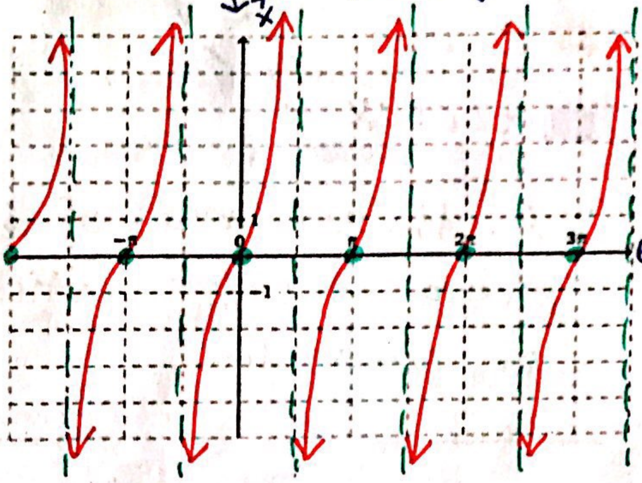
Unit 6 Packet – Trig Inverses and Applications Name: Key

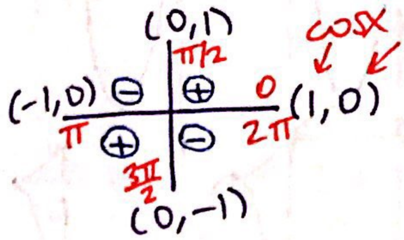
Day 1 – Other Trig Graphs and Simple Harmonic Motion

Locate the ① vertical asymptotes and ② zeros, then sketch the graph of each function.

1. y = tan (x) → y, x Think: $y = \tan\theta$

Range: $(-\infty, \infty)$





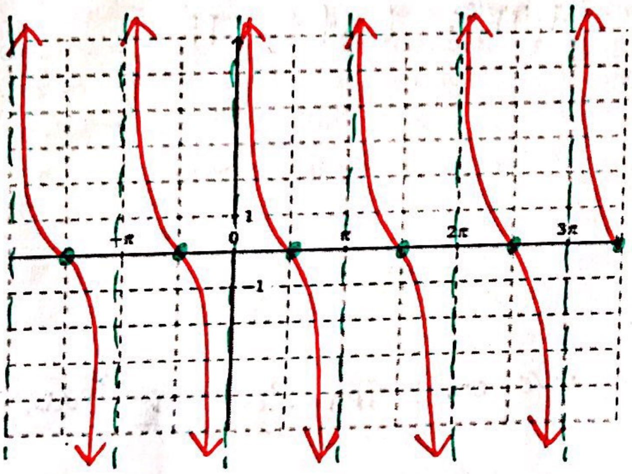
$\tan x = \dfrac{\sin x}{\cos x}$

x	0	π/2	π	3π/2	2π
sin x	0	1	0	−1	0
cos x	1	0	−1	0	1
tan x	0	und.	0	und.	0

vert. asymptotes roots

2. y = cot(x)

Range: $(-\infty, \infty)$

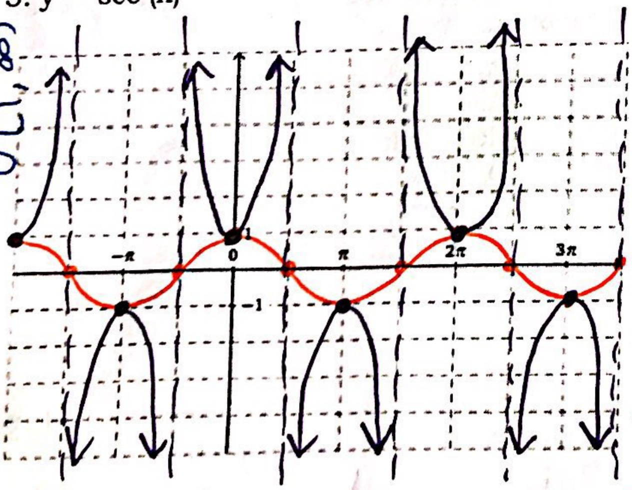


$\cot x = \dfrac{1}{\tan x} = \dfrac{\cos x}{\sin x}$

x	0	π/2	π	3π/2	2π
cot x	und.	0	und.	0	und.

