3.5 EQUATION SOLVING AND MODELING

Learning Targets:
1. Solve exponential and logarithmic equations.

When you solve an equation, you "undo" what has been done ... addition to undo subtraction, multiplication to undo division. Since exponents and logarithms are inverses of each other, it follows that in order to solve a logarithmic equation, you can write it as an exponent to "undo" the logarithm, and if you are solving for an exponent, you write the equation as a logarithm.

NOTE: You can only switch between exponential and logarithmic forms when you have $\log_b y = x$ or $b^x = y$

① Isolate ② Rewrite ③ Solve ④ CYA

Example 1: Solve the following equations:

a) $e^{x-2} = 19$ (ln ln)

$x - 2 = \ln 19$

$x = \ln 19 + 2$

$x =$

b) $3 - 5(2)^{3x} = -14$

$-5(2)^{3x} = -17$

$2^{3x} = \frac{17}{5}$

$\frac{\log_2(17/5)}{3} = \frac{3x}{3}$

$x =$

c) $\log_2(x) = 4$

$2^4 = x$

$\boxed{x = 16}$

d) $7 - 3\log(x) = -5$

$-3\log x = -12$

$\frac{-3}{-3}$

$\log x = 4$

$10^4 = x$

$\boxed{x = 10000}$

e) $\log x + \log(x+21) = 2$

$\log(x(x+21)) = 2$

$\log(x^2 + 21x) = 2$

$10^2 = x^2 + 21x$

$0 = x^2 + 21x - 100$

$0 = (x-4)(x+25)$

$\boxed{x = 4}$ $x = -25$ ↑ not allowed!!

f) $\log_3(x+4) - \log_3(x-5) = 2$

$\log_3\left(\frac{x+4}{x-5}\right) = 2$

$3^2 = \frac{x+4}{x-5}$

$9(x-5) = x+4$

$9x - 45 = x + 4$

$8x - 49 = 0$

$8x = 49$

$\boxed{x = \frac{49}{8}}$

Newton's Law of Cooling

Use Newton's Law of cooling to find the answer to the scenario described below.

$$T = C + (T_0 - C)e^{-kt}$$

What do each of the variables represent?

T = Final temperature

C = Surrounding temperature (constant)

T_0 = Initial temperature

k = Negative constant associated with the cooling object

t = Time (typically in minutes)

Example: *At 9:00 a.m., a coroner arrived at the home of a person who had died during the night. The temperature of the room was 70° F, and it is assumed that at the time of death the person had a body temperature of 98.6° F. The coroner took the body's temperature at 9:30 a.m., at which time it was 85.6° F, and again at 10:00 a.m., when it was 82.7° F. At what time did the person die?* $T(?) = 98.6°F$

$T = 82.7$ (10:00)

$C = 70$

$T_0 = 85.6$ (9:30)

$k = ?$

$t = 30$ (min.)

① Find k first!

$82.7 = 70 + (85.6 - 70)e^{-k(30)}$

$-70 \quad -70$

$\frac{12.7}{15.6} = \frac{(85.6-70)e^{-k(30)}}{15.6}$

$\ln\left(\frac{12.7}{15.6}\right) = \ln e^{-k(30)}$

$\frac{\ln\left(\frac{12.7}{15.6}\right)}{-30} = \frac{-k(30)}{-30} \rightarrow k = 0.00686$

② Now find t when $T_0 = 98.6$!

$82.7 = 70 + (98.6 - 70)e^{-kt}$

$12.7 = 28.6e^{-kt}$

$\ln\left(\frac{12.7}{28.6}\right) = \ln e^{-kt}$

$\frac{\ln\left(\frac{12.7}{28.6}\right)}{-k} = t \rightarrow t = 118$ minutes ago

From 10:00 am → 8:02 am

$$T = C + (T_0 - C)e^{-kt}$$

2. Teresa was late getting ready for a party, and the liter of soft drinks she bought were still at room temperature (73 F) with guests due to arrive in 15 minutes. If she puts these in her freezer at – 10 F, will the drinks be cold enough (35 F) for her guests? Assume k = 0.031

$T = 35$??
$C = -10$
$T_0 = 73$
$k = 0.031$
$t = 15$

$-10 + (73 - -10)e^{-0.031(15)}$
$= 42.1^\circ F$
$\therefore$ NOT cold enough!!

Follow up Q: How long should Thersa leave the drinks in the freezer!

$35 = -10 + (73+10)e^{-0.031t} \rightarrow t \approx$ **20 min.**

3. A pizza is taken from a 425 F (T_0) oven and placed on the counter to cool. If the temperature in the kitchen is 75 F (C) and the cooling rate for this type of pizza is $k = 0.35$,

a) What is the temperature (to the nearest degree of the pizza 2 minutes later? ($t = 2$)

$T = 75 + (425 - 75)e^{-0.35(2)} \approx$ **249°F**

b) To the nearest minute, how long until the pizza has cooled to a temperature below 90 F? ($T = 90$)

$90 = 75 + (425 - 75)e^{-0.35t}$

$t \approx$ **9 minutes**

c) If Matt and Tyler like to eat their pizza at a temperature of about 110 F, how many minutes should they wait to dig in? ($T = 110$)

$110 = 75 + (425 - 75)e^{-0.35t}$

$t \approx$ **6.5 minutes**