Evaluate each of the following -- exact values only.

$\tan(0) - 6\sin\left(\frac{\pi}{2}\right)$

$0 - 6(1) = \boxed{-6}$

$\sin^2\left(\frac{2\pi}{3}\right) + \cos^2\left(\frac{2\pi}{3}\right)$

$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{-1}{2}\right)^2$

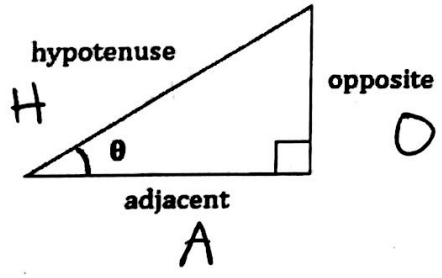
$= \frac{3}{4} + \frac{1}{4} = \boxed{1}$

$\sin^2\left(\frac{5\pi}{4}\right) - \cos^2\left(\frac{3\pi}{2}\right) + \tan\left(\frac{4\pi}{3}\right)$

$\left(-\frac{\sqrt{2}}{2}\right)^2 - 0^2 + \sqrt{3} = \boxed{\frac{1}{2} + \sqrt{3}}$

THE 6 TRIG FUNCTIONS

Recall SOH-CAH-TOA?



Reciprocals: "FLIPPED" Trig Functions

$\sin\theta = \frac{O}{H}$ $\quad \cos\theta = \frac{A}{H}$ $\quad \tan\theta = \frac{O}{A}$

$\csc\theta = \frac{H/O}{}$ "cosecant" $\quad \sec\theta = \frac{H/A}{}$ "secant" $\quad \cot\theta = \frac{A/O}{}$ "cotangent"

**** Let's find each Trig Function's "Happy Place" (in which quadrants they are each positive) and label our Unit Circle (back of agenda) with our findings. ****

Ex 1: Angle θ has a terminal side that passes through the point (3, 2). Find ALL 6 trig ratios for this angle. $2^2+3^2=c^2$

$\sin\theta = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$

$\cos\theta = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$

$\tan\theta = \frac{2}{3}$

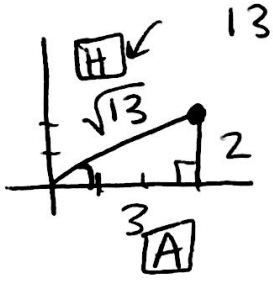
$\csc\theta = \frac{\sqrt{13}}{2}$

$\sec\theta = \frac{\sqrt{13}}{3}$

$\cot\theta = \frac{3}{2}$

FLIPS

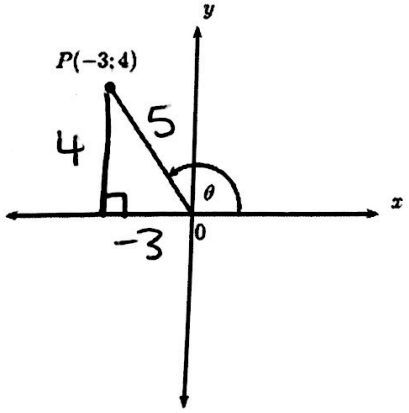
$13 = c^2$



change

Ex 2: Find the ~~three~~ trig values. $(\sin\theta, \cos\theta, \tan\theta)$

Sidebar: How *would* you find theta?



$\sin\theta = \frac{4}{5}$

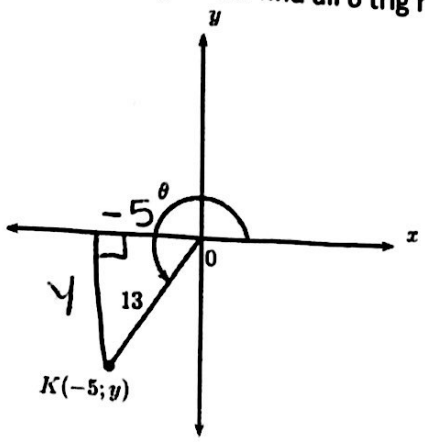
$\cos\theta = \frac{-3}{5}$

$\tan\theta = \frac{-4}{3}$

$\sin^{-1}(\sin\theta) = \sin^{-1}(4/5)$

$\theta = ??$

Ex 3: Find the value of y. Then find all 6 trig ratios.



