Unit 7 Packet – Identities Continued...
Pre-Calculus Honors

Name: Key!!

Unit 6 Notes – SOLVING TRIG EQUATIONS

RECALL: We have solved basic trig equations before (they were on the last test).

Ex 1: Solve: sin Θ = ½. (ask yourself, "Self, where does sine = ½?") *Sine is ⊕ in I, II*

$\theta = \frac{\pi}{6} \pm 2\pi k$

and $\frac{5\pi}{6} \pm 2\pi k$

2πk takes into account k number of full rotations!

Ex 2: Solve: $2\sin^2\theta - \sin\theta - 1 = 0$ (this has a squared term and a linear term, so factor)

Let $y = \sin\theta$.

$2y^2 - y - 1 = 0$

$(y - \frac{2}{2})(y + \frac{1}{2}) = 0$

$(y-1)(2y+1) = 0$

$y - 1 = 0 \quad 2y + 1 = 0$

$y = 1 \quad y = -1/2$

$\sin\theta = 1 \quad \sin\theta = -1/2$ *sine is ⊖ in III, IV*

$\theta = \frac{\pi}{2} + 2\pi k$ and $\frac{7\pi}{6} + 2\pi k$ and $\frac{11\pi}{6} + 2\pi k$

"BLOB" PROBLEMS:

These problems will be very similar to Example 1. The main difference is the angle.

Ex 3: Solve: $\sin\boxed{2\theta} = 1$ Let $\boxed{} = 2\theta$

To do: Treat the problem like sin( ) = 1. ( ) = blob (represents an unknown angle)

So, find the angle whose sine = 1 → π/2 ← where sine = 1

You know that the unknown angle must represent π/2 in order for the equation to be true...

consider possible full circle rotations

Therefore, ( ) = π/2 + 2πk In the original problem, we did not have a BLOB, we had 2Θ.

Therefore, $\frac{2\theta}{2} = \frac{\pi/2}{2} + \frac{2\pi k}{2}$ Now, you can solve for theta by dividing *everything* by 2.

Answer: $\boxed{\theta = \pi/4 + \pi k}$ $\frac{\pi}{2} \div 2 = \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$

Ex 4: Solve: cos Θ/4 = 1

$\boxed{} = \frac{\theta}{4}$

Rewrite with a blob: $\cos\boxed{} = 1$

Find out what angle has a cosine of 1: $0 + 2\pi k = 2\pi k$

Set blob contents = to that angle: $4 \cdot \frac{\theta}{4} = 2\pi k \cdot 4$

Solve for theta:

Answer: $\boxed{8\pi k} = \theta$

Ex 5: Solve: $\tan 3\theta = \sqrt{3}$ * remember for tan, use "+ πk" in answer

jumps back and forth between QI and QIII

$\tan\boxed{} = \frac{\sqrt{3}}{1} \begin{matrix} \leftarrow y \\ \leftarrow x \end{matrix}$ $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ or $\left(-1/2, -\sqrt{3}/2\right)$

$\frac{\boxed{3\theta}}{3} = \frac{\pi/3}{3} + \frac{\pi k}{3} \rightarrow \boxed{\theta = \frac{\pi}{9} + \frac{\pi k}{3}}$

5.3 Solving Trig Equations Practice Worksheet #1
Pre-calculus

Name: ____________________
Date: ____________________ Block: ____

Solve for the unknown variable on the interval $0 \le x < 2\pi$.

1. $4\cos^2 x - 3 = 0$

$4x^2 - 3 = 0$

$\sqrt{x^2} = \pm\sqrt{\frac{3}{4}}$

$x = \frac{\pm\sqrt{3}}{2}$

$\cos\theta = \frac{\pm\sqrt{3}}{2}$

$\boxed{\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

2. $\sqrt{2}\sin\boxed{2x} = 1$

$\frac{\sqrt{2}}{\sqrt{2}} y\square = \frac{1}{\sqrt{2}}$

$y\square = \frac{1\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\boxed{2x} = \frac{\pi}{4}$, $\boxed{2x} = \frac{3\pi}{4} \cdot \frac{1}{2}$

