Pre-Calculus HONORS
Unit 6 Test Review

Name: Key

****THE ENTIRE TEST IS CALCULATOR FREE, therefore, this review sheet is also calculator free!**

Find the exact values of the following. If the answer is an angle, state the angle in RADIANS. (NO CALCULATOR)

1. $\tan(\cos^{-1} 1) = 0$ ($\cos^{-1} 1 = 0$)
2. $\cos(\tan^{-1} 1) = \sqrt{2}/2$ ($\tan^{-1} 1 = \pi/4$)
3. $\cos[\csc^{-1}(-2)] = \sqrt{3}/2$ ($\csc^{-1}(-2) = -\pi/6$)
4. $\tan(\sin^{-1} \frac{1}{2}) = \sqrt{3}/3$ ($\sin^{-1}\frac{1}{2} = \pi/6$)
5. $\cos^{-1}(\cos\frac{7\pi}{4}) = \pi/4$ ($\cos\frac{7\pi}{4} = \sqrt{2}/2$)
6. $\sin(\sin^{-1} 2) =$ NO solution
7. $\cos^{-1}(\cos 0) = 0$
8. $\tan^{-1}(-\sqrt{3}) = -\pi/3$ ($y = -\sqrt{3}$, $x = 1$)
9. $\cot^{-1}(-\sqrt{3}) = -\pi/6$ ($\frac{-\sqrt{3}}{1} = \frac{x}{y}$)

Use your general knowledge of trig and IDENTITIES to solve the following (NO CALCULATOR).

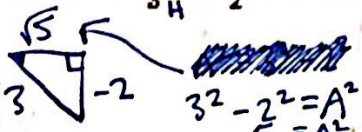
10. $\sec 20° \sin 70° = 1$

$= \frac{1}{\cos(90°-70°)} \cdot \frac{\sin(70°)}{1}$

$= \frac{\sin(70°)}{\sin(70°)} = 1$

11. Given $\sin\alpha = \frac{-2}{3}$ (O, H) and $\frac{3\pi}{2} \le \alpha \le 2\pi$. a) $\cos\left(\frac{\pi}{2} - \alpha\right) = -2/3 = \sin\alpha$

b) $\tan\alpha = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$ ($\frac{O}{A}$)



$3^2 - 2^2 = A^2$

$5 = A^2$

12. Simplify: $\csc(-x)\sin(-x)$

$\frac{1}{\sin(-x)} \cdot \frac{\sin(-x)}{1}$

$= 1$

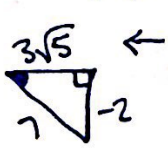
13. Simplify: $\tan\theta\sin\theta + \cos\theta$

$\frac{\sin\theta}{\cos\theta} \cdot \frac{\sin\theta}{1} + \cos\theta$

$= \frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta}$ → $= \frac{1}{\cos\theta} = \sec\theta$

Find the exact values of the following (NO CALCULATOR).

14. $\cos(\sin^{-1}(-2/7)) = \frac{A}{H} = \frac{3\sqrt{5}}{7}$ (O, H)

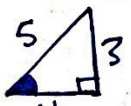


$7^2 - 2^2 = A^2$

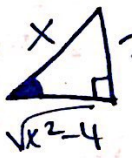
$49 - 4 = A^2$

$45 = A^2$

15. $\sin(\tan^{-1}\frac{3}{4}) = \frac{O}{H} = \frac{3}{5}$ (O, A)



16. $\cos(\sin^{-1}\frac{2}{x}) = \frac{A}{H} = \frac{\sqrt{x^2-4}}{x}$ (O, H)



$x^2 - 2^2 = A^2$

$\sqrt{x^2-4} = A$

Prove the following equations (NO CALCULATOR):

17. $\sin x + \cos x \cot x = \csc x$

$\sin x + \frac{\cos x \cos x}{\sin x}$ → $= \frac{\sin^2 x + \cos^2 x}{\sin x}$

$= \sin x + \frac{\cos^2 x}{\sin x}$ → $= \frac{1}{\sin x} = \csc x$ Q.E.D.

18. $\sec x - \cos x = \sin x \tan x$

$\frac{1}{\cos x} - \frac{\cos x}{1}$ → $\frac{\sin^2 x}{\cos x} = \frac{\sin x \sin x}{\cos x}$

$= \frac{1 - \cos^2 x}{\cos x}$ → $= \sin x \tan x$ Q.E.D.

19. $(1 + \sin\theta)^2 = 2(1 + \sin\theta) - \cos^2\theta$

$2 + 2\sin\theta - \cos^2\theta = 2 + 2\sin\theta - (1 - \sin^2\theta)$

$= \sin^2\theta + 2\sin\theta + 1$

$= (1 + \sin\theta)^2$ Q.E.D.

[1 hint: Let $x = \sin\theta$
$x^2 + 2x + 1$
$(x+1)(x+1) = (x+1)^2$]

20. $\cos^4 x - \sin^4 x = \cos^2 x - \sin^2 x$ Diff. of Squares!

$(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$

$= 1(\cos^2 x - \sin^2 x)$

Q.E.D.

21. $\frac{1 + \tan\theta + \csc\theta}{\sec\theta} = \sin\theta + \cos\theta + \cot\theta$

$\cos\theta\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\sin\theta}\right)$

$= \cos\theta + \frac{\sin\theta\cos\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \sin\theta + \cos\theta + \cot\theta$ Q.E.D.

22. $\tan x (\sin x + \cot x \cos x) = \sec x$

$\sin x \cdot \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \cdot \frac{\cos x}{1} \cdot \frac{\sin x}{\cos x}$

$= \frac{\sin^2 x}{\cos x} + \frac{\cos x}{1} = \frac{\sin^2 x + \cos^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$

Q.E.D.