$(-5)^2 + y^2 = 13^2$

$y^2 = 169 - 25$

$\pm\sqrt{y^2} = \pm\sqrt{144}$

$y = \ominus 12$

↑ QIII

$y = -12$

$\sin\theta = \frac{O}{H} = \frac{-12}{13}$

$\cos\theta = \frac{A}{H} = \frac{-5}{13}$

$\tan\theta = \frac{O}{A} = \frac{12}{5}$

$\csc\theta = \frac{H}{O} = \frac{-13}{12}$

$\sec\theta = \frac{H}{A} = \frac{-13}{5}$

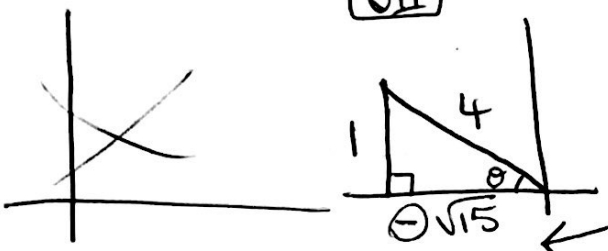
$\cot\theta = \frac{A}{O} = \frac{5}{12}$

Evaluate the following without using a calculator:

1. **Find cos θ and cot θ if sin θ = ¼ and tan θ < 0.** (+) (−)

~~I~~ II II IV

QII



$\sin\theta = \frac{1}{4}\ \frac{O}{H}$

$1^2 + A^2 = 4^2$

$A^2 = 16 - 1 = 15$

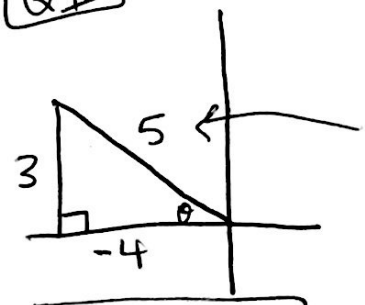
$\cos\theta = \frac{A}{H} = \frac{-\sqrt{15}}{4}$

$\cot\theta = \frac{-\sqrt{15}}{1}$

2. **Find sec θ and csc θ if cot θ = −4/3 and cos θ < 0.**

II IV II III

QII



$\cot\theta = \frac{-4}{3}\ \frac{A}{O}$

$3^2 + (-4)^2 = H^2$

$9 + 16 = H^2$

$25 = H^2$

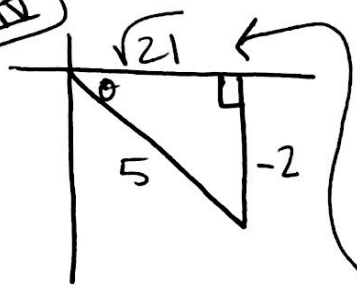
$\sec\theta = \frac{H}{A} = \frac{5}{-4}$

$\csc\theta = \frac{H}{O} = \frac{5}{3}$

3. **Find tan θ and sec θ if sin θ = −2/5 and cos θ > 0.**

III IV I IV

QIV



$\sin\theta = \frac{-2}{5}\ \frac{O}{H}$

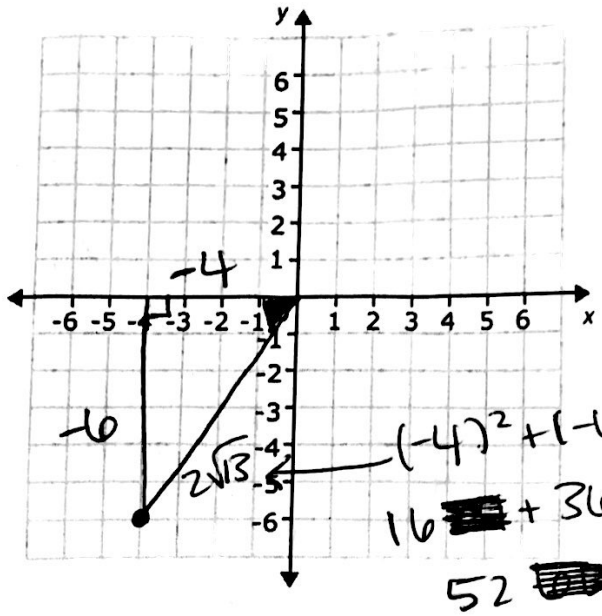
$A^2 + (-2)^2 = 5^2$

$A^2 = 25 - 4 = 21$

$\tan\theta = \frac{O}{A} = \frac{-2}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \frac{-2\sqrt{21}}{21}$

