Day 1 HW:

Find the radius of convergence and interval of convergence of the series.

1. $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ R: 1 I: [-1, 1)

2. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$

3. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n^3}$ R: 1 I: [-1, 1]

4. $\sum_{n=1}^{\infty} \sqrt{n} x^n$

5. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ R: ∞ I: $(-\infty, \infty)$

6. $\sum_{n=1}^{\infty} n^n x^n$

7. $\sum_{n=1}^{\infty} (-1)^n n 4^n x^n$ R: 1/4 I: (-1/4, 1/4)

8. $\sum_{n=1}^{\infty} \frac{x^n}{n 3^n}$

9. $\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{\sqrt[4]{n}}$ R: 1/2 I: (-1/2, 1/2]

10. $\sum_{n=1}^{\infty} \frac{x^n}{5^n n^5}$

11. $\sum_{n=2}^{\infty} (-1)^n \frac{x^n}{4^n \ln n}$ R: 4 I: (-4, 4]

12. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$

13. $\sum_{n=0}^{\infty} \sqrt{n}(x-1)^n$ R: 1 I: (0, 2)

14. $\sum_{n=0}^{\infty} n^3 (x-5)^n$

15. $\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n 2^n}$ R: 2 I: (-4, 0]

16. $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}} (x+3)^n$

17. $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^n}$ R: ∞ I: $(-\infty, \infty)$ *could use Root Test!*

18. $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n 3^n}$

19. $\sum_{n=1}^{\infty} (x-a)^n \frac{n}{b^n}$, $b > 0$ R: b I: (-b+a, b+a)

20. $\sum_{n=1}^{\infty} \frac{n(x-4)^n}{n^3+1}$

21. $\sum_{n=1}^{\infty} n!(2x-1)^n$ R: 0 I: $\{\frac{1}{2}\}$

22. $\sum_{n=1}^{\infty} \frac{n^2 x^n}{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2n)}$

23. $\sum_{n=1}^{\infty} \frac{(4x+1)^n}{n^2}$ R: 1/4 I: [-1/2, 0]

24. $\sum_{n=1}^{\infty} (-1)^n \frac{(2x+3)^n}{n \ln n}$

25. $\sum_{n=2}^{\infty} \frac{x^n}{(\ln n)^n}$ R: ∞ I: $(-\infty, \infty)$

26. $\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2n)}{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)} x^n$

Day 2 HW:

Find a power series representation for the function and determine the interval of convergence.

1. $f(x)=\frac{1}{1+x}$ $=\sum_{n=0}^{\infty}(-1)^n x^n$, $(-1,1)$

2. $f(x)=\frac{3}{1-x^4}$ $=\sum_{n=0}^{\infty}3x^{4n}$, $(-1,1)$

3. $f(x)=\frac{1}{1-x^3}$ $=\sum_{n=0}^{\infty}x^{3n}$, $(-1,1)$

4. $f(x)=\frac{1}{1+9x^2}$ $=\sum_{n=0}^{\infty}(-1)^n(9x^2)^n$, $\cancel{(-1/3,1/3)}$ $(-1/9,1/9)$

5. $f(x)=\frac{1}{x-5}$ $=-\sum_{n=0}^{\infty}\frac{x^n}{5^{n+1}}$, $(-5,5)$

6. $f(x)=\frac{x}{4x+1}$ $=\sum_{n=0}^{\infty}(-1)^n 4^n x^{n+1}$, $(-1/4,1/4)$

7. $f(x)=\frac{x}{9+x^2}$ $=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{9^{n+1}}$, $\cancel{(-3,3)}$ $(-9,9)$

8. $f(x)=\frac{x^2}{a^3-x^3}$ $=\sum_{n=0}^{\infty}\frac{x^{3n+2}}{a^{3n+3}}$, $(-a^3,a^3)$

Day 3 HW:

Find a power series representation for the function and determine the radius of convergence.

9. $f(x)=\ln(5-x)$

$\ln(5)-\sum_{n=1}^{\infty}\left(\frac{x}{5}\right)^n\frac{1}{n}$, $R=5$

10. $f(x)=\frac{x^2}{(1-2x)^2}$

11. $f(x)=\frac{x^3}{(x-2)^2}$ $\sum_{n=1}^{\infty}n\frac{x^{n+2}}{2^{n+1}}$, $R=2$

12. $f(x)=\arctan\left(x/3\right)$

Find a power series representation for f, and graph f as several partial sums $s_n(x)$ on the same screen. What happens as n increases?

