

Unit 5: Sequences and Series

**Day 1 - Factorial Practice**

Simplify and evaluate. No calculators!

1.  $6!$

2.  $\frac{5!}{3!}$

3.  $\frac{6!}{4!}$

4.  $\frac{6!}{4!2!}$

5.  $\frac{5!}{2!2!}$

6.  $\frac{7!}{3!2!}$

7.  $\frac{6!}{(5-3)!3!}$

8.  $\frac{7!}{(7-4)!4!}$

9.  $\frac{4!}{(4-1)!0!}$

Simplify. (There should be no more factorials left)

10.  $\frac{n!}{(n-2)!}$

11.  $\frac{n!}{(n+1)!}$

12.  $\frac{n!}{(n-3)!}$

13.  $\frac{(2n)!}{(2n+1)!}$

14.  $\frac{(2n+1)!}{(2n+3)!}$

15.  $\frac{[2(n+1)]!}{(2n)!}$

Evaluate the limit.

16.  $\lim_{n \rightarrow \infty} \frac{n!}{(n+1)!}$

17.  $\lim_{n \rightarrow \infty} \frac{(2n+1)!}{(2n-1)!}$

18.  $\lim_{n \rightarrow \infty} \frac{x^n}{n!}$

Find the derivative. No calculators. You may use factorials in your answer.

19.  $f^{10}$  of  $x^{10}$

20.  $f^{15}$  of  $3x^{15}$

21.  $f^{40}$  of  $5x^{40}$

## Day 2 – Sequences

- (a) What is a sequence?  
(b) What does it mean to say that  $\lim_{n \rightarrow \infty} a_n = 8$ ?  
(c) What does it mean to say that  $\lim_{n \rightarrow \infty} a_n = \infty$ ?
- (a) What is a convergent sequence? Give two examples.  
(b) What is a divergent sequence? Give two examples.

List the first five terms of the sequence.

- $a_n = 1 - (0.2)^n$
- $a_n = \frac{n+1}{3n-1}$
- $a_n = \frac{3(-1)^n}{n!}$
- $\{2 \cdot 4 \cdot 6 \cdots (2n)\}$
- $a_1 = 3, a_{n+1} = 2a_n - 1$
- $a_1 = 4, a_{n+1} = \frac{a_n}{a_n - 1}$

Find a formula for the general term  $a_n$  of the sequence, assuming that the pattern of the first few terms continues.

- $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots\right\}$
- $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \dots\right\}$
- $\{2, 7, 12, 17, \dots\}$
- $\left\{-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, \dots\right\}$
- $\left\{1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \dots\right\}$
- $\{5, 1, 5, 1, 5, 1, \dots\}$

Determine whether the sequence converges or diverges. If it converges, find the limit.

- $a_n = n(n-1)$
- $a_n = \frac{n+1}{3n-1}$
- $a_n = \frac{3+5n^2}{n+n^2}$
- $a_n = \frac{\sqrt{n}}{1+\sqrt{n}}$
- $a_n = \frac{2^n}{3^{n+1}}$
- $a_n = \frac{n}{1+\sqrt{n}}$
- $a_n = \frac{(-1)^{n-1}n}{n^2+1}$
- $a_n = \frac{(-1)^n n^3}{n^3+2n^2+1}$
- $a_n = \cos\left(\frac{n}{2}\right)$
- $a_n = \cos\left(\frac{2}{n}\right)$
- $\left\{\frac{(2n-1)!}{(2n+1)!}\right\}$
- $\{\arctan 2n\}$
- $\left\{\frac{e^n + e^{-n}}{e^{2n} - 1}\right\}$
- $\left\{\frac{\ln n}{\ln 2n}\right\}$
- $\{n^2 e^{-n}\}$
- $\{n \cos n\pi\}$
- $a_n = \frac{\cos^2 n}{2^n}$
- $a_n = \ln(n+1) - \ln n$
- $a_n = n \sin\left(\frac{1}{n}\right)$
- $a_n = \sqrt{n} - \sqrt{n^2 - 1}$
- $a_n = \left(1 + \frac{2}{n}\right)^{1/n}$
- $a_n = \frac{\sin 2n}{1+\sqrt{n}}$
- $\{0, 1, 0, 0, 1, 0, 0, 0, 1, \dots\}$
- $\left\{\frac{1}{1}, \frac{1}{3}, \frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{4}, \frac{1}{6}, \dots\right\}$
- $a_n = \frac{n!}{2^n}$
- $a_n = \frac{(-3)^n}{n!}$

