Logistic Functions:

no "is"

Do you think it is reasonable for a populations to grow exponentially indefinitely? NO!

Logistic Growth Functions ... functions that model situations where exponential growth is limited.

An equation of the form $f(x) = \frac{c}{1+ab^x}$ or $f(x) = \frac{c}{1+ae^{-kx}}$.

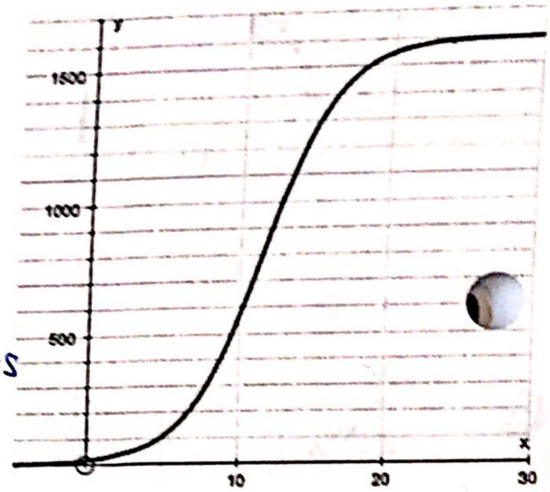
where c = growth limit.

The graph of a logistic function looks like an exponential function at first, but then "levels off" at $y = c$. The logistic function has two HA: $y = 0$ and $y = c$.

Example of modeling with the logistic function:

The number of students infected with flu after t days at Springfield High School is modeled by the following function:

$$P(t) = \frac{1600}{1+99e^{-0.4t}}$$



a) What was the initial number of infected students t = 0?

$P(0) = \frac{1600}{1+99e^{-0.4(0)}} = \frac{1600}{1+99} = \frac{1600}{100} = \boxed{16}$

b) After 5 days, how many students will be infected?

$P(5) = 111.125 \rightarrow$ about $\boxed{111}$ students

c) What is the maximum number of students that will be infected?

$c = \boxed{1600}$ students

d) According to this model, when will the number of students infected be 800?

$800 = \frac{1600}{1+99e^{-0.4t}} \rightarrow 1+99e^{-0.4t} = \frac{1600}{800} \rightarrow 1+99e^{-0.4t} = 2$

$99e^{-0.4t} = 1$

$\ln(e^{-0.4t}) = \ln(1/99)$

$\frac{-0.4t}{-0.4} = \frac{\ln(1/99)}{-0.4}$

$\boxed{t = 11.49 \text{ days}}$

Analyzing Logistic Functions:

$$f(x) = \frac{9}{1+2(0.6)^x}$$

D: $(-\infty, \infty)$

R: $(0, 9)$

Inc.: $(-\infty, \infty)$

Dec.: —

Boundedness: Above/Below

Extrema: none

Asy.: $y=0$, $y=9$

End Behavior: L: 0, R: 9

$$f(x) = \frac{8}{1+4e^{-x}}$$

D: $(-\infty, \infty)$ R: $(0, 8)$

Inc.: $(-\infty, \infty)$ Dec.: —

Bounded (above + below)

Extrema: none

Asy.: $y=0$, $y=8$

End Behavior:

$\lim_{x\to-\infty} f(x) = 0$ $\lim_{x\to\infty} f(x) = 8$

MODELING CONT. $\boxed{y=a(1\pm r)^t}$

Example 1: The population of Glenbrook in the year 1910 was 4200. Assume the population increased at the rate of 2.25% per year.

$t=0$ $a=4200$ $b>1$ growth

$r=0.0225$

a) Write an exponential model for the population of Glenbrook. Define your variables.

y= population t=years after 1910 $\boxed{y=4200(1+0.0225)^t}$

b) Determine the population in 1930 and 1900.

$t=20$ $t=-10$ $\boxed{f(20)=6554}$ $\boxed{f(-10)=3362}$

c) Determine when the population is double the original amount.

$\frac{8400}{4200}=(1+0.0225)^t \rightarrow 2=(1+.0225)^t$ * look at table * $y=2(4200)=8400$

Between $t=31+32$ $\boxed{1941-1942}$

Example 2: The half-life of a certain radioactive substance is 14 days. There are 10 grams present initially.

$b=1/2$ $t \div 14$ $a=10$

a) Express the amount of substance remaining as an exponential function of time. Define your variables.

y: amount of substance remaining

t: days

$\boxed{y=10(0.5)^{t/14}}$

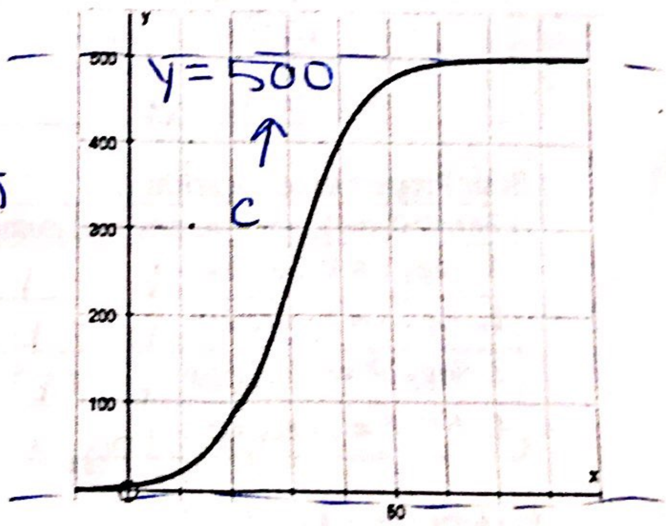
b) When will there be less than 1 gram remaining?

Let $y_1 = 10(.5)^\wedge(t/14)$ $t=47$ $\boxed{\text{After 46 days}}$

Table:

X	y_1
46	1.0254
47	0.9759

Example 3: Find a logistic equation of the form $y=\frac{c}{1+ae^{-bx}}$ that fits the graph below, if the y-intercept is (0, 5) and the point (24, 135) is on the curve. * Replace ALL constants *



$(0,5)$

$5=\frac{500}{1+ae^{-k(0)}}$

$5=\frac{500}{1+a}$

$\frac{5(1+a)}{5}=\frac{500}{5}$

~~5+5a=500~~

$1+a=100$

$a=99$

$(24,135)$

$135=\frac{500}{1+99e^{-k(24)}}$

$\ldots k=0.15$

$$\boxed{y=\frac{500}{1+99e^{-0.15x}}}$$