13. $f(x)=\ln(3+x)$

$\ln(3)+\sum_{n=1}^{\infty}(-1)^n\left(\frac{1}{3}\right)^{n+1}\frac{x^{n+1}}{n+1}$, $R=3$?

14. $f(x)=\frac{1}{x^2+25}$

Prop. of Logs. 15. $f(x)=\ln\left(\frac{1+x}{1-x}\right)$

$\sum_{n=0}^{\infty}\frac{2x^{2n+1}}{2n+1}$, $R=1$

16. $f(x)=\tan^{-1}(2x)$

Evaluate the indefinite integral as a power series. What is the radius of convergence?

17. $\int\frac{t}{1-t^8}dt$

$C+\sum_{n=0}^{\infty}\frac{t^{8n+2}}{8n+2}$, $R=1$

18. $\int\frac{\ln(1-t)}{t}dt$

19. $\int\tan^{-1}(x^2)dx$

$C+\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n-1}}{4n^2-1}$, $R=1$

20. $\int\frac{x-\tan^{-1}x}{x^3}dx$

Use a power series to approximate the definite integral to six decimal places.

21. $\int_0^{0.2}\frac{1}{1+x^5}dx$ 0.199989

22. $\int_0^{0.4}\ln(1+x^4)dx$

23. $\int_0^{1/3}x^2\tan^{-1}(x^4)dx$

0.000065

24. $\int_0^{0.5}\frac{dx}{1+x^6}$

Day 4 HW:

Free Response Practice
Calc. Active:

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with

$$\frac{dx}{dt} = 12t - 3t^2 \text{ and } \frac{dy}{dt} = \ln\left(1 + (t-4)^4\right).$$

At time $t = 0$, the object is at position (-13, 5). At time $t = 2$, the object is at point P with x-coordinate 3.

a) Find the acceleration vector at time $t = 2$ and speed at $t = 2$.

$|v(2)| = \sqrt{(24-12)^2 + ((\ln(17))^2}$

$=$

$a(t) = \left\langle 12 - 6t, \frac{4(t-4)^3}{1+(t-4)^4} \right\rangle$

$a(2) = \left\langle 12-12, \frac{4(-8)}{1+(-2)^4} \right\rangle = \boxed{\left\langle 0, \frac{-32}{17} \right\rangle}$

b) Find the y-coordinate of P.

$y(3) = 5 + \int_0^2 (12t - 3t^2)\vec{i} + (\ln(1+(t-4)^4))\vec{j}\,dt = 5 + [(6t^2 - t^3)\vec{i} + ($

c) Write an equation for the line tangent to the curve at P. $(t=2)$

$y - \quad = \quad (x-3)$

$\left.\frac{dy}{dx}\right|_{t=2} = \left.\frac{dy/dt}{dx/dt}\right|_{t=2} = \frac{(\ln(17))^2}{12}$

d) For what value of t, if any, is the object at rest? Explain your reasoning.

NON – Calc. Active:

Consider the curve given by $y^2 = 2 + xy$

a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

$2y\frac{dy}{dx} = y + x\frac{dy}{dx}$

$\frac{dy}{dx}(2y - x) = y$

$\frac{dy}{dx} = \frac{y}{2y-x}$

b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

Day 5-6 HW:

Find the Maclaurin series for $f(x)$. Also, find the associated radius of convergence. [Hint: Use the series expansion for e^x, $\sin x$, $\cos x$ (See bottom of pg. 5!) when possible to find the Maclaurin series for $f(x)$.]

1. $f(x)=\cos x$

R=∞ $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}\cdots \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$

2. $f(x)=\sin 2x$

3. $f(x)=(1+x)^{-3}$

R=1 $1-3x+6x^2-10x^3+15x^4 \ldots \sum_{n=0}^{\infty}\frac{(n+1)(n+2)}{2}x^n(-1)^n$

4. $f(x)=\ln(1+x)$

5. $f(x)=e^{5x}$

$1+5x+\frac{5^2x^2}{2!}+\frac{5^3x^3}{3!}+\frac{5^4x^4}{4!}+\ldots \sum_{n=0}^{\infty}\frac{(5x)^n}{n!}$

6. $f(x)=xe^x$

7. $f(x)=\cos \pi x$

8. $f(x)=e^{-x/2}$

9. $f(x)=x\tan^{-1}x$

10. $f(x)=\sin x^4$

11. $f(x)=x^2e^{-x}$

12. $f(x)=x\cos 2x$

Find the Taylor series for $f(x)$ centered at the given value of a.