### Day 3 – Series

1. (a) What is the difference between a sequence and a series?

(b) What is a convergent series? What is a divergent series?

2. Explain what it means to say that  $\sum_{n=1}^{\infty} a_n = 5$ .

3. Let  $a_n = \frac{2n}{3n+1}$

(a) Determine whether  $\{a_n\}$  is convergent

(b) Determine whether  $\sum_{n=1}^{\infty} a_n$  is convergent.

**Determine whether the series is convergent or divergent. If it is convergent, find its sum.**

4.  $3 + 2 + \frac{4}{3} + \frac{8}{9} + \dots$

5.  $\frac{1}{8} - \frac{1}{4} + \frac{1}{2} - 1 + \dots$

6.  $-2 + \frac{5}{2} - \frac{25}{8} + \frac{125}{32} - \dots$

7.  $1 + 0.4 + 0.16 + 0.064 + \dots$

8.  $\sum_{n=1}^{\infty} 5 \left( \frac{2}{3} \right)^{n-1}$

9.  $\sum_{n=1}^{\infty} \frac{(-6)^{n-1}}{5^{n-1}}$

10.  $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$

11.  $\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$

12.  $\sum_{n=0}^{\infty} \frac{\pi^n}{3^{n+1}}$

13.  $\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$

14.  $\sum_{n=1}^{\infty} \frac{n}{n+5}$

15.  $\sum_{n=1}^{\infty} \frac{3}{n}$

16.  $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$

17.  $\sum_{n=1}^{\infty} \frac{(n+1)^2}{n(n+2)}$

18.  $\sum_{n=2}^{\infty} \frac{n^2}{n^2-1}$

19.  $\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3}$

20.  $\sum_{n=1}^{\infty} \frac{3^n + 2^n}{6^n}$

21.  $\sum_{n=1}^{\infty} [(0.8)^{n-1} - (0.3)^n]$

22.  $\sum_{n=1}^{\infty} \sqrt[n]{2}$

23.  $\sum_{n=1}^{\infty} \ln \left( \frac{n}{2n+5} \right)$

24.  $\sum_{n=1}^{\infty} \arctan n$

25.  $\sum_{n=1}^{\infty} (\cos 1)^n$

26.  $\sum_{n=1}^{\infty} \left( \frac{3}{n(n+3)} + \frac{5}{4^n} \right)$

27.  $\sum_{n=1}^{\infty} \left( \frac{3}{5^n} + \frac{2}{n} \right)$

## Day 4 - Integral/P-Series Test

1. Draw a picture to show that

$$\sum_{n=2}^{\infty} \frac{1}{n^{13}} < \int_1^{\infty} \frac{1}{x^{13}} dx$$

What can you conclude about the series?

