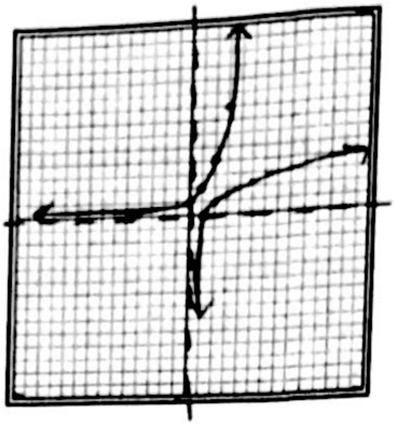
Logarithmic Functions

$\log_b x = \frac{\log x}{\log b}$

Make a table & then graph these 2 functions:

$y_1 = 2^x$		$y_2 = \log_2 x$		
x	y	x	y	
0	1	0	Error	
1	2	1	0	
2	4	2	1	
3	8	3	1.59	$\log_2 3$
5	32	5	2.32	$\log_2 5$



So 2^x and $\log_2 x$ are INVERSES!!

A **LOGARITHM** is an exponent

If $y = b^x$, then $\log_b y = x$.

Note: 1. You can't take the log of 0 or a negative number ($y \geq 0$) ✱✱✱
2. $b \neq 1$ $b > 0$

Ex) $\log_2 16 = x \rightarrow 2^x = 16 \rightarrow \boxed{x = 4}$ "2 times itself ? times will equal 16??"

Ex) $\log_5 \sqrt[3]{25} = x \rightarrow 5^x = \sqrt[3]{25} \rightarrow 5^x = 25^{1/3} \rightarrow 5^x = (5)^{2(1/3)} \rightarrow \boxed{x = 2/3}$

Writing in different forms:

Exponential Form	Logarithmic Form
$2^3 = 8$	$\log_2 8 = 3$
$4^2 = 16$	$\log_4 16 = 2$
$5^{-2} = 1/25$	$\log_5 (1/25) = -2$
$3^{1/2} = \sqrt{3}$	$\log_3 (\sqrt{3}) = 1/2$

Basic Properties of Logarithms

For $y \geq 0$, $b \neq 1$, $b > 0$, for any real number x...	
1. $\log_b 1 = 0$ because	$b^0 = 1$
2. $\log_b b = 1$ because	$b^1 = b$
3. $\log_b b^y = y$ because	$b^y = b^y$
4. $b^{\log_b x} = x$ because	$\log_b x = \log_b x$

Ex) $\cancel{6}^{\cancel{\log_6} 11} =$ 11

Ex) $\cancel{\log_8} \cancel{8}^f =$ f

A common logarithm is a log in base 10.

If $y = 10^x$, then $\log y = x$ (same as $\log_{10} y = x$).

Basic Properties of Common Logarithms

Let x & y be real numbers with x>0...
1. $\log_{10} 1 = 0$ because $10^0 = 1$
2. $\log_{10} 10 = 1$ because $10^1 = 10$
3. $\log_{10} 10^y = y$ because $10^y = 10^y$
4. $10^{\log_{10} x} = x$ because $\log_{10} x = \log_{10} x$

Ex) $\log \sqrt[5]{10} = \cancel{\log_{10}} (\cancel{10})^{1/5} = 1/5$

Ex) $\log \frac{1}{1000} = \cancel{\log_{10}(10)}^{-3} = -3$

*These can also be done easily in the calculator.

use the $\boxed{\text{LOG}}$ button :)

A natural logarithm is a log in base "e".

If $y = e^x$, then $\ln y = x$. $\log_e y = x$

Basic Properties of Natural Logarithms

Let x & y be real numbers with x>0...
1. $\ln 1 = 0$ because $e^0 = 1$
2. $\ln e = 1$ because $e^1 = e$
3. $\ln e^y = y$ because $e^y = e^y$
4. $e^{\ln x} = x$ because $\log_e x = \ln x$

Ex) $\ln \sqrt{e} = \cancel{\log_e e}^{1/2} = 1/2$

Ex) $\ln e^5 = \cancel{\log_e e}^5 = 5$

Ex) $e^{\ln 4} =$ $\cancel{e^{\log_e}} 4 = 4$

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Properties of Logarithmic Functions

Properties of Logarithms

M,N, b are positive numbers where b≠1...
1. Product Rule: $\log_b(MN) = \log_b M + \log_b N$
2. Quotient Rule: $\log_b(M/N) = \log_b M - \log_b N$
3. Power Rule: $\log_b M^N = N \cdot \log_b M$

Expanding Logs: "Bring everything to the 'ground' and separate!"

EX) $\log(xy^3)$

$= \log x + \log y^3$

$= \boxed{\log x + 3\log y}$

EX) $\ln\left(\frac{x^2 \sqrt[3]{q}}{y\sqrt{t}}\right)$

$= \ln(x^2) + \ln(q^{1/3}) - \ln(y) - \ln(t^{1/2})$

$= \boxed{2\ln x + \frac{1}{3}\ln q - \ln y - \frac{1}{2}\ln t}$

Condensing Logs: "Send to the sky and write 'log' ONCE only!"

EX) $3\ln x - \frac{1}{2}\ln y + 5\ln z$

$= \ln(x^3) - \ln(y^{1/2}) + \ln(z^5)$

$= \boxed{\ln\left(\frac{x^3 z^5}{\sqrt{y}}\right)}$ $\uparrow$ $y^{1/2} = \sqrt{y}$

EX) $\frac{4}{3}\log_2 27 - 2\log_2 9$

$= \log_2 \sqrt[3]{27^4} - \log_2 9^2$

$= \log_2\left(\frac{81}{81}\right) = \boxed{\log_2 1}$

Your calculator (unless you have the new TI-84+ operating system) will only evaluate a logarithm with base 10 or *e*. If you need to evaluate a logarithm with a different with a different base........

To change base (for those not in common log):
$\log_b x = \frac{\log x}{\log b}$ *and* $\log_b x = \frac{\ln x}{\ln b}$

More examples:

$\log_7 5$

$= \frac{\log 5}{\log 7}$

$= \boxed{0.827}$

$\log_{0.4} 8$

$= \frac{\ln 8}{\ln 0.4}$

$= \boxed{-2.269}$