$\sum_{n=0}^{\infty}\frac{\boxed{f^n(a)}}{n}(x-a)^n$

13. $f(x)=1+x+x^2,\ a=2$

$7+\frac{5}{\cancel{1}}(x-2)+(x-2)^2$

14. $f(x)=x^3,\ a=-1$

15. $f(x)=e^x,\ a=3$

$\sum_{n=0}^{\infty}\frac{(x-3)^n}{n!}$

16. $f(x)=\ln x,\ a=2$

17. $f(x)=\cos x,\ a=\pi$

18. $f(x)=\sin x,\ a=\frac{\pi}{2}$

✗ 19. $f(x)=1/\sqrt{x},\ a=9$

20. $f(x)=x^{-2},\ a=1$

21. For a given function $h(x)$, a 12th-order Taylor polynomial is written in ascending powers of x, so that the last term of the polynomial is $\frac{x^{12}}{3096}$. Which is the value of $h^{12}(0)$, the 12th derivative of h at $x=0$?

3960

A) 9! B) 3(8!) C) 12! D) 3960^2 E) Cannot be determined

Day 7 HW:

$$R_n \le \left| f^{(n+1)}(c) \cdot \frac{(x-a)^{n+1}}{(n+1)!} \right|$$

a) Approximate f by a Taylor polynomial with degree n at the number a
b) Use Taylor's Inequality to estimate the accuracy of the approximation $f(x) \approx T_n(x)$ when x lies in the given interval.

1. $f(x) = \sqrt{x}$ $a = 4$ $n = 2$ $4 \le x \le 4.2$

n	d.	a=4
0	$x^{1/2}$	2
1	$\frac{1}{2}x^{-1/2}$	1/4
2	$-\frac{1}{4}x^{-3/2}$	-1/32

$\boxed{a}$ $2 + \frac{(x-4)}{4} - \frac{(x-4)^2}{64}$ $\boxed{b}$ $R_n \approx 0.0000015625$

2. $f(x) = x^{-2}$ $a = 1$ $n = 2$ $0.9 \le x \le 1.1$

$\boxed{a}$ $1 - 2(x-1) + 3(x-1)^2$

3. $f(x) = \cos(x)$ $a = \pi/3$ $n = 4$ $0 \le x \le 2\pi/3$

n	der.	a=π/3
0	cos x	1/2
1	−sin x	$-\sqrt{3}/2$
2	−cos x	−1/2
3	sin x	$\sqrt{3}/2$
4	cos x	1/2

$\boxed{a}$ $\frac{1}{2} - \frac{\sqrt{3}(x-\pi/3)}{2} - \frac{(x-\pi/3)^2}{4} + \frac{\sqrt{3}(x-\pi/3)^3}{12} + \frac{(x-\pi/3)^4}{48}$

$\boxed{b}$ $R_n \approx 0.010495$

4. $f(x) = \ln(1+2x)$ $a = 1$ $n = 3$ $0.5 \le x \le 1.5$

$\boxed{a}$ $\ln(3) + \frac{2(x-1)}{3} - \frac{2(x-1)^2}{9} + \frac{8(x-1)^3}{81}$

5. $f(x) = x\ln x$ $a = 1$ $n = 3$ $.5 \le x \le 1.5$

$\boxed{a}$ $(x-1) + \frac{(x-1)^2}{2} - \frac{(x-1)^3}{6}$

Taylor/Maclaurin Series you should know (yes, memorize):

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$ for x = all real #s

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$ for x = all real #s

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$ for x = all real #s

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1} \frac{x^n}{n} + \cdots = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$$ $-1 < x \le 1$

1. What is the radius of convergence of the series $\sum_{n=0}^{\infty}\frac{(x-2)^n}{n!}$

E

(A) 0 (B) 1 (C) 2 (D) 3 (E) ∞

2. For what values of r does $\sum_{n=0}^{\infty}\frac{3r^{2n}}{5^n}$ converge?

A

(A) $-2.24<r<2.24$ (B) $-1.67<r<1.67$ (C) $-5<r<5$
(D) For all values of r (E) $r=0$ only