3. y = sec (x)

Range: $(-\infty, -1] \cup [1, \infty)$



$\sec x = \dfrac{1}{\cos x}$

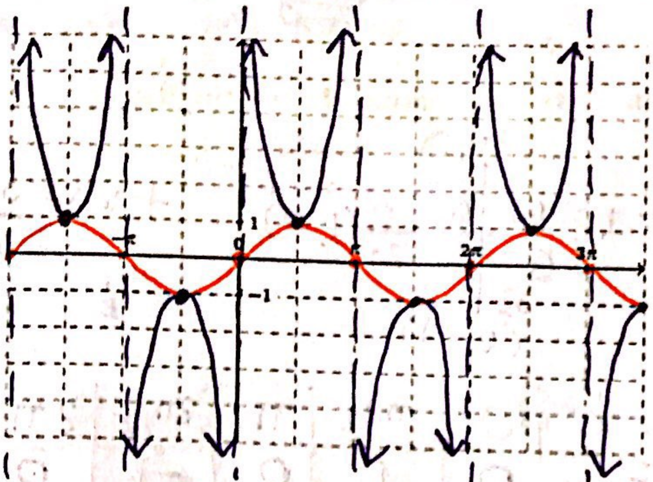
= cos x

= sec x

- VA: through roots of cos x
- Extrema of cos x = vertices of each parabola

Range: $(-\infty, -1] \cup [1, \infty)$

4. y = csc x



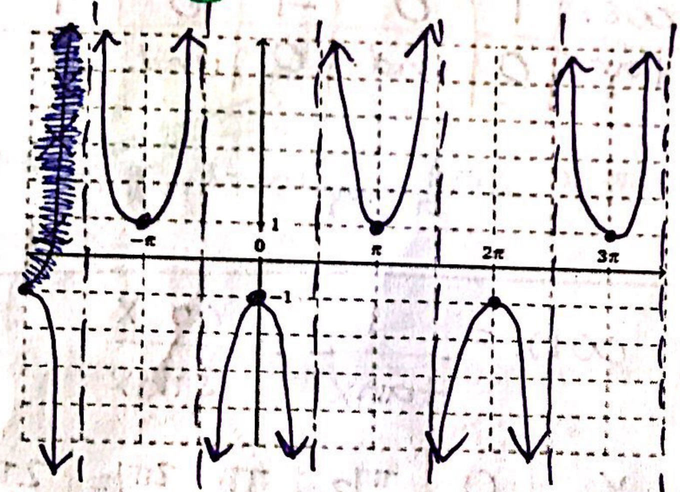
$\csc x = \dfrac{1}{\sin x}$

■ = sin x

■ = csc x

- V.A. through roots of sin x
- Extrema of sin x = vertices of parabolas

5. y = sec(x + π)



+π : Left π

* Unless otherwise indicated, we are in Radian $\boxed{\text{MODE}}$ *

Harmonic Motion

The periodic nature of the trigonometric functions is useful for describing the motion of a point on an object that oscillates, rotates, or is moved by wave motion.

Simple-Harmonic Motion

A point that moves on a coordinate line is said to be in simple harmonic motion if its distance/displacement, d, from the origin at time t is given by either

$\boxed{d = a\sin(bt)}$ *or* $\boxed{d = a\cos(bt)}$

Where a and b are real numbers such that $b > 0$.

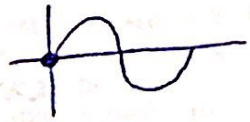
The motion of the object has the following properties:

Amplitude = $|a|$

Period = $\dfrac{2\pi}{b}$ Time in seconds, minutes, etc.

Frequency = $\dfrac{b}{2\pi}$ cycles per "time" unit

Examples:



1. Given the equation for the simple harmonic motion $d = 6\sin(\frac{3\pi}{4}t)$,

   a) Find the maximum displacement. (amp) $= \boxed{6 \text{ units}}$

   b) Find the frequency. $= \frac{b}{2\pi} = \frac{3\pi}{4} \cdot \frac{1}{2\pi} = \boxed{\frac{3}{8} \text{ cycles per } t}$

   c) Find the value of d when t = 4.

   $d = 6\sin(3\pi/4 \cdot 4) = 6\sin(3\pi) = 6(0) = \boxed{0 \text{ units}}$

   d) Find the least ~~positive~~ value of t for which d=0.

   non-negative

   $\frac{0}{6} = \frac{6}{6}\sin(3\pi/4\, t) \rightarrow 0 = \sin(3\pi/4 \cdot t)$
   $\sin^{-1}(0) = 3\pi/4 \cdot t$
   $0 = \frac{3\pi}{4} \cdot t$
   $\boxed{t = 0}$

2. Write an equation for the simple harmonic motion of a ball with a maximum displacement of 10cm, where the period is 4 seconds. What is the $\boxed{\text{frequency}}$ of the motion?