2. Suppose  $f$  is a continuous positive decreasing function for  $x \geq 1$  and  $a_n = f(n)$ . By drawing a picture, rank the following three quantities in increasing order:

$$\int_1^6 f(x) dx \qquad \sum_{i=1}^5 a_i \qquad \sum_{i=2}^6 a_i$$

**Use the Integral Test to determine whether the series is convergent or divergent.**

3.  $\sum_{n=1}^{\infty} \frac{1}{n^4}$       4.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n}}$       5.  $\sum_{n=1}^{\infty} \frac{1}{3n+1}$

6.  $\sum_{n=1}^{\infty} e^{-n}$       7.  $\sum_{n=1}^{\infty} ne^{-n}$       8.  $\sum_{n=1}^{\infty} \frac{n+2}{n+1}$

**Determine whether the series is convergent or divergent.**

9.  $\sum_{n=1}^{\infty} \frac{2}{n^{0.85}}$       10.  $\sum_{n=1}^{\infty} (n^{-1.4} + 3n^{-1.2})$

11.  $1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \frac{1}{125} + \dots$       12.  $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \dots$

13.  $\sum_{n=1}^{\infty} \frac{5-2\sqrt{n}}{n^3}$       14.  $\sum_{n=3}^{\infty} \frac{5}{n-2}$

15.  $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$       16.  $\sum_{n=1}^{\infty} \frac{3n+2}{n(n+1)}$

17.  $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$       18.  $\sum_{n=1}^{\infty} \frac{1}{n^2-4n+5}$

19.  $\sum_{n=1}^{\infty} ne^{-n^2}$       20.  $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

21.  $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$       22.  $\sum_{n=1}^{\infty} \frac{n}{n^4+1}$

### BONUS (THESE ARE HARD!!)

23.  $\sum_{n=1}^{\infty} \frac{1}{n^3+n}$  (use non-linear partial fractions)      24.  $\sum_{n=3}^{\infty} \frac{1}{n \ln n \ln(\ln n)}$

**25-26: Find the values of  $p$  for which the series is convergent.**

25.  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$       26.  $\sum_{n=1}^{\infty} \frac{1}{n \ln n [\ln(\ln n)]^p}$

**Day 5 - Review for Quick Quiz 1**

**Sequences, Geometric Series,  
Test for Divergence, Telescoping,  
Integral and P-Series Tests**

**I. Sequences – Find the limit of the sequence or explain why it does not exist.**

1.  $a_k = \left(\frac{-3}{2}\right)^k$

2.  $a_k = \left(\frac{\sqrt{26}}{17}\right)^k$

3.  $\left\{\frac{k^2}{k^2+1}\right\}$

4.  $\left\{\frac{k!}{(k+1)!}\right\}$

5.  $a_k = \ln\left(\frac{k}{k+1}\right)$

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**II. Series – Do the following converge or diverge? WHY – what test or logic did you use?**

6.  $\sum_{n=0}^{\infty} \frac{n+1}{2n+1}$

7.  $\sum_{j=0}^{\infty} (\ln 2)^j$

8.  $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$

9.  $\frac{1}{100} + \frac{1}{200} + \frac{1}{300} + \dots$

CV / DV

CV / DV

CV / DV

CV / DV

Why? \_\_\_\_\_

Why? \_\_\_\_\_

Why? \_\_\_\_\_

Why? \_\_\_\_\_

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10.  $\sum_{k=1}^{\infty} k^{-11/10}$

11.  $\sum_{k=2}^{\infty} \frac{2}{k \ln k}$

12.  $\sum_{k=1}^{\infty} \frac{\tan^{-1} k}{1+k^2}$

13.  $\sum_{k=1}^{\infty} \frac{9}{k(k+3)}$

CV / DV

CV / DV

CV / DV

CV / DV

Why? \_\_\_\_\_

Why? \_\_\_\_\_

Why? \_\_\_\_\_

Why? \_\_\_\_\_

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14. What is the sum of the series  $\frac{3}{2} - \frac{3}{8} + \frac{3}{32} - \frac{3}{128} + \dots$ ?

