Unit 4 Test Review Name: Key 11

Pre-Calculus Honors *** = NO CALCULATOR**

1. Solve: $7^x < 8^x$ $\boxed{(0, \infty)}$

*2. Rewrite in exponential form: $\log_m(5x) = s \rightarrow \boxed{m^s = 5x}$

*3. Rewrite in logarithmic form & solve for w: $m - n = e^{5w-3} \rightarrow \log_e(m-n) = 5w-3$

$\boxed{w = \frac{\ln(m-n)+3}{5}}$

*4. Evaluate: $\log_3\left(\frac{1}{\sqrt[3]{81}}\right) = \log_3\left(\frac{1}{81^{1/3}}\right) = \log_3(81)^{-1/3}$

$= \log_3(3^4)^{-1/3}$

$= \cancel{\log_3}(3^{-4/3}) = \boxed{-4/3}$

*5. Evaluate: $e^{\ln 1347986}$ $\boxed{1347986}$!!

*6. Condense into a single logarithm:

a) $7\log(x^4) - 2\log(x^2 y)$

$\log\left(\frac{x^{28}}{x^4 y^2}\right) = \boxed{\log\left(\frac{x^{24}}{y^2}\right)}$

b) $-3\ln(ab) + 5\ln(ab)$

$\ln(a^{-3}b^{-3}a^5b^5) = \boxed{\ln(a^2b^2)}$

*7. Expand the logarithm: $\log_3 \frac{a^4 b^{3/2}}{(4c)^2} = \boxed{4\log_3 a + \underbrace{\tfrac{3}{2}\log_3 b}_{} - 2\log_3 4 - 2\log_3 c}$

or $3\log_3\sqrt{b}$

*8. Evaluate: $\log_4(-5)$ $\boxed{\text{undefined}}$

*9. Write the function of the form $f(x) = a \cdot b^x$ that goes through the points (0, 5) and (4, 20)

$x=0,\ y=5$
$5 = ab^0$
$5 = a$

$x = 4,\ y = 20$
$20 = 5b^4$
$\sqrt[4]{4} = b$

$f(x) = 5\left(\sqrt[4]{4}\right)^x$
$\boxed{f(x) = 5 \cdot 4^{x/4}}$

10. Given a population modeled by the function: $P(t) = \frac{2300}{1+4e^{-5t}}$.

a) Find the carrying capacity. $\boxed{c = 2300}$
b) What is the population at time 0? $\boxed{P(0) = 460}$
c) When will the population reach 2000? $\boxed{t \approx 0.657}$
d) What are the asymptotes for this function?

$\boxed{y = 0 \quad y = 2300}$

SOLVE & SHOW ALL NECESSARY WORK

11. $3^{2x-1} = 21$

$\log_3 21 = 2x - 1$
$2.77 = 2x - 1$
$\boxed{x \approx 1.886}$

*12. $2^{x+3} = 8^{2-x}$

$\cancel{8}^{x+3} = \cancel{8}^{3(2-x)}$
$x + 3 = 6 - 3x$
$4x = 3$
$\boxed{x = 3/4}$

*13. $\log_4 x = 3$

$x = 4^3$
$\boxed{x = 64}$

*14. $\log_2 \frac{1}{32} = x$

$2^x = \frac{1}{32}$
$2^x = 2^{-5}$
$\rightarrow \boxed{x = -5}$

*15. $\log(x-9) + \log x = 1$

$\log(x^2 - 9x) = 1$
$10^1 = x^2 - 9x$
$0 = x^2 - 9x - 10$
$0 = (x-10)(x+1)$
$\boxed{x = 10}$

$\cancel{x = -1}$
extraneous

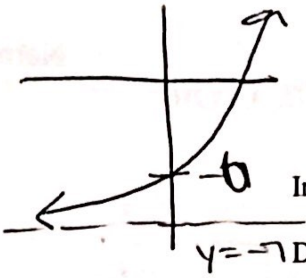
*16. $\log 2x = \log(12x^4 - 6x^2) - \log(3x^2)$

$\cancel{\log} 2x = \cancel{\log}(4x^2 - 2)$
$0 = 4x^2 - 2x - 2$
$0 = 2(2x^2 - x - 1)$
$0 = 2(x-1)(2x+1)$
$\boxed{x = 1}$

$\cancel{x = -1/2}$
extraneous

Sketch a graph to help in your answer.

17. Analyze $f(x) = 3^x - 7$



D: $(-\infty, \infty)$

R: $(-7, \infty)$

VA: —

HA/EBA: $y=-7$

x – intercept(s): $(1.77, 0)$

y – intercept: $(0, -6)$

Extrema: —

Increasing: $(-\infty, \infty)$

Decreasing: —

Even/Odd: neither

Boundedness: below

Continuity: yes

End Behavior: $\lim_{x\to-\infty} f(x) = -7$

$\lim_{x\to\infty} f(x) = \infty$

18. Evaluate: $\log_7 68$

$\boxed{2.168}$

*19. Simplify: $\ln e^w$

$\boxed{w}$

20. The number of North Carolina cows infected with the mad cow disease after t days is modeled by the function $P(t) = \frac{578}{1+46e^{-.43t}}$. When will the number of cows be 473? SHOW ALL WORK.