$\boxed{x = \frac{\pi}{8}, \frac{3\pi}{8}}$

3. $3\cot^2 x - 1 = 0$

$\cot^2 x = 1/3$

$\cot x = \pm\sqrt{1/3} = \frac{\pm 1}{\sqrt{3}}$

$\tan x = \pm\sqrt{3}$

$\boxed{x = \frac{\pi}{3}, \frac{4\pi}{3}}$

4. $\cos^3 x = \cos x$

$\cos^3 x - \cos x = 0$

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x+1)(x-1) = 0$

$x = 0$ $x + 1 = 0$ $x - 1 = 0$

$\cos x = 0$ $\cos x = -1$ $\cos x = 1$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$ $\boxed{x = \pi}$ $\boxed{x = 0}$

5. $\sin x - 2\sin x\cos x = 0$

$\sin x(1 - 2\cos x) = 0$

$\sin x = 0$ $1 - 2\cos x = 0$

$\boxed{x = 0, \pi}$ $\cos x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

6. $2\sin^2 x - \sin x - 3 = 0$

$2y^2 - y - 3 = 0$

$(y - \frac{3}{2})(y + \frac{2}{2}) = 0$

$(2y - 3)(y + 1) = 0$

$2\sin x - 3 = 0$ $\sin x + 1 = 0$

$\sin x = 3/2$ $\sin x = -1$

$x =$ DNE $\boxed{x = \frac{3\pi}{2}}$

7. $\csc^2 x - \csc x - 2 = 0$

$y^2 - y - 2 = 0$

$(y + 1)(y - 2) = 0$

$\csc x = -1$ $\csc x = 2$

$\frac{1}{\sin x} = -1$ $\frac{1}{\sin x} = 2$

$\sin x = -1$ $\sin x = 1/2$

$\boxed{x = \frac{3\pi}{2}}$ $\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

8. $\cos^2 x = 1 - \sin x$

$1 - \sin^2 x = 1 - \sin x$

$-1 + \sin x$ $-1 + \sin x$

$-\sin^2 x + \sin x = 0$

$-\sin x(\sin x - 1) = 0$

$-\sin x = 0$ $\sin x = 1$

$\boxed{x = 0, \pi}$ $\boxed{x = \frac{\pi}{2}}$

Solve for the unknown variable on the given interval.

9. $\sqrt{3} + \tan\boxed{2x} = 0$ on $[0, 2\pi)$.

$\tan\square = -\sqrt{3}$

$\frac{\boxed{2x}}{2} = \frac{\frac{2\pi}{3}}{2}$ $\frac{\boxed{2x}}{2} = \frac{\frac{5\pi}{3}}{2}$

$\boxed{x = \frac{\pi}{3} \quad x = \frac{5\pi}{6}}$

10. $\cos(\pi x) = 0.5$ on $[0, 2)$.

$\cos\square = 1/2$

$\frac{\boxed{\pi x}}{\pi} = \frac{\frac{\pi}{3}}{\pi}$ $\frac{\boxed{\pi x}}{\pi} = \frac{\frac{5\pi}{3}}{\pi}$

$\boxed{x = \frac{1}{3} \quad x = \frac{5}{3}}$

11. $\sin\left(\frac{x}{2}\right) - 1 = 0$ on $[0, 8\pi)$.

= BLOB

$+1 \quad +1$

$y\square = 1$

$2\boxed{x/2} = \frac{\pi}{2} + 2\pi k$

$x = \pi + 4\pi k$

$\boxed{x = \pi, 5\pi}$

5.3 Solving Trig Equations – Worksheet #2
Pre-calculus

Name: ____________
Date: ____________ Block: ____

Part 1: Solve for the unknown variable. Give all of the exact general solutions.