- (A)  $\frac{6}{7}$       (B)  $\frac{9}{8}$       (C)  $\frac{6}{5}$       (D)  $\frac{15}{8}$       (E) 2

15.

$$\sum_{k=0}^{\infty} \left( \sin\left(\frac{\pi}{6}\right) \right)^k =$$

- (A) 1      (B) 2      (C)  $\frac{1}{1 - \frac{\sqrt{3}}{2}}$       (D)  $\frac{\frac{\sqrt{3}}{2}}{1 - \frac{\sqrt{3}}{2}}$       (E) Does not converge

16.

$$\sum_{n=1}^{\infty} \left( \frac{1}{2} \right)^{2n} =$$

- (A)  $\frac{1}{3}$       (B)  $\frac{1}{2}$       (C) 1      (D) 2      (E)  $\infty$

17. Which of the following series are convergent?

- (A) I only  
 (B) II only  
 (C) I and II only  
 (D) II and III only  
 (E) I, II, and III

I.  $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \dots$   
 II.  $\frac{1}{1 \bullet 2} + \frac{1}{2 \bullet 3} + \frac{1}{3 \bullet 4} + \dots$   
 III.  $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$

18.

$$\sum_{k=0}^{\infty} \left( \frac{-\pi}{3} \right)^k =$$

- (A)  $\frac{1}{1 - \frac{\pi}{3}}$       (B)  $\frac{\frac{\pi}{3}}{1 - \frac{\pi}{3}}$       (C)  $\frac{3}{3 + \pi}$       (D)  $\frac{\pi}{3 + \pi}$       (E) Does not converge

**Remember, the only two that tell you what the sum IS are geometric and telescoping (partial fractions)...the others only indicate CV or DV!**

Determine whether the series converges or diverges. There may be more than one correct way to determine convergence or divergence of a given series.

1.  $\sum_{n=1}^{\infty} \frac{5}{n+1}$

2.  $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n}}$

3.  $\sum_{n=2}^{\infty} \frac{\ln n}{n}$

4.  $\sum_{n=1}^{\infty} \frac{1}{2n-1}$

5.  $\sum_{n=1}^{\infty} \frac{1}{(\ln 2)^n}$

6.  $\sum_{n=1}^{\infty} \frac{1}{(\ln 3)^n}$

7.  $\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$

Find the sum of the telescoping series.

8.  $\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$

9.  $\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)}$

10.  $\sum_{n=1}^{\infty} \left( \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$

### Day 6 - Direct and Limit Comparison Tests

- Suppose  $\sum a_n$  and  $\sum b_n$  are series with positive terms and  $\sum b_n$  is known to be convergent.
  - If  $a_n > b_n$  for all  $n$ , what can you say about  $\sum a_n$ ? Why?
  - If  $a_n < b_n$  for all  $n$ , what can you say about  $\sum a_n$ ? Why?
- Suppose  $\sum a_n$  and  $\sum b_n$  are series with positive terms and  $\sum b_n$  is known to be divergent.
  - If  $a_n > b_n$  for all  $n$ , what can you say about  $\sum a_n$ ? Why?
  - If  $a_n < b_n$  for all  $n$ , what can you say about  $\sum a_n$ ? Why?

