

Unit 6 Notes – SOLVING TRIG EQUATIONS

RECALL: We have solved basic trig equations before (they were on the last test).

Ex 1: Solve: $\sin \theta = \frac{1}{2}$. (ask yourself, "Self, where does sine = $\frac{1}{2}$?") *Sine is \oplus in I, II*

$$\theta = \frac{\pi}{6} \pm 2\pi k$$

$$\text{and } \frac{5\pi}{6} \pm 2\pi k$$

2 πk takes into account
k number of full rotations!

Ex 2: Solve: $2\sin^2 \theta - \sin \theta - 1 = 0$ (this has a squared term and a linear term, so factor)

Let $y = \sin \theta$.

$$2y^2 - y - 1 = 0$$

$$(y - \frac{2}{2})(y + \frac{1}{2}) = 0$$

$$(y - 1)(2y + 1) = 0$$

$$y - 1 = 0 \quad 2y + 1 = 0$$

$$y = 1 \quad y = -\frac{1}{2}$$

$$\sin \theta = 1 \quad \sin \theta = -\frac{1}{2}$$

Sine is \ominus in III, IV

$$\theta = \frac{\pi}{2} + 2\pi k \text{ and } \frac{7\pi}{6} + 2\pi k \text{ and } \frac{11\pi}{6} + 2\pi k$$

"BLOB" PROBLEMS:

These problems will be very similar to Example 1. The main difference is the angle.

Ex 3: Solve: $\sin \square = 1$ Let $\square = 2\theta$

To do: Treat the problem like $\sin \square = 1$. $\square = \text{blob}$ (represents an unknown angle)
So, find the angle whose sine = 1 $\rightarrow \pi/2$ where sine = 1

You know that the unknown angle must represent $\pi/2$ in order for the equation to be true...

consider possible full circle rotations

Therefore, $\square = \pi/2 + 2\pi k$ In the original problem, we did not have a BLOB, we had 2θ .

Therefore, $\frac{2\theta}{2} = \frac{\pi/2 + 2\pi k}{2}$ Now, you can solve for theta by dividing everything by 2.

Answer: $\theta = \frac{\pi}{4} + \pi k$ $\frac{\pi}{2} \div 2 = \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$

Ex 4: Solve: $\cos \theta/4 = 1$

$\square = \frac{\theta}{4}$ Rewrite with a blob: $\cos \square = 1$
Find out what angle has a cosine of 1: $0 + 2\pi k = 2\pi k$
Set blob contents = to that angle: $\frac{\theta}{4} = 2\pi k \cdot 4$
Solve for theta:
Answer: $\theta = 8\pi k$

Ex 5: Solve: $\tan 3\theta = \sqrt{3}$ *remember for tan, use " $+\pi k$ " in answer

$\tan \square = \frac{\sqrt{3}}{1} \leftarrow y \right\} (\frac{1}{2}, \frac{\sqrt{3}}{2})$
 $\frac{3\theta}{3} = \frac{\pi/3 + \pi k}{3}$ or $(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$ jumps back and forth between QI and QIII
 $\theta = \frac{\pi}{9} + \frac{\pi k}{3}$

5.3 Solving Trig Equations Practice Worksheet #1
Pre-calculus

Name: _____
Date: _____ Block: _____

Solve for the unknown variable on the interval $0 \leq x < 2\pi$.

1. $4 \cos^2 x - 3 = 0$

$$4x^2 - 3 = 0$$

$$\sqrt{x^2} = \pm \sqrt{\frac{3}{4}}$$

$$x = \pm \frac{\sqrt{3}}{2}$$

$$\cos \theta = \pm \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

2. $\sqrt{2} \sin \theta = 1$

$$\frac{\sqrt{2} y}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$y = \frac{1}{\sqrt{2} \sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\frac{\theta}{2} = \frac{\pi}{4} \quad \frac{\theta}{2} = \frac{3\pi}{4} \cdot \frac{1}{2}$$

$$x = \frac{\pi}{8}, \frac{3\pi}{8}$$

3. $3 \cot^2 x - 1 = 0$

$$\cot^2 x = \frac{1}{3}$$

$$\cot x = \pm \sqrt{\frac{1}{3}} = \pm \frac{1}{\sqrt{3}}$$

$$\rightarrow \tan x = \pm \sqrt{3}$$

$$x = \frac{\pi}{3}, \frac{4\pi}{3}$$

4. $\cos^3 x = \cos x$

$$\cos^3 x - \cos x = 0$$

$$x^3 - x = 0$$

$$x(x^2 - 1) = 0$$

$$x(x+1)(x-1) = 0$$

$$x = 0 \quad x+1=0 \quad x-1=0$$

$$\cos x = 0 \quad \cos x = -1 \quad \cos x = 1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad x = \pi \quad x = 0$$