1. $\sin\theta = \frac{\sqrt{2}}{2}$

$\theta = \boxed{\frac{\pi}{4} \pm 2\pi k}$

and $\boxed{\frac{3\pi}{4} \pm 2\pi k}$

2. $\cos\theta = \sin\theta$

$\frac{\cos\theta}{\sin\theta} = \frac{\sin\theta}{\sin\theta}$

$\tan\theta = 1$

$\boxed{\theta = \frac{\pi}{4} \pm \pi k}$

3. $\tan\theta = 1$

$\boxed{\theta = \frac{\pi}{4} \pm \pi k}$

$\leftarrow$ "

4. $1 + \sin\theta = 2\cos^2\theta$

$1 + \sin\theta = 2(1 - \sin^2 x)$

$1 + \sin\theta = 2 - 2\sin^2 x$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(\sin\theta + \frac{2}{2})(\sin\theta - \frac{1}{2}) = 0$

$\sin\theta = -1 \quad \sin\theta = 1/2$

$\boxed{\theta = \frac{3\pi}{2} \pm 2\pi k, \frac{\pi}{6} \pm 2\pi k, \frac{5\pi}{6} \pm 2\pi k}$

5. $2\cos^2\theta + \cos\theta = 0$

$\cos\theta(2\cos\theta + 1) = 0$

$\cos\theta = 0 \quad \cos\theta = -1/2$

$\boxed{\theta = \frac{\pi}{2} \pm 2\pi k, \frac{3\pi}{2} \pm 2\pi k}$ $\boxed{\theta = \frac{2\pi}{3} \pm 2\pi k, \frac{4\pi}{3} \pm 2\pi k}$

6. $\sin 3\theta = -1$

$\sin\boxed{} = -1$

$\frac{\boxed{3\theta}}{3} = \frac{\frac{3\pi}{2}}{3} \pm \frac{2\pi k}{3}$

$\boxed{\theta = \frac{\pi}{2} \pm \frac{2\pi k}{3}}$

7. $\sin^2\theta - 1 = 0$

$\sqrt{\sin^2\theta} = \pm\sqrt{1}$

$\sin\theta = \pm 1$

$\boxed{\theta = \frac{\pi}{2} \pm 2\pi k \text{ or } \frac{3\pi}{2} \pm 2\pi k}$

8. $\cos\boxed{2\theta} = ½$

$\cos\boxed{} = 1/2$

$\frac{\boxed{2\theta}}{2} = \frac{\frac{\pi}{3}}{2} + \cancel{\frac{2\pi k}{2}}$

$\boxed{\theta = \frac{\pi}{6} \pm \pi k}$

$\frac{\boxed{2\theta}}{2} = \frac{\frac{5\pi}{3}}{2} + \cancel{\frac{2\pi k}{2}}$

$\boxed{\theta = \frac{5\pi}{6} \pm \pi k}$

9. $2\sin^2\theta - \sin\theta - 1 = 0$

$2x^2 - x - 1 = 0$

$(x - \frac{2}{2})(x + \frac{1}{2}) = 0$

$(\sin\theta - 1)(2\sin\theta + 1) = 0$

$\sin\theta = 1 \quad \sin\theta = -1/2$

$\boxed{\theta = \frac{\pi}{2} \pm 2\pi k, \frac{7\pi}{6} \pm 2\pi k, \frac{11\pi}{6} + 2\pi k}$

10. $\tan\boxed{4\theta} = -1$ II, IV

$\frac{y}{x}\boxed{} = \theta_1$

$\frac{\boxed{4\theta}}{4} = \frac{\frac{3\pi}{4}}{4} \pm \frac{\pi k}{4}$

$\boxed{\theta = \frac{3\pi}{16} \pm \frac{\pi k}{4}}$

11. $\tan^2\boxed{3x} = 3$

$\pm\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\sqrt{\tan^2\boxed{}} = \pm\sqrt{3}$

$\tan\boxed{} = \frac{\pm\sqrt{3}}{1} = \frac{y}{x}$

$\frac{\boxed{3x}}{3} = \frac{\pi}{3}\cdot\frac{1}{3} \pm \frac{\pi k}{2}\cdot\frac{1}{3}$