   amp = 10, $a = \pm 10$

   $4 = \frac{2\pi}{b}$, $4b = 2\pi$, $b = \pi/2$

   $\boxed{y = \pm 10\sin(\frac{\pi}{2}t)}$ (or cos)

   $\frac{b}{2\pi} = \frac{\pi}{2} \cdot \frac{1}{2\pi} = \boxed{\frac{1}{4}}$ (Reciprocal of Per = 4)

3. Suppose a ball at the end of a spring undergoes simple harmonic motion where 8 cm is the maximum displacement. In addition, it takes 8 seconds for the ball to move from its maximum displacement above zero to its maximum displacement below zero and back again. Assume the initial position is at equilibrium. (use sinx)

   $a = 8$ per = 8 $\Rightarrow 8 = \frac{2\pi}{b} \Rightarrow 8b = 2\pi$, $b = \frac{\pi}{4}$

   a) Write an equation for the simple harmonic motion of the mass.

   $\boxed{d = 8\sin\frac{\pi}{4}t}$

   b) Find the displacement of the mass at 20 seconds. ($t = 20$)

   $d = 8\sin(\frac{\pi}{4} \cdot 20) = 8\sin(5\pi) = 8(0) = \boxed{0 \text{ cm}}$

   (@ equillibrium)

   c) What is the least positive value of t for which d = 3cm ?

   $\frac{3}{8} = \frac{8}{8}\sin(\frac{\pi}{4}t) \rightarrow \sin^{-1}(3/8) = \frac{\pi}{4}t$

   $.384\ldots(4/\pi) = t$

   $\approx \boxed{.489 \text{ sec.}}$

   d) What is the frequency of this motion in cycles per second?

   $\frac{b}{2\pi} = \frac{\pi}{4} \cdot \frac{1}{2\pi} = \boxed{\frac{1}{8} \text{ cycles per sec.}}$

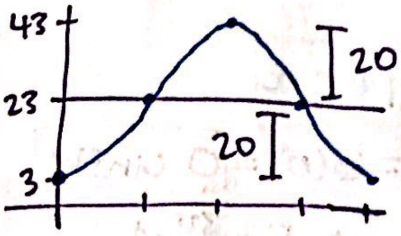
   Reciprocal of Period (8)

40, 3

4. You are on a ferris wheel that has a diameter of 40 feet and is 3 feet off the ground. You travel at a rate of 6 revolutions per minute. Assume the ride starts at the bottom. ← use $-\cos x$

$6 = b/2\pi \rightarrow b = 12\pi$

a) Find an equation to model your height above the ground as a function of time.



$$\boxed{d = -20\cos(12\pi t) + 23}$$

d=23 (middle)

b) How high above the ground are you at 42 seconds?

$t = 42$ sec = ? min?

$$\frac{60 \text{ sec}}{1 \text{ min}} = \frac{42 \text{ sec}}{x \text{ min}}$$

$60x = 42 \rightarrow x = .7$ min

$d = -20\cos(12\pi \cdot 0.7) + 23$

$\approx$ $\boxed{16.82 \text{ ft}}$

c) Find the least positive value of t for which your displacement is 18 feet.

d=18

$18 = -20\cos(12\pi t) + 23$

$-23 \qquad -23$

$\frac{-5}{-20} = \frac{-20}{-20}\cos(12\pi t)$

$\frac{\cos^{-1}(1/4)}{12\pi} = \frac{12\pi t}{12\pi}$

$t \approx \boxed{0.035 \text{ min}}$

In exercises 5-7, an object moves in simple harmonic motion described by the given equations where t is measured in seconds and d in inches. In each exercise, find the following:

a) The maximum displacement
b) Distance from rest position at t = 0 (and whether it's above or below equilibrium)
c) Direction of initial movement
d) Time required for one cycle (Period)

5. $d = 5\cos\frac{\pi}{2}t$

a) $\boxed{5}$ inches

b) $5\cos(0) = \boxed{5}$ (above equil.)

c) downward

d) $\frac{2\pi}{\pi/2} = \boxed{4 \text{ sec.}}$

6. $d = -6\cos 2\pi t$

a) $\boxed{6}$ inches

b) $-6\cos(0) = \boxed{-6}$ (below equil.)

c) upward

d) $\frac{2\pi}{2\pi} = \boxed{1 \text{ sec.}}$

7. $d = -5\sin\frac{2\pi}{3}t$

a) $\boxed{5}$ inches

b) $-5\sin(0) = \boxed{0}$ (@ equilibrium)

c) downward

d) $\frac{2\pi}{2\pi/3} = \boxed{3 \text{ sec.}}$