BC Calculus – L'Hospital's Rule **NAME:** _____

Given that

$\lim_{x\to a} f(x)=0$ $\lim_{x\to a} g(x)=0$ $\lim_{x\to a} h(x)=1$ $\lim_{x\to a} p(x)=\infty$ $\lim_{x\to a} q(x)=\infty$

Which of the following limits are indeterminate forms? For those that are not an indeterminate form, evaluate the limit where possible.

1. a) $\lim_{x\to a}\frac{f(x)}{g(x)}$ ind. b) $\lim_{x\to a}\frac{f(x)}{p(x)}$ 0 c) $\lim_{x\to a}\frac{h(x)}{p(x)}$ 0 d) $\lim_{x\to a}\frac{p(x)}{f(x)}$ ind. e) $\lim_{x\to a}\frac{p(x)}{q(x)}$ ind.

2. a) $\lim_{x\to a}[f(x)p(x)]$ ind. b) $\lim_{x\to a}[h(x)p(x)]$ ∞ c) $\lim_{x\to a}[p(x)q(x)]$ ∞

3. a) $\lim_{x\to a}[f(x)-p(x)]$ $0-\infty$ b) $\lim_{x\to a}[p(x)-q(x)]$ ind. c) $\lim_{x\to a}[p(x)+q(x)]$ ∞

4. a) $\lim_{x\to a}[f(x)]^{g(x)}$ ind. b) $\lim_{x\to a}[f(x)]^{p(x)}$ 0 c) $\lim_{x\to a}[h(x)]^{p(x)}$ ind. d) $\lim_{x\to a}[p(x)]^{f(x)}$ ind. e) $\lim_{x\to a}[p(x)]^{q(x)}$ ∞ f) $\lim_{x\to a}\sqrt[q(x)]{p(x)}$ ∞

Find the limit. Use l'Hospital's rule if needed.

5. $\lim_{x\to -1}\frac{x^2-1}{x+1}$ -2 6. $\lim_{x\to -2}\frac{x+2}{x^2+3x+2}$ -1 7. $\lim_{x\to 1}\frac{x^9-1}{x^5-1}$ $\frac{9}{5}$ 8. $\lim_{x\to 1}\frac{x^a-1}{x^b-1}$ $\frac{a}{b}$ 9. $\lim_{x\to(\pi/2)^+}\frac{\cos x}{1-\sin x}$ $-\infty$

10. $\lim_{x\to 0}\frac{x+\tan x}{\sin x}$ 2 11. $\lim_{t\to 0}\frac{e^t-1}{t^3}$ ∞ 12. $\lim_{t\to 0}\frac{e^{3t}-1}{t}$ 3 13. $\lim_{x\to 0}\frac{\tan px}{\tan qx}$ $\frac{p}{q}$ 14. $\lim_{\theta\to\pi/2}\frac{1-\sin\theta}{\csc\theta}$ 1

15. $\lim_{x\to\infty}\frac{\ln x}{x}$ 0 16. $\lim_{x\to\infty}\frac{e^x}{x}$ 0 17. $\lim_{x\to 0^+}\frac{\ln x}{x}$ $-\infty$ 18. $\lim_{x\to\infty}\frac{\ln\ln x}{x}$ 0 19. $\lim_{t\to 0}\frac{5^t-3^t}{t}$ $\ln\left(\frac{5}{3}\right)$

20. $\lim_{x\to 1}\frac{\ln x}{\sin\pi x}$ $\frac{-1}{\pi}$ 21. $\lim_{x\to 0}\frac{e^x-1-x}{x^2}$ $\frac{1}{2}$ 22. $\lim_{x\to 0}\frac{e^x-1-x-(x^2/2)}{x^3}$ 23. $\lim_{x\to\infty}\frac{e^x}{x^3}$ ∞ 24. $\lim_{x\to 0}\frac{\sin x}{\sinh x}$ 1