Solve the following equations (NO CALCULATOR): Solve in the interval $[0, 2\pi)$.

23. $\sec x + \tan^2 x = -1$

$\sec x + \tan^2 x + 1 = 0$

$\sec x + \sec^2 x = 0$

$\sec x(1 + \sec x) = 0$

$\sec x = 0$ DNE $\sec x = -1$

$x = \pi$

24. $4\cos x \sin^2 x - 3\cos x = 0$

$\cos x(4\sin^2 x - 3) = 0$

$\cos x = 0$ $x = \frac{\pi}{2}, \frac{3\pi}{2}$

$4\sin^2 x = 3$

$\sin^2 x = 3/4$

$\sin x = \pm\sqrt{3}/2$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

25. $2\cos^4 x - 3\cos^2 x + 1 = 0$

Let $x = \cos^2 x$.

$2x^2 - 3x + 1 = 0$

$(x - 1/2)(x - 1) = 0$

$(\cos^2 x - 1/2)(\cos^2 x - 1) = 0$

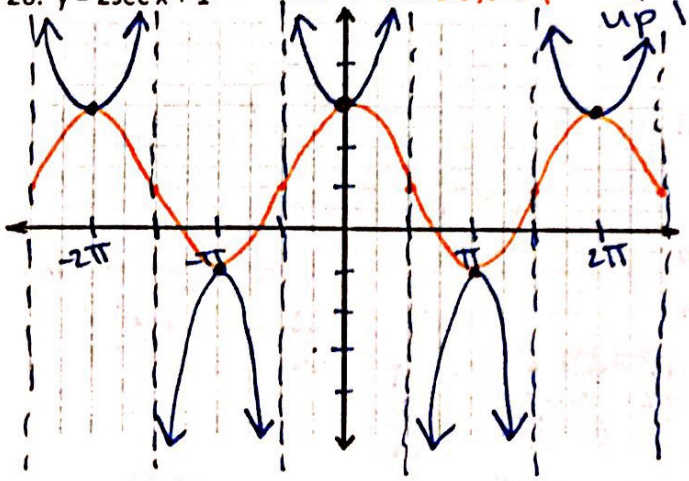
$\cos x = \pm\sqrt{2}/2$ or $\cos x = \pm 1$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, 0, \frac{\pi}{2}, \pi$

Graph the following trig functions (NO CALCULATOR):

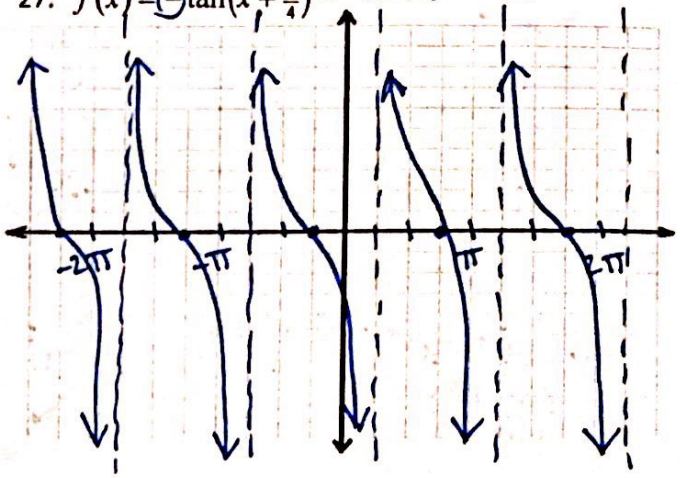
26. $y = 2\sec x + 1$

$= 2\cos x + 1$ amp = 2 up 1



27. $f(x) = -\tan\left(x + \frac{\pi}{4}\right)$ ← left π/4

flip over x-axis



28. An object moves in simple harmonic motion described by the equation $d = 20\cos\frac{\pi}{4}t$, where t is measured in seconds and d in centimeters. Find the following:

A) the maximum displacement amp = 20 cm

B) the frequency $= \frac{b}{2\pi} = \frac{\pi/4}{2\pi} = \frac{\pi}{4} \cdot \frac{1}{2\pi} =$ $\frac{1}{8}$ cycles per sec

C) the time required to complete one cycle

period = 8 sec

29. An object is attached to a coiled spring. The object is initially at its rest position. (+ sin θ) After that, it is pulled down (− sin θ) and then released. Write an equation for the distance of the object from it rest position after t seconds given the following information.

$d = -\frac{1}{4}\sin\left(\frac{2\pi}{5}t\right)$

Distance from rest position at t = 0 is 0 no vertical shift

Amplitude is ¼ inch $a = -1/4$

Period is 5 seconds. $5 = \frac{2\pi}{b} \rightarrow b = \frac{2\pi}{5}$

30. An object in simple harmonic motion has a frequency of ¼ oscillation per minute and an amplitude of 8 feet. Write an equation in the form ***d = a sin ωt*** for the object's simple harmonic motion.

$d = 8\sin\frac{\pi}{2}t$ $*\omega = b*$ $\frac{1}{4} = \frac{b}{2\pi} \rightarrow b = \frac{\pi}{2}$

31. A person seated on a Ferris Wheel of a radius of 100ft (amp = 100) makes one rotation every 30 seconds (per = 30). The center of the wheel is 105ft above the ground (up 105). Find ~~and graph~~ a function to represent a person's height above the ground at any time of a 2 min ride. Assume uniform speed from the beginning to the end of the ride and that a person is at the level of the center of the wheel and headed up (+ sin θ) when the ride begins.

$30 = \frac{2\pi}{b} \rightarrow b = \frac{\pi}{15}$

$h = 100\sin\left(\frac{\pi}{15}t\right) + 105$