5. $\sin x - 2 \sin x \cos x = 0$

$$\sin x (1 - 2 \cos x) = 0$$

$$\sin x = 0 \quad 1 - 2 \cos x = 0$$

$$x = 0, \pi \quad \cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

6. $2 \sin^2 x - \sin x - 3 = 0$

$$2y^2 - y - 3 = 0$$

$$(y - \frac{3}{2})(y + 2) = 0$$

$$(2y - 3)(y + 1) = 0$$

$$2 \sin x - 3 = 0 \quad \sin x + 1 = 0$$

$$\sin x = \frac{3}{2} \quad \sin x = -1$$

$x = \text{DNE}$

$$x = \frac{3\pi}{2}$$

8. $\cos^2 x = 1 - \sin x$

$$1 - \sin^2 x = 1 - \sin x$$

$$1 + \sin x - 1 + \sin x$$

$$-\sin^2 x + \sin x = 0$$

$$-\sin x (\sin x - 1) = 0$$

$$-\sin x = 0 \quad \sin x = 1$$

$$x = 0, \pi \quad x = \frac{\pi}{2}$$

7. $\csc^2 x - \csc x - 2 = 0$

$$y^2 - y - 2 = 0$$

$$(y+1)(y-2) = 0$$

$$\csc x = -1 \quad \csc x = 2$$

$$\frac{1}{\sin x} = -1 \quad \frac{1}{\sin x} = 2$$

$$\sin x = -1 \quad \sin x = \frac{1}{2}$$

$$x = \frac{3\pi}{2} \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$$

Solve for the unknown variable on the given interval.

9. $\sqrt{3} + \tan(2x) = 0$ on $[0, 2\pi)$.

$$\tan \square = -\sqrt{3}$$

$$\frac{2x}{2} = \frac{2\pi}{3} \quad \frac{2x}{2} = \frac{5\pi}{3}$$

$$x = \frac{\pi}{3} \quad x = \frac{5\pi}{6}$$

10. $\cos(\pi x) = 0.5$ on $[0, 2)$.

$$\cos \square = \frac{1}{2}$$

$$\frac{\pi x}{\pi} = \frac{\pi}{3} \quad \frac{\pi x}{\pi} = \frac{5\pi}{3}$$

$$x = \frac{1}{3} \quad x = \frac{5}{3}$$

11. $\sin(\frac{x}{2}) - 1 = 0$ on $[0, 8\pi)$.

$$\sin \square = 1$$

$$\frac{x}{2} = \frac{\pi}{2} + 2\pi k$$

$$x = \pi + 4\pi k$$

$$x = \pi, 5\pi$$

Part 1: Solve for the unknown variable. Give all of the exact general solutions.

1. $\sin \theta = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4} \pm 2\pi k$

and $\frac{3\pi}{4} \pm 2\pi k$

4. $1 + \sin \theta = 2 \cos^2 \theta$

$1 + \sin \theta = 2(1 - \sin^2 \theta)$

$1 + \sin \theta = 2 - 2\sin^2 \theta$

$2\sin^2 \theta + \sin \theta - 1 = 0$

$(\sin \theta + \frac{1}{2})(\sin \theta - 1) = 0$

$\sin \theta = -1 \quad \sin \theta = 1/2$

$\theta = \frac{3\pi}{2} \pm 2\pi k, \frac{\pi}{6} \pm 2\pi k, \frac{5\pi}{6} \pm 2\pi k$

7. $\sin^2 \theta - 1 = 0$

$\sqrt{\sin^2 \theta} = \pm \sqrt{1}$

$\sin \theta = \pm 1$

$\theta = \frac{\pi}{2} \pm 2\pi k$

or

$\frac{3\pi}{2} \pm 2\pi k$

10. $\tan 4\theta = -1$

II, IV

$\frac{y}{x} = -1$

$\frac{4\theta}{4} = \frac{3\pi}{4} \pm \frac{\pi k}{4}$

$\theta = \frac{3\pi}{16} \pm \frac{\pi k}{4}$

2. $\frac{\cos \theta}{\sin \theta} = \frac{\sin \theta}{\sin \theta}$

$\tan \theta = 1$

$\theta = \frac{\pi}{4} \pm \pi k$

5. $2 \cos^2 \theta + \cos \theta = 0$

$\cos \theta (2 \cos \theta + 1) = 0$

$\cos \theta = 0$

$\cos \theta = -1/2$

$\theta = \frac{\pi}{2} \pm 2\pi k, \frac{3\pi}{2} \pm 2\pi k$

$\theta = \frac{2\pi}{3} \pm 2\pi k, \frac{4\pi}{3} \pm 2\pi k$

3. $\tan \theta = 1$

$\theta = \frac{\pi}{4} \pm \pi k$



6. $\sin 3\theta = -1$

$\sin \square = -1$

$\frac{3\theta}{3} = \frac{3\pi}{2} \pm \frac{2\pi k}{3}$

$\theta = \frac{\pi}{2} \pm \frac{2\pi k}{3}$

8. $\cos 2\theta = 1/2$

$\cos \square = 1/2$

$\frac{2\theta}{2} = \frac{\pi}{3} \pm \frac{2\pi k}{2}$

$\theta = \frac{\pi}{6} \pm \pi k$

$\frac{2\theta}{2} = \frac{5\pi}{3} \pm \frac{2\pi k}{2}$

$\theta = \frac{5\pi}{6} \pm \pi k$

9. $2 \sin^2 \theta - \sin \theta - 1 = 0$

$2x^2 - x - 1 = 0$

$(x - \frac{1}{2})(x + 1) = 0$

$(\sin \theta - 1/2)(\sin \theta + 1) = 0$

$\sin \theta = 1/2 \quad \sin \theta = -1$

$\theta = \frac{\pi}{6} \pm 2\pi k, \frac{5\pi}{6} \pm 2\pi k, \frac{3\pi}{2} \pm 2\pi k$