3. What is the radius of convergence of the series $\sum_{n=0}^{\infty}\frac{x^n}{2\ln n}$

B

(A) 0 (B) 1 (C) 2 (D) e (E) ∞

4. The first four terms of the Taylor series around 0 of $\frac{1}{1+2x}$ are

A

(A) $1-2x+4x^2-8x^3$ (B) $1+2x+4x^2+8x^3$ (C) $1-x+2x^2-4x^3$
(D) $1+x+2x^2+4x^3$ (E) $1-2x^2+4x^4-8x^6$

5. The first four terms of the Taylor series around 0 of $\cos\left(\sqrt{x}\right)$ are

A

(A) $1-\frac{x}{2}+\frac{x^2}{4!}-\frac{x^3}{6!}$ (B) $1+\frac{x}{2}+\frac{x^2}{4!}+\frac{x^3}{6!}$ (C) $1-\frac{x}{2}+\frac{x^2}{4}-\frac{x^3}{6}$
(D) $1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{6}$ (E) $1-\frac{x}{2}-\frac{x^2}{4!}-\frac{x^3}{6!}$

6. The Taylor polynomial of order 3 at $x=0$ for $f(x)=\sqrt{1+x}$ is

B

(A) $1+\frac{x}{2}-\frac{x^2}{4}+\frac{3x^3}{8}$ (B) $1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}$ (C) $1-\frac{x}{2}+\frac{x^2}{8}-\frac{x^3}{16}$
(D) $1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{8}$ (E) $1-\frac{x}{2}+\frac{x^2}{4}-\frac{3x^3}{8}$

7. The Taylor polynomial of order 3 at $x=1$ for e^x is

D

(A) $1+(x-1)+\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}$ (B) $e\left[1+(x-1)+\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}\right]$

(C) $e\left[1+(x+1)+\frac{(x+1)^2}{2!}+\frac{(x+1)^3}{3!}\right]$ (D) $e\left[1+(x-1)+\frac{(x-1)^2}{2!}+\frac{(x-1)^3}{3!}\right]$

(E) $e\left[1-(x-1)+\frac{(x-1)^2}{2!}+\frac{(x-1)^3}{3!}\right]$

8. The coefficient of $\left(x-\frac{\pi}{4}\right)^3$ in the Taylor series about $\frac{\pi}{4}$ of $f(x)=\cos x$ is

D

(A) $\frac{\sqrt{3}}{2}$ (B) $-\frac{1}{12}$ (C) $\frac{1}{12}$ (D) $\frac{1}{6\sqrt{2}}$ (E) $-\frac{1}{3\sqrt{2}}$

9. The Taylor polynomial of order 3 at $x=0$ for $(1+x)^p$, where p is a constant, is

C

(A) $1+px+p(p-1)x^2+p(p-1)(p-2)x^3$ (B) $1+px+\frac{p(p-1)}{2}x^2+\frac{p(p-1)(p-2)}{3}x^3$

(C) $1+px+\frac{p(p-1)}{2!}x^2+\frac{p(p-1)(p-2)}{3!}x^3$ (D) $px+\frac{p(p-1)}{2!}x^2+\frac{p(p-1)(p-2)}{3!}x^3$

(E) None of these

10. The Taylor series for $\ln(1+2x)$ about $x=0$ is

D

(A) $2x-\frac{(2x)^2}{2}+\frac{(2x)^3}{3}-\frac{(2x)^4}{4}+\ldots$ (B) $2x-2x^2+8x^3-16x^4+\ldots$