$\frac{473}{1} = \frac{578}{1+46e^{-.43t}}$

$1+46e^{-.43t} = \frac{578}{473}$

$-1 \qquad -1$

$\frac{46e^{-.43t}}{46} = \frac{.22199..}{46}$

$\frac{-.43t}{-.43} = \frac{\ln(.0048)}{-.43}$

$\rightarrow t \approx \boxed{12.4 \text{ days}}$

21. Use the data below to find the ***exponential*** regression. Predict the population in Punxsutawney for 2015.

u	Year	Punxsutawney Population
0	1935	980
10	1945	1040
33	1968	1178
44	1979	1253
58	1993	1355
67	2002	1423

a) Population function:

$\boxed{f(x) = 981.61(1.0056)^x}$

b) Population in 2015: $(x=80)$

$\approx \boxed{1530 \text{ people}}$

22. A virus spreads according to $N = N_0e^{0.0345t}$ where time is measured in days. If <u>10 people are currently infected</u> (N_0), how long does it take for <u>100 people to be infected?</u> (N) SHOW ALL WORK.

$100 = 10e^{.0345t}$

$\ln 10 = \ln e^{.0345t}$

$\frac{\ln 10}{.0345} = \frac{.0345t}{.0345}$

$\rightarrow t \approx \boxed{66.7 \text{ days}}$

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C: surrounding

23. Use Newton's Law of Cooling: $T(t) = T_m + (T_0 - T_m)\,e^{-kt}$; A 6 pack of Coca Cola at room temperature (72°F) is placed in a fridge (36°F). After 23 minutes it's temperature is down to 60°F. $\boxed{\text{How long}}$ will it take the soda to cool to 38°F?

T_o

T_m t T_{23} $t=?$ T

① Find k:

$$60 = 36 + (72-36)e^{-23k}$$

$\boxed{\text{STO}} \rightarrow k \approx 0.018\ldots$

② Find t:

STORED @ X

$$38 = 36 + (72-36)e^{-kt}$$

$$t \approx \boxed{163.96 \text{ minutes}}$$

24. Use Newton's Law of Cooling: $T(t) = T_m + (T_0 - T_m)\,e^{-kt}$; A cake comes out of a 350°F oven and is set in a 70°F room. After 10 minutes it's temperature has cooled to 280°F. How long will it take the cake to cool to 120°F?

$T_o =$ $C =$ $T =$

① Find k:

$$280 = 70 + (350-70)e^{-k(10)}$$

$\boxed{\text{STO}} \rightarrow k \approx 0.0288\ldots$

② $t = ?$ $T_{Final} = 120$

$$120 = 70 + (350-70)\,e^{-k\boxed{t}}$$

$$t \approx \boxed{59.88 \text{ minutes}}$$

25. In 1995, Ms. Murphy deposited \$2000 into an account at State Employees Credit Union that paid 4.7% interest compounded monthly.

$P=$ $n=12$ $1995 \rightarrow 2017$ (22 years) $r = .047$

a) How much money does she have in that account now?

$A = ?$ $A = 2000\left(1 + \frac{.047}{12}\right)^{12(22)} = \boxed{\$5613.24}$

b) What if it were compounded weekly instead?

$n=52$ $A = \boxed{\$5621.96}$

c) What interest rate (still compounded monthly) would she need to have received in order to have \$10,000 in the account now?

$r = ?$ $n=12$ $A = 10000$

$$10000 = 2000\left(1 + \frac{r}{12}\right)^{264}$$

$$\rightarrow r = 0.073 \rightarrow \boxed{7.3\%}$$

26. David contributes \$50 per month into a Hoffbrau Fund that earns 15.5% annual interest. What is the value of his investment after 20 years?

PMT $t=20$ $I=15.5$ $n=1$ $FV=?$ $PV=0$

$$FV = \boxed{\$5435.50}$$

27. Find Ms. Hutchin's quarterly payment if she wants to save \$30,000 for her son's college fund that he will need in eighteen years. Her account will pay 5.3% compounded quarterly.

$n=4$ $PMT=?$ $FV=?$ $PV=0$ $t=18$ $I=5.3$

$$PMT = \boxed{\$251.60}$$

28. What is Carly's monthly $\boxed{\text{payment}}$ for a 3-year \$9000 car loan with a monthly compounded interest rate of 10.25% from BB&T?

$PMT=?$ $n=12$ $t=3$ $PV=9000$ $FV=0$ $I=10.25$

$$PMT = \boxed{\$291.46}$$