)$

$= \dfrac{\tan(5\pi/6) + \tan(\pi/4)}{1 - \tan(5\pi/6)(\pi/4)} = \dfrac{\frac{-\sqrt{3}}{3} + 1 \cdot 3}{3 \cdot 1 + (\frac{+\sqrt{3}}{3}) \cdot 1}$

$= \dfrac{(-\sqrt{3} + 3)}{(3 \cdot (+\sqrt{3}))} \cdot \dfrac{(3 - \sqrt{3})}{(3 - \sqrt{3})} \quad \begin{matrix} F \\ O \\ I \\ L \end{matrix} = \dfrac{-3\sqrt{3} + 3 + 9 \boxed{-3\sqrt{3}}}{9 - 3}$

$= \dfrac{-6\sqrt{3} + 12}{6}$

$= \boxed{-\sqrt{3} + 2}$

25) $\sin \frac{\pi}{12}$

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$

26) $\cos -\frac{7\pi}{12}$

$$\frac{\sqrt{2} - \sqrt{6}}{4}$$

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