25. $\lim_{x\to 0}\frac{\sin^{-1}x}{x}$ 1 26. $\lim_{x\to 0}\frac{\sin x-x}{x^3}$ $\frac{-1}{6}$ 27. $\lim_{x\to 0}\frac{1-\cos x}{x^2}$ $\frac{1}{2}$ 28. $\lim_{x\to\infty}\frac{(\ln x)^2}{x}$ ∞ 29. $\lim_{x\to 0}\frac{x+\sin x}{x+\cos x}$ 0

30. $\lim_{x\to 0}\frac{\cos mx-\cos nx}{x^2}$ $\frac{n^2-m^2}{2}$ 31. $\lim_{x\to\infty}\frac{x}{\ln(1+2e^x)}$ 1 32. $\lim_{x\to 0}\frac{x}{\tan^{-1}(4x)}$ 1 33. $\lim_{x\to 1}\frac{1-x+\ln x}{1+\cos\pi x}$ $\frac{-1}{\pi^2}$ 34. $\lim_{x\to\infty}\frac{\sqrt{x^2+2}}{\sqrt{2x^2+1}}$ $\frac{1}{2}$

35. $\lim_{x\to 1}\frac{x^a-ax+a-1}{(x-1)^2}$ $\frac{a(a-1)}{2}$ 36. $\lim_{x\to 0}\frac{1-e^{-2x}}{\sec x}$ ∞ 37. $\lim_{x\to 0^+}\sqrt{x}\ln x$ 0 38. $\lim_{x\to-\infty}x^2e^x$

AP Calculus BC - Improper Integrals

Keep in mind that any integration technique is fair game. Keep u-Substitution, Integration by Parts, Integration with Partial Fractions, and Inverse Trig functions in mind.

1. $\int_3^\infty \frac{4}{x^5}dx =$

$(A)\frac{1}{81}$ $(B)\frac{4}{81}$ $(C)\frac{1}{27}$ $(D)\frac{2}{9}$ $(E)81$

2. $\int_3^\infty \frac{dx}{x^3} =$

$(A)\frac{1}{27}$ $(B)\frac{1}{18}$ $(C)\frac{1}{9}$ $(D)9$ $(E)18$

3. $\int_2^\infty \frac{\pi dx}{x^4} =$

$(A)\frac{\pi}{24}$ $(B)\frac{\pi}{12}$ $(C)\pi$ $(D)3\pi$ $(E)16\pi$

4. $\int_0^\infty e^{-3x}dx =$

$(A)\frac{2}{3}$ $(B)\frac{1}{3}$ $(C)\frac{e}{3}$ $(D)\frac{3}{e}$ $(E)\frac{1}{e}$

5. $\int_{-\infty}^0 e^{4x}dx =$

$(A)\frac{2}{e}$ $(B)\frac{1}{4}$ $(C)\frac{e}{4}$ $(D)\frac{4}{e}$ $(E)\frac{1}{\sqrt{e}}$

6. $\int_{-\infty}^{-1} \frac{1}{x^5} =$

$(A)\frac{-2}{e}$ $(B)\frac{-1}{4}$ $(C)\frac{e}{5}$ $(D)\frac{5}{e}$ $(E)\frac{5}{\sqrt{e}}$

For #7-10: Determine whether or not the following integrals converge or diverge. Evaluate the integral if it converges.

7. $\int_1^\infty \frac{1}{x^{\frac{2}{3}}}dx$

(A) converges, 1
(B) converges, $\frac{1}{2}$
(C) converges, $\frac{2}{3}$
(D) converges, $\frac{1}{3}$
(E) diverges

8. $\int_4^\infty \frac{1}{x^2-1}dx$

(A) Converges, 1
(B) converges, $\frac{1}{2}$
(C) converges, $\ln\frac{5}{3}$
(D) converges, $\frac{1}{2}\ln\frac{5}{3}$
(E) diverges

9. $\int_0^\infty e^{-3x}dx$

(A) converges, 1
(B) converges, 2
(C) converges, 3
(D) converges, $\frac{1}{3}$
(E) diverges

10. $\int_1^\infty \frac{\ln x}{x}dx$

(A) diverges
(B) converges, e
(C) converges, e^2
(D) converges, $\frac{e}{2}$
(E) converges, $\frac{1}{e}$