11. $\tan^2 3x = 3$

$\sqrt{\tan^2 \square} = \pm \sqrt{3}$

$\tan \square = \pm \sqrt{3} = y$
 $1 = x$

$\frac{3x}{3} = \frac{\pi}{3} \pm \frac{\pi k}{3}$

$x = \frac{\pi}{9} \pm \frac{\pi k}{9}$

12. $\cos \frac{x}{2} = \frac{\sqrt{2}}{2}$

$\cos \square = \frac{\sqrt{2}}{2}$

$\frac{x/2}{2} = \frac{2\pi}{4} \pm \frac{2\pi k}{2}$

$x = \frac{\pi}{2} \pm 4\pi k$

$\frac{x/2}{2} = \frac{7\pi}{4} \pm \frac{2\pi k}{2}$

$x = \frac{7\pi}{2} \pm 4\pi k$

Part 2: Solve by approximating the solutions on the interval $[0, 2\pi)$.

13. $2\sin^2 x + 3\sin x + 1 = 0$

$2y^2 + 3y + 1 = 0$

$(y + \frac{1}{2})(y + 2) = 0$

$(2y + 1) = 0 \Rightarrow y + 1 = 0$

$\sin x = -\frac{1}{2} \Rightarrow \sin x = -1$

$x = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}$

16. $\frac{\cos x \cot x}{1 - \sin x} = 3$

$\cos x \cot x = 3(1 - \sin x) \cdot \sin x$

$\cos^2 x = 3\sin x(1 - \sin x)$

$1 - \sin^2 x = 3\sin x - 3\sin^2 x$

$2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$\sin x = \frac{1}{2} \Rightarrow \sin x = 1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

$x = 0, \pi$

OMTTH

14. $4\sin^2 x = 2\cos x + 1$

$4(1 - \cos^2 x) = 2\cos x + 1$

$4(1 - x^2) = 2x + 1$

$4x - 4x^2 - 2x - 1 = 0$

$+4x^2 + 2x + 1 = 0$

Quadratic Formula $x = \frac{-1 \pm \sqrt{13}}{4}$

$x = \cos^{-1}\left(\frac{-1 \pm \sqrt{13}}{4}\right)$

17. $\sec^2 x + 0.5 \tan x = 1$

$\sec^2 x + \frac{1}{2}(\sec^2 x - 1) = 1$

$\sec^2 x + \frac{1}{2}\sec^2 x - \frac{1}{2} = 1$

$\frac{3}{2}\sec^2 x - \frac{3}{2} = 0$

$3\frac{1}{2}(\sec^2 x - 1) = 0$

$\tan^2 x = 0$

15. $\csc x + \cot x = 1$

$\frac{1}{\sin x} + \frac{\cos x}{\sin x} = 1$

$\frac{1 + \cos x}{\sin x} = 1 \Rightarrow 1 + \cos x = \sin x$

$(1 + \cos x)^2 = (\sin x)^2$

$1 + 2\cos x + \cos^2 x = \sin^2 x$

$1 + 2\cos x + \cos^2 x = 1 - \cos^2 x$

$2\cos x + 2\cos^2 x = 0$

$2x + 2x^2 = 0$
 $2x(1 + x) = 0$

$\cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$1 + \cos \theta = 0 \Rightarrow \cos \theta = -1 \Rightarrow \theta = \pi$

Part 3: Use the calculator's inverse trig functions to approximate the solutions. Remember that you must also find the other solution by either adding π or subtracting the value from π , or subtracting the value from 2π .

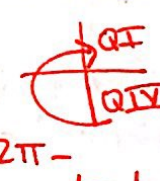
18. $\tan \theta = 4$



$\tan^{-1} 4 = \theta$

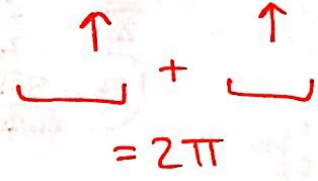
$\theta = 1.3258, 4.4674$

19. $\cos \theta = 0.84$

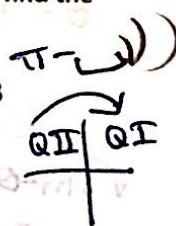


$\cos^{-1}(0.84) = \theta$

$\theta = 0.5735, 5.7097$



20. $\sin \theta = 0.63$



$\sin^{-1}(0.63) = \theta$

$\theta = 0.6816, 2.46$

