Day 1 Homework

In Exercises 1-4, use graphs and tables to find (a) $\lim_{x\to\infty} f(x)$, (b) $\lim_{x\to-\infty} f(x)$, and

(c) Identify all horizontal asymptotes.

1. $f(x) = \cos\left(\frac{1}{x}\right)$

a) 1
b) 1
c) $y=1$

2. $f(x) = \frac{\sin 2x}{x}$

a) 2
b) 2
c) $y=2$

3. $f(x) = \frac{e^{-x}}{x}$

a) 0
b) 0
c) $y=0$

4. $f(x) = \frac{3x^3 - x + 1}{x+3}$

a) ∞
b) ∞
c) $\varnothing$

In Exercises 5-10, find $\lim_{x\to\infty} y$ and $\lim_{x\to-\infty} y$.

5. $y = \left(2 - \frac{x}{x+1}\right)\left(\frac{x^2}{5+x^2}\right)$

a) $(2-1)(1) = \boxed{1}$
b) $(2+1)(1) = \boxed{3}$

6. $y = \left(\frac{2}{x} + 1\right)\left(\frac{5x^2 - 1}{x^2}\right)$

a) $(1)(5) = \boxed{5}$
b) $(1)(5) = \boxed{5}$

7. $y = \frac{\cos(1/x)}{1 + (1/x)}$

a) $\frac{1}{1} = \boxed{1}$
b) $\frac{1}{1} = \boxed{1}$

8. $y = \frac{2x + \sin x}{x}$

a) $2+0 = \boxed{2}$
b) $2+0 = \boxed{2}$

9. $y = \frac{\sin x}{2x^2 + x}$

a) $0 \cdot 0 = \boxed{0}$
b) $0 \cdot 0 = \boxed{0}$

10. $y = \frac{x \sin x + 2\sin x}{2x^2} = \frac{\sin x\,(x+2)}{2x^2} = \left(\frac{\sin x}{x}\right)\left(\frac{x+2}{2x}\right)$

a) $(0)(1/2) = \boxed{0}$
b) $(0)(1/2) = \boxed{0}$

Evaluate the following integrals (disregard numbering…).

10. $\int \sec^2(x+2)\,dx$

$u = x+2$
$du = dx$
$\boxed{\tan(x+2) + C}$

11) $\int \sqrt{\tan x}\,\sec^2 x\,dx$

$u = \tan x$
$du = \sec^2 x\,dx$
$\boxed{\frac{2}{3}\sqrt{\tan^3 x} + C}$

17) $\int \frac{\ln^6 x}{x}\,dx$

$u = \ln x$
$du = dx/x$
$\boxed{\frac{1}{7}\ln^7 x + C}$

18. $\int \tan^7 \frac{x}{2} \sec^2 \frac{x}{2}\,dx$

$u = \tan(x/2)$
$du = \frac{1}{2}\sec^2(x/2)\,dx$
$\boxed{\frac{1}{4}\tan^8(x/2) + C}$

19) $\int s^{\frac{1}{3}} \cos(s^{\frac{4}{3}} - 8)\,ds$

$u = x^{4/3} - 8$
$du = 4/3\, x^{1/3}\,dx$
$\boxed{\frac{3}{4}\sin(x^{4/3} - 8) + C}$

26) $\int_0^5 \frac{40\,dx}{x^2 + 25}$

$= 40\tan^{-1}(5)\Big|_0^5$
$\approx \boxed{54.936}$

27) $\int \frac{dx}{\cot 3x}$

$u = \sin(3x)$
$du = 3\cos(3x)\,dx$
$\boxed{\frac{1}{3}\ln|\sin(3x)| + C}$

37) $\int_0^1 \sqrt{t^5 + 2t}\,(5t^4 + 2)\,dt$

$= \frac{2}{3}(t^5 + 2t)^{3/2}\Big|_0^1$
$= \boxed{\frac{4\sqrt{3}}{3}}$

1. $\int x \sin x\,dx$

$u = x \quad dv = \sin x\,dx$
$du = dx \quad v = -\cos x$
$\boxed{-x\cos x + \sin x + C}$

2. $\int x^2 \cos x\,dx$ (IBP x2)

$u = x^2 \quad dv = \cos x\,dx$
$du = 2x\,dx \quad v = \sin x$
$\boxed{(x^2 - 2)\sin x + 2x\cos x + C}$

3. $\int y \ln y\,dy$

$u = \ln y \quad dv = y\,dy$
$du = dy/y \quad v = y^2/2$
$\boxed{\frac{y^2}{2}\ln y - \frac{y^2}{4} + C}$