### Determine whether the series converges or diverges.

- $\sum_{n=1}^{\infty} \frac{1}{n^2 + n + 1}$
- $\sum_{n=1}^{\infty} \frac{2}{n^3 + 4}$
- $\sum_{n=1}^{\infty} \frac{5}{2 + 3^n}$
- $\sum_{n=1}^{\infty} \frac{1}{n - \sqrt{n}}$
- $\sum_{n=1}^{\infty} \frac{n+1}{n^2}$
- $\sum_{n=1}^{\infty} \frac{4+3^n}{2^n}$
- $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2 + 1}$
- $\sum_{n=1}^{\infty} \frac{n^2 - 1}{3n^4 + 1}$
- $\sum_{n=1}^{\infty} \frac{n^2 + 1}{n^3 - 1}$
- $\sum_{n=0}^{\infty} \frac{1 + \sin n}{10^n}$
- $\sum_{n=1}^{\infty} \frac{n-1}{n4^n}$
- $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n-1}$
- $\sum_{n=1}^{\infty} \frac{2 + (-1)^n}{n\sqrt{n}}$
- $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 1}}$
- $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2 + 1}}$
- $\sum_{n=1}^{\infty} \frac{1}{2n+3}$
- $\sum_{n=1}^{\infty} \frac{2^n}{1+3^n}$
- $\sum_{n=1}^{\infty} \frac{1+2^n}{1+3^n}$
- $\sum_{n=1}^{\infty} \frac{1}{1+\sqrt{n}}$
- $\sum_{n=1}^{\infty} \frac{n+2}{(n+1)^3}$
- $\sum_{n=1}^{\infty} \frac{5+2n}{(1+n^2)^2}$
- $\sum_{n=1}^{\infty} \frac{n^2 - 5n}{n^3 + n + 1}$
- $\sum_{n=1}^{\infty} \frac{1+n+n^2}{\sqrt{1+n^2+n^6}}$
- $\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$
- $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^2 e^{-n}$
- $\sum_{n=1}^{\infty} \frac{2n^2 + 7n}{3^n (n^2 + 5n - 1)}$
- $\sum_{n=1}^{\infty} \frac{1}{n!}$
- $\sum_{n=1}^{\infty} \frac{n!}{n^n}$
- $\sum_{n=1}^{\infty} \sin \frac{1}{n}$
- $\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$



## **Day 7 - Alternating Series**

- (a) What is an alternating series?  
(b) Under what conditions does an alternating series converge?  
(c) If these conditions are satisfied, what can you say about the remainder after  $n$  terms?

### **Test the series for convergence or divergence.**

$$2. -\frac{1}{3} + \frac{2}{4} - \frac{3}{5} + \frac{4}{6} - \frac{5}{7} + \dots$$

$$3. \frac{4}{7} - \frac{4}{8} + \frac{4}{9} - \frac{4}{10} + \frac{4}{11} - \dots$$

$$4. \frac{1}{\ln 2} - \frac{1}{\ln 3} + \frac{1}{\ln 4} - \frac{1}{\ln 5} + \frac{1}{\ln 6} - \dots$$