$\sec\theta = \frac{H}{A} = \frac{5}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$

1. Evaluate all six trig functions of the angle that terminates at the point (-4, -6) in the coordinate plane.



$(-4)^2+(-6)^2=H^2$

$16+36=H^2$

$52=H^2$

$2\sqrt{13}=H$

$\sin\theta=\frac{O}{H}=\frac{-6}{2\sqrt{13}}=\frac{-3\sqrt{13}}{13}$

$\cos\theta=\frac{A}{H}=\frac{-4}{2\sqrt{13}}=\frac{-2\sqrt{13}}{13}$

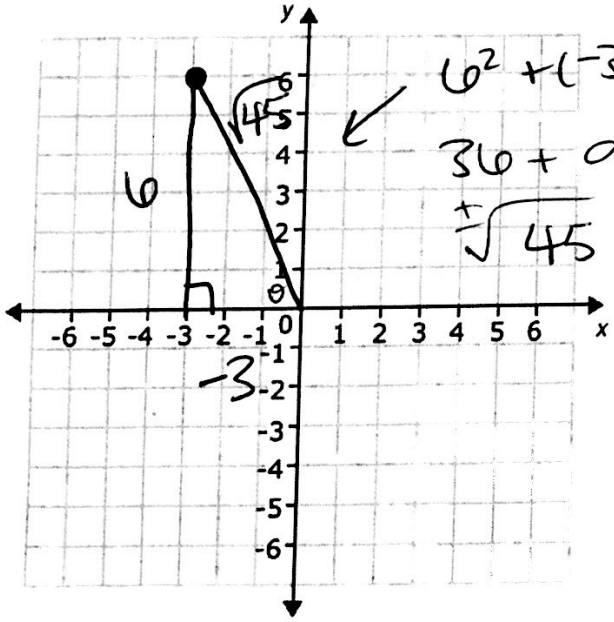
$\tan\theta=\frac{O}{A}=\frac{-6}{-4}=\frac{3}{2}$

$\csc\theta=\frac{H}{O}=\frac{2\sqrt{13}}{-6}=\frac{-\sqrt{13}}{3}$

$\sec\theta=\frac{H}{A}=\frac{2\sqrt{13}}{-4}=\frac{-\sqrt{13}}{2}$

$\cot\theta=\frac{O}{A}=\frac{2}{3}$

2. Evaluate all six trig functions of the angle that terminates at the (-3, 6) in the coordinate plane.



$6^2+(-3)^2=H^2$

$36+9=H^2$

$\pm\sqrt{45}=\pm\sqrt{H^2}$

$O=6$

$A=-3$

$H=\sqrt{45}=3\sqrt{5}$

$\sin\theta=\frac{6}{\sqrt{45}}=\frac{6\sqrt{45}}{45}=\frac{2\sqrt{5}}{5}$

$\cos\theta=\frac{-3}{3\sqrt{5}}=\frac{-\sqrt{5}}{5}$

$\tan\theta=\frac{6}{-3}=-2$

$\csc\theta=\frac{\sqrt{5}}{2}$

$\sec\theta=-\sqrt{5}$

$\cot\theta=\frac{-1}{2}$

Evaluating Trig Functions

If. calculator Active:

$\csc\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, $\cot\theta = \frac{1}{\tan\theta}$

Find the exact value of each trigonometric function.

1) $\csc -900°$

Undefined

2) $\cos -\frac{11\pi}{6}$

$\frac{\sqrt{3}}{2}$

3) $\csc -990°$

1

4) $\cot \frac{35\pi}{6}$

$-\sqrt{3}$

5) $\tan -675°$

1

6) $\sin 300°$

$-\frac{\sqrt{3}}{2}$

7) $\csc \frac{3\pi}{4}$

$\sqrt{2}$

8) $\sec \pi$

-1

9) $\sec 840°$

-2

10) $\sin 840°$

$\frac{\sqrt{3}}{2}$

11) $\cot -2\pi$

Undefined

12) $\cot -\frac{7\pi}{4}$

1

Solve each equation for $0 \le \theta < 2\pi$.