9. $\int x^3 \ln x\,dx$

$u = \ln x \quad dv = x^3\,dx$
$du = dx/x \quad v = x^4/4$
$\boxed{\frac{x^4}{4}\ln x - \frac{x^4}{16} + C}$

10. $\int x^4 e^{-x}\,dx$

x^4	+	e^{-x}
$4x^3$	−	$-e^{-x}$
$12x^2$	+	e^{-x}
$24x$	−	$-e^{-x}$
24	+	e^{-x}
0		$-e^{-x}$

$\boxed{-e^{-x}(x^4 + 4x^3 + 12x^2 + 24x + 24)}$

11. $\int (x^2 - 5x) e^x\,dx$ (IBP x2)

$u = x^2 - 5x \quad dv = e^x\,dx$
$du = 2x - 5\,dx \quad v = e^x$
$\boxed{(x^2 - 7x - 3)e^x + C}$

Day 2 Homework

☐ Revert Back!

☐ change 3→4

In Exercises 1-4, estimate the limit graphically. Use L'Hôpital's Rule to confirm your estimate.

1. $\lim_{x\to 2} \frac{x-2}{x^2-4} = \lim_{x\to 2} \frac{1}{x+2} = \boxed{\frac{1}{4}}$

2. $\lim_{x\to 0} \frac{\sin 5x}{x} = \lim_{x\to 0} \frac{5\sin(5x)}{5x} = 5\lim_{x\to 0} \frac{\sin(5x)}{5x} = 5(1) = \boxed{5}$

3. $\lim_{x\to 0^+} (1+\frac{1}{x})^x$ $\boxed{1}$ (on back)

4. $\lim_{x\to\infty} \frac{5x^2-3x}{7x^2+1} = \boxed{\frac{5}{7}}$

In Exercises 5-8, use L'Hôpital's Rule to evaluate the limit.

5. $\lim_{x\to 1} \frac{x^3-1}{4x^3-x-3} = \lim_{x\to 1} \frac{3x^2}{12x^2-1} = \frac{3}{12-1} = \boxed{\frac{3}{11}}$

6. $\lim_{x\to 0} \frac{1-\cos x}{x^2} = \lim_{x\to 0} \frac{\sin x}{2x} = \lim_{x\to 0} \frac{\cos x}{2} = \boxed{\frac{1}{2}}$

7. $\lim_{x\to 0^+} (e^x+x)^{1/x}$ $\boxed{e^2}$ (on back)

8. $\lim_{x\to\infty} \frac{2x^2+3x}{x^3+x+1}$ $\boxed{0}$

In Exercises 9 and 10 (a) complete the table and estimate the limit. (b) use L'Hôpital's Rule to confirm your estimate.

9. $\lim_{x\to\infty} f(x), f(x) = \frac{\ln x^5}{x}$ (a) $\approx \boxed{0}$ (b) $\lim_{x\to\infty} \frac{5\ln x}{x} = \lim_{x\to\infty} \frac{\frac{5}{x}}{1} = \frac{0}{1} = \boxed{0}$

X	10	10^2	10^3	10^4	10^5
f(x)	1.1513	0.2303	0.0354	0.0046	0.00058

10. $\lim_{x\to 0^+} f(x), f(x) = \frac{x-\sin x}{x^3}$ (a) $\approx \boxed{\frac{1}{6}}$ (b) $\lim_{x\to 0^+} \frac{1-\cos x}{3x^2} = \lim_{x\to 0^+} \frac{\sin x}{6x} = \lim_{x\to 0^+} \frac{\cos x}{6} = \boxed{\frac{1}{6}}$

X	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
f(x)	0.1585	0.1666	0.1667	0.1667	0.1667

In Exercises 11-14, use tables to estimate the limit. Confirm your estimate using L'Hôpital's Rule.

11. $\lim_{\theta \to 0} \dfrac{\sin 3\theta}{\sin 4\theta} = \lim_{\theta \to 0} \dfrac{3\cos(3\theta)}{4\cos(3\theta)}$

$= \dfrac{3(1)}{4(1)} = \boxed{3/4}$

12. $\lim_{t \to 0} \left(\dfrac{1}{\sin t} - \dfrac{1}{t}\right) = \lim_{t \to 0} \dfrac{t - \sin t}{t \sin t}$

$= \lim_{t \to 0} \dfrac{1 - \cos t}{\sin t + t\cos t} = \lim_{t \to 0} \dfrac{\sin t}{2\cos t - t\sin t}$

$= \boxed{0}$

13. $\lim_{x \to \infty} (1+x)^{1/x}$ $\boxed{1}$

(on back)

14 $\lim_{x \to \infty} \dfrac{x - 2x^{2}}{3x^{2} + 5x} = \boxed{\dfrac{-2}{3}}$

In Exercises 15-42, use L'Hôpital's Rule to evaluate the limit.