$$5. \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$$

$$6. \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3n-1}$$

$$7. \sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$$

$$8. \sum_{n=1}^{\infty} (-1)^n \frac{2n}{4n^2+1}$$

$$9. \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n^2+1}$$

$$10. \sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{1+2\sqrt{n}}$$

$$11. \sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^3+4}$$

$$12. \sum_{n=1}^{\infty} (-1)^n \frac{e^{1/n}}{n}$$

$$13. \sum_{n=1}^{\infty} (-1)^n \frac{n}{\ln n}$$

$$14. \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\ln n}{n}$$

$$15. \sum_{n=1}^{\infty} \frac{\cos n\pi}{n^{3/4}}$$

$$16. \sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{n!}$$

$$17. \sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{\pi}{n}\right)$$

$$18. \sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{\pi}{n}\right)$$

$$19. \sum_{n=1}^{\infty} (-1)^n \frac{n^n}{n!}$$

$$20. \sum_{n=1}^{\infty} \left(-\frac{n}{5}\right)^n$$

## Day 8 - Absolute and Conditional Convergence

1. What can you say about the series  $\sum a_n$  in each of the following cases?

(a)  $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 8$

(b)  $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 0.8$

(c)  $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$

**Determine whether the series is absolutely convergent, conditionally convergent, or divergent.**

2.  $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

3.  $\sum_{n=1}^{\infty} \frac{(-10)^n}{n!}$

4.  $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^n}{n^4}$

5.  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[4]{n}}$

6.  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$

7.  $\sum_{n=1}^{\infty} (-1)^n \frac{n}{5+n}$

8.  $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+1}$

9.  $\sum_{n=1}^{\infty} \frac{1}{(2n)!}$

10.  $\sum_{n=1}^{\infty} e^{-n} n!$

11.  $\sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{n^3}$

12.  $\sum_{n=1}^{\infty} \frac{\sin 4n}{4^n}$

13.  $\sum_{n=1}^{\infty} \frac{n(-3)^n}{4^{n-1}}$

14.  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2 2^n}{n!}$

15.  $\sum_{n=1}^{\infty} \left( \frac{10^n}{(n+1)4^{2n+1}} \right)$

16.  $\sum_{n=1}^{\infty} \frac{3 - \cos n}{n^3 - 2}$

17.  $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

18.  $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

19.  $\sum_{n=1}^{\infty} \frac{\cos(n\pi/3)}{n!}$

20.  $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n}$

21.  $\sum_{n=1}^{\infty} \frac{n^n}{3^{1+3n}}$

22.  $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

23.  $\sum_{n=1}^{\infty} \left( \frac{n^2+1}{2n^2+1} \right)^n$

**Day 9 - All Types of Tests Mixed Up!**

Test the series for convergence or divergence.

1. 
$$\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + n}$$

2. 
$$\sum_{n=1}^{\infty} \frac{n-1}{n^2 + n}$$

3. 
$$\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$$

4. 
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n-1}{n^2 + n}$$

5. 
$$\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$$

6. 
$$\sum_{n=1}^{\infty} \left( \frac{3n}{1+8n} \right)^n$$

7. 
$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

8. 
$$\sum_{k=1}^{\infty} \frac{2^k k!}{(k+2)!}$$

9. 
$$\sum_{k=1}^{\infty} k^2 e^{-k}$$

10. 
$$\sum_{n=1}^{\infty} n^2 e^{-n^3}$$

11. 
$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n \ln n}$$

12. 
$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 25}$$

13. 
$$\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$$

14. 
$$\sum_{n=1}^{\infty} \sin n$$

15. 
$$\sum_{n=0}^{\infty} \frac{n!}{2 \cdot 5 \cdot 8 \cdots (3n+2)}$$

16. 
$$\sum_{n=1}^{\infty} \frac{n^2 + 1}{n^3 + 1}$$

17. 
$$\sum_{n=1}^{\infty} (-1)^n 2^{1/n}$$

18. 
$$\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n} - 1}$$

19. 
$$\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$$

20. 
$$\sum_{k=1}^{\infty} \frac{k+5}{5^k}$$

21. 
$$\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$$

22. 
$$\sum_{n=1}^{\infty} \frac{\sqrt{n^2 - 1}}{n^3 + 2n^n + 5}$$

23. 
$$\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$$

24. 
$$\sum_{n=1}^{\infty} \frac{\cos(n/2)}{n^2 + 4n}$$

25. 
$$\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$$

26. 
$$\sum_{n=1}^{\infty} \frac{n^2 + 1}{5^n}$$

27. 
$$\sum_{n=1}^{\infty} \frac{k \ln k}{(k+1)^3}$$

28. 
$$\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$$

29. 
$$\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n\sqrt{n}}$$

30. 
$$\sum_{j=1}^{\infty} (-1)^j \frac{\sqrt{j}}{j+5}$$

31. 
$$\sum_{k=1}^{\infty} \frac{5^k}{3^k + 4^k}$$

32. 
$$\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$$

33. 
$$\sum_{n=1}^{\infty} \frac{\sin\left(\frac{1}{n}\right)}{\sqrt{n}}$$

34. 
$$\sum_{n=1}^{\infty} \frac{1}{n + n \cos^2 n}$$

35. 
$$\sum_{n=1}^{\infty} \left( \frac{n}{n+1} \right)^{n^2}$$

36. 
$$\sum_{n=1}^{\infty} \frac{1}{(\ln n)^{\ln n}}$$

37. 
$$\sum_{n=1}^{\infty} (\sqrt[3]{2} - 1)^n$$

38. 
$$\sum_{n=1}^{\infty} (\sqrt[3]{2} - 1)$$