(C) $2x-4x^2+16x^3+\ldots$ (D) $2x-2x^2+\frac{8}{3}x^3-4x^4+\ldots$

(E) $2x-\frac{(2x)^2}{2!}+\frac{(2x)^3}{3!}-\frac{(2x)^4}{4!}+\ldots$

11. What are the first four nonzero terms in the power series expansion of e^{-4x} about $x=0$?

D

(A) $1+x+\frac{x^2}{2}+\frac{x^3}{3}$ (B) $1-4x+8x^2-32x^3$

(C) $1-4x-2x^2-\frac{2}{3}x^3$ (D) $1-4x+8x^2-\frac{32}{3}x^3$

(E) $1-4x+8x^2-\frac{64}{3}x^3$

12. What are all values of x for which the series $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots$ converges?

C

(A) $-1\le x\le 1$ (B) $-1\le x<1$ (C) $-1<x\le 1$

(D) $-1<x<1$ (E) All real x

13. $\sum_{n=0}^{\infty}\frac{(-1)^n(\pi)^{2n}}{(2n)!}=$

B

(A) 1 (B) -1 (C) π (D) $\frac{\pi}{2}$ (E) e^{π}

14. Let $T(x)=\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^{n}\frac{(x-3)^k}{k!}$ be the Taylor series for a function f. What is the value of $f^{(10)}(3)$, the tenth derivative of f at $x=3$?

C

(A) 0 (B) 2.691×10^{-10} (C) 9.766×10^{-4}

(D) 9.766×10^{-5} (E) 0.5

15. The first three non-zero terms in the Taylor series about $x = 0$ for $f(x) = \cos(x)$

C

(A) $x + \frac{x^3}{3!} + \frac{x^5}{5!}$ (B) $x - \frac{x^3}{3!} + \frac{x^5}{5!}$ (C) $1 - \frac{x^2}{2!} + \frac{x^4}{4!}$

(D) $1 - \frac{x^2}{2!} - \frac{x^4}{4!}$ (E) $1 + \frac{x^2}{2!} + \frac{x^4}{4!}$

16. If $f(x) = \sum_{k=1}^{\infty} (\cos^2 x)^k$, then $f\left(\frac{\pi}{4}\right)$ is

D

(A) –2 (B) –1 (C) 0 (D) 1 (E) 2

17. The function f has derivatives of all orders for all real numbers x. Assume $f(2) = -3, f'(2) = 5, f''(2) = 3,$ and $f'''(2) = -8$.

(a) Write the third-degree Taylor polynomial for f about $x = 2$ and use it to approximate $f(1.5)$.

(b) The fourth derivative of f satisfies the inequality $\left|f^{(4)}(x)\right| \le 3$ for all x in the closed interval $[1.5, 2]$. Use the Lagrange error bound on the approximation to $f(1.5)$ found in part (a) to explain why $f(1.5) \ne -5$.

(c) Write the fourth-degree Taylor polynomial, $P(x)$, for $g(x) = f(x^2 + 2)$ about $x = 0$. Use P to explain why g must have a relative minimum at $x = 0$.

18. A function f is defined by: $f(x) = \frac{1}{3} + \frac{2}{3^2}x + \frac{3}{3^3}x^2 + \ldots + \frac{n+1}{3^{n+1}}x^n + \ldots$ for all x in the interval of convergence of the given power series.

(a) Find the interval of convergence for this power series. Show the work that leads to your answer.

(b) Find $\lim_{x \to 0} \frac{f(x) - \frac{1}{3}}{x}$.

(c) Write the first three nonzero terms and the general term for an infinite series that represents $\int_0^1 f(x)\,dx$.

(d) Find the sum of the series determined in part (c).

Power series Review Answers:

1. E
2. A
3. B
4. A
5. A
6. B
7. D
8. D
9. C
10. D
11. D
12. C
13. B
14. C
15. C
16. D

17.A) $T_3(x) = -3 + 5(x-2) + \frac{3}{2}(x-2)^2 - \frac{4}{3}(x-2)^3$

Approx at 1.5 = -4.958

B) Error Bound = .00728125

F(1.5) > -4.9583 - .0078125 = -4.966 . –5

Thus, f(1.5) does not = -5

18. A) Use Ratio Test (-3, 3)

B) 2/9

C)

$$\int_0^1 f(x)dx = \int_0^1 \left(\frac{1}{3} + \frac{2}{3^2}x + \frac{3}{3^3}x^2 + \ldots + \frac{n+1}{3^{n+1}}x^n + \ldots\right)dx$$

$$= \left(\frac{1}{3}x + \frac{1}{3^2}x^2 + \frac{1}{3^3}x^3 + \ldots + \frac{1}{3^{n+1}}x^{n+1} + \ldots\right) \mathit{from}\, x = 0 \mathit{\ to\ } 1$$

$$= \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots + \frac{1}{3^{n+1}} + \ldots$$

D) Geometric Series -> a = 1/3 r = 1/3 Answer = 1/2