1

15 $\lim_{\theta \to 0} \dfrac{\sin \theta^2}{\theta} = \lim_{\theta \to 0} \dfrac{2\sin\theta\cos\theta}{1}$

$= \dfrac{0}{1} = \boxed{0}$

16. $\lim_{\theta \to \pi/2} \dfrac{1 - \sin\theta}{1 + \cos 2\theta} = \lim_{\theta \to \frac{\pi}{2}} \dfrac{+\cos\theta}{+2\sin(2\theta)}$

$= \lim_{\theta \to \frac{\pi}{2}} \dfrac{-\sin\theta}{4\cos(2\theta)} = \boxed{\dfrac{1}{4}}$

17. $\lim_{t \to 0} \dfrac{\cos t - 1}{e^t - t - 1} = \lim_{t \to 0} \dfrac{-\sin t}{e^t - 1}$

$= \lim_{t \to 0} \dfrac{-\cos t}{e^t} = \dfrac{-1}{1} = \boxed{-1}$

18. $\lim_{t \to 1} \dfrac{t - 1}{\ln t - \sin \pi t} = \lim_{t \to 1} \dfrac{1}{\frac{1}{t} - \pi\cos(\pi t)}$

$= \boxed{\dfrac{1}{1+\pi}}$

19. $\lim_{x \to \infty} \dfrac{\ln(x+1)}{\log_2 x} = \lim_{x \to \infty} \dfrac{\frac{1}{x+1}}{\frac{1}{x\ln 2}}$

$= \lim_{x \to \infty} \dfrac{x\ln 2}{x+1} = \lim_{x \to \infty} \dfrac{\ln 2}{1} = \boxed{\ln 2}$

20. $\lim_{x \to \infty} \dfrac{\log_2 x}{\log_3(x+3)} = \lim_{x \to \infty} \dfrac{\frac{1}{x\ln 2}}{\frac{1}{(x+3)\ln 3}}$

$= \lim_{x \to \infty} \dfrac{(x+3)\ln 3}{x\ln 2} = \lim_{x \to \infty} \dfrac{\ln 3}{\ln 2}$

$= \boxed{\dfrac{\ln 3}{\ln 2}}$

21. $\lim_{x \to 0^+} \dfrac{\ln(y^2 + 2y)}{\ln y} = \lim_{x \to 0^+} \dfrac{\frac{2y+2}{y^2+2y}}{\frac{1}{y}} = \lim_{x \to 0^+} \dfrac{2y^2 + 2y}{y^2 + 2y}$

y $\square$

$= \lim_{x \to 0^+} \dfrac{4y+2}{2y+2} = \dfrac{2}{2} = \boxed{1}$

$= \ln(1)$

$= \boxed{0}$

Day 3 Homework

In Exercises 1-5, (a) state why the integral is improper or involves improper integrals. Then, (b) determine whether the integral converges or diverges, and (c) evaluate the integral if it converges.

1. $\int_0^{\infty} \frac{dx}{x^2+1}$

a) infinite limit of integration
b) converges
c) $\pi/2$

3. $\int_{-8}^{1} \frac{dx}{x^{1/3}}$

a) $f(x)$ DNE @ $x=0 \in [-8,1]$
b) converges
c) $-9/2$

5. $\int_0^{\ln 2} x^{-2}e^{1/x}\,dx$

a) $f(x)$ DNE @ $x=0$, lower bound
b) diverges

In Exercises 7-26, evaluate the integral or state that it diverges.

7. $\int_1^{\infty} \frac{dx}{x^{1.001}}$ 1000

9. $\int_0^{4} \frac{dr}{\sqrt{4-r}}$ 4

11. $\int_0^{1} \frac{dx}{\sqrt{1-x^2}}$ $\frac{\pi}{2}$

13. $\int_{-\infty}^{-2}\left[\frac{1}{x-1}-\frac{1}{x+1}\right]dx$ $+\ln(3)$

15. $\int_{0}^{1}\frac{\theta+1}{\sqrt{\theta^2+2\theta}}d\theta$ $\sqrt{3}$

19. $