$\boxed{x = \frac{\pi}{9} \pm \frac{\pi k}{6}}$

12. $\cos\boxed{\frac{x}{2}} = \frac{\sqrt{2}}{2}$ I IV

$\cos\boxed{} = \frac{\sqrt{2}}{2}$

$2 \cdot \boxed{x/2} = \frac{\pi}{4}\cdot 2 \pm 2\pi k \cdot 2$

$\boxed{x = \frac{\pi}{2} \pm 4\pi k}$

$2 \cdot \boxed{x/2} = \frac{7\pi}{4}\cdot 2 \pm 2\pi k \cdot 2$

$\boxed{x = \frac{7\pi}{2} \pm 4\pi k}$?

Part 2: Solve by approximating the solutions on the interval [0,2π).

13. $2\sin^2 x + 3\sin x + 1 = 0$

$2y^2+3y+1=0$

$(y+\frac{1}{2})(y+\frac{2}{2})=0$

$(2y+1)=0 \quad y+1=0$

III IV $\sin x = \frac{-1}{2} \quad \sin x = -1$

$\boxed{x=\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}}$

OMIT!! * 14. $4\sin^2 x = 2\cos x + 1$ *

$4(1-\cos^2 x)=2\cos x+1$

$4(1-x^2)=2x+1$

$4x-4x^2-2x-1=0$

$+4x^2+2x+3=0$

Quad Formula $x=\frac{1\pm\sqrt{13}}{-4}$

$\boxed{x=\cos^{-1}\left(\frac{1\pm\sqrt{13}}{-4}\right)}$

15. $\csc x + \cot x = 1$

$\frac{1}{\sin x}+\frac{\cos x}{\sin x}=1$

$\sin x \cdot \frac{1+\cos x}{\sin x} = 1(\sin$

$(1+\cos x)^2=(\sin x)^2$

$1+2\cos x+\cos^2 x=\sin$

$1+2\cos x+\cos^2 x=1-\cos$
$-1 \quad + \cos^2 x \quad -1 + \cos$

$2\cos x+2\cos^2 x = 0$

$2x+2x^2=0$

$2x(1+x)=0$

$2\cos\theta=0 \quad 1+\cos\theta=0$

$\cos\theta=-1$

$\boxed{\theta=\frac{\pi}{2},\frac{3\pi}{2}} \quad \boxed{\theta=\pi}$

16. $\frac{\cos x \cot x}{1-\sin x} = 3$

$\sin x \cdot \frac{\cos x\cos x}{\sin x} = 3(1-\sin x) \cdot \sin x$

$\cos^2 x = 3\sin x(1-\sin x)$

$1-\sin x^2 = 3\sin x - 3\sin^2 x$

$2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x-1)(\sin x-1)=0$

$\sin x = 1/2 \quad \sin x = 1 \quad \boxed{x=\pi/2, \pi/6, 5\pi/6}$

17. $\sec^2 x + 0.5\tan x = 1$

$\sec^2 x+\frac{1}{2}(\sec^2 x-1)=1$

$\sec^2 x+\frac{1}{2}\sec^2 x-\frac{1}{2}=1$

$\frac{3}{2}\sec^2 x-\frac{3}{2}=0$

$3/2(\sec^2 x-1)=0$

$\tan^2 x=0$

$\boxed{x=0,\pi}$

Part 3: Use the calculator's inverse trig functions to approximate the solutions. Remember that you must also find the other solution by either adding π, subtracting the value from π, or subtracting the value from 2π.

tanx   sinx   cosx

18. $\tan\theta = 4$

+π   QI   QIII

$\tan^{-1}4=\theta$

$\boxed{\theta=1.3258, 4.4674}$

+π

19. $\cos\theta = 0.84$

QI   QIV

2π − ___

$\cos^{-1}(0.84)=\theta$

$\boxed{\theta=0.5735, 5.7097}$

___ + ___ = 2π

20. $\sin\theta = 0.63$

π − ___

QII   QI

$\sin^{-1}(0.63)=\theta$

$\boxed{\theta=0.6816, 2.46}$

___ + ___ = π