13) $\sec \theta = 2$

$\frac{\pi}{3}, \frac{5\pi}{3}$

14) $\cos \theta = -\frac{\sqrt{3}}{2}$

$\frac{5\pi}{6}, \frac{7\pi}{6}$

15) $-1 = \csc \theta$

$\frac{3\pi}{2}$

16) $\tan \theta = \sqrt{3}$

$\frac{\pi}{3}, \frac{4\pi}{3}$

17) $\sqrt{3} = \cot \theta$

$\frac{\pi}{6}, \frac{7\pi}{6}$

18) $\sin \theta = -\sqrt{2}$

NO Solution

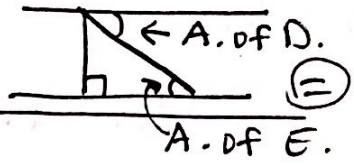
Angle of Elevation: measured from ground up, always INSIDE the triangle

Angle of Depression: measured from top looking down, OUTSIDE the triangle!

Name _____ Date _____

Angle of Elevation & Depression Trig Worksheet

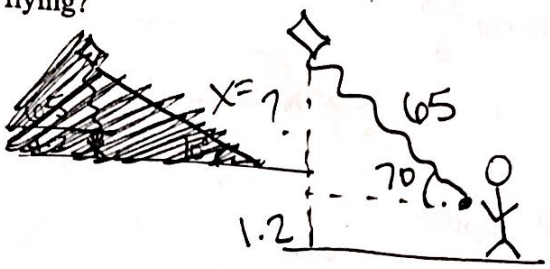
***Draw and label a picture for each problem**



* Calc. MODE = degrees!! *

S O/H
C A/H
T O/A

1. Brian's kite is flying above a field at the end of 65 m of string. If the angle of elevation to the kite measures 70°, and Brian is holding the kite 1.2 m off the ground. How high above the ground is the kite flying?



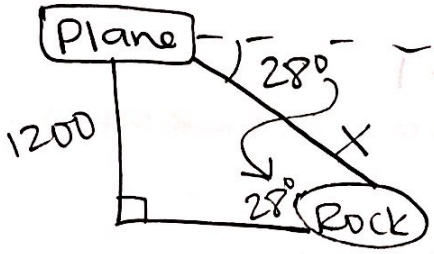
$\sin(70) = \frac{x}{65}$

$x = 65\sin(70)$

$x \approx \boxed{61.08\text{ m}}$

S O/H
C A/H
T O/A

2. From an airplane at an altitude (height) of 1200 m, the angle of depression to a rock on the ground measures 28°. Find the distance from the plane to the rock.

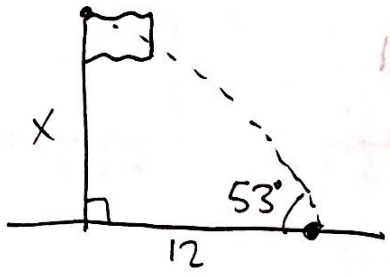


$\sin(28) = \frac{1200}{x}$

$x = \frac{1200}{\sin(28)} \approx \boxed{2556.07\text{ m}}$

S O/H
C A/H
T O/A

3. From a point on the ground 12 ft from the base of a flagpole, the angle of elevation of the top of the pole measures 53°. How tall is the flagpole?



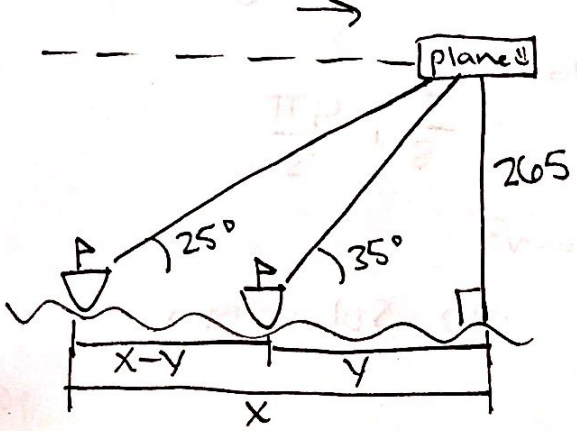
$\tan(53) = \frac{x}{12}$

$x = 12 \cdot \tan(53)$

$x \approx \boxed{15.92\text{ ft}}$

4. From a plane flying due east at 265 m above sea level, the angles of depression of two ships sailing due east measure 35° and 25 °. How far apart are the ships?

= ∠ of elev.



$\tan(35) = \frac{265}{y}$

$y = \frac{265}{\tan(35)} \approx 378.46\text{m}$

$\tan(25) = \frac{265}{x} \rightarrow x \approx 568.29$

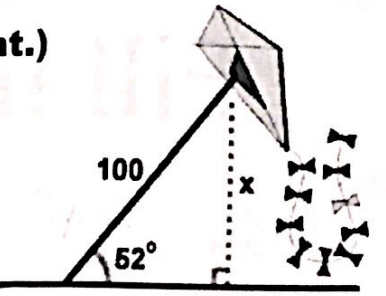
$x - y \approx \boxed{189.83\text{ m}}$

Name_____ Date_____

Angle of Elevation & Depression Worksheet (Cont.)

Find all values to the nearest tenth.

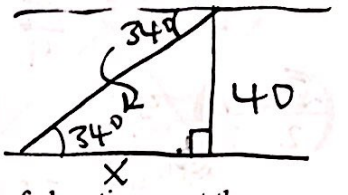
5. A man flies a kite with a 100 foot string. The angle of elevation of the string is 52°. How high off the ground is the kite?



$\sin(52) = \frac{x}{100}$

$x = 100 \cdot \sin(52) \approx$ 78.8 ft

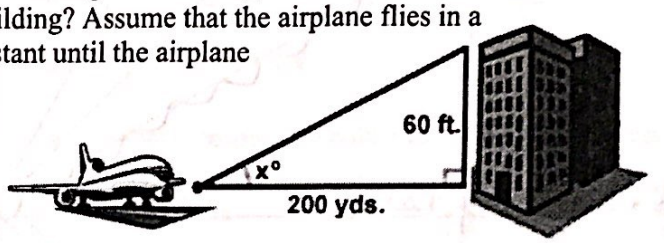
6. From the top of a vertical cliff 40 m high, the angle of depression of an object that is level with the base of the cliff is 34°. How far is the object from the base of the cliff?



$\tan(34) = \frac{40}{x}$

$x = \frac{40}{\tan(34)} \approx$ 59.3 m

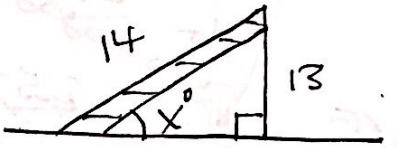
7. An airplane takes off 200 yards in front of a 60 foot building. At what angle of elevation must the plane take off in order to avoid crashing into the building? Assume that the airplane flies in a straight line and the angle of elevation remains constant until the airplane flies over the building.



$\tan x = \frac{60}{200}$

$x = \tan^{-1}(.3) \approx$ 16.7°

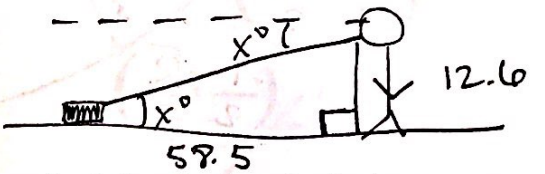
8. A 14 foot ladder is used to scale a 13 foot wall. At what angle of elevation must the ladder be situated in order to reach the top of the wall?



$\sin x = \frac{13}{14}$

$x = \sin^{-1}(.929\ldots) \approx$ 68.2°

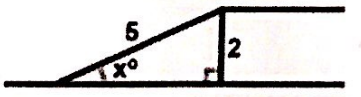
9. A person stands at the window of a building so that his eyes are 12.6 m above the level ground. An object is on the ground 58.5 m away from the building on a line directly beneath the person. Compute the angle of depression of the person's line of sight to the object on the ground.



$\tan(x) = \frac{12.6}{58.5}$

$x = \tan^{-1}(.215\ldots) \approx$ 12.15°

10. A ramp is needed to allow vehicles to climb a 2 foot wall. The angle of elevation in order for the vehicles to safely go up must be 30° or less, and the longest ramp available is 5 feet long. Can this ramp be used safely?



$\sin x = \frac{2}{5}$ yes!!

$x = \sin^{-1}(.4) \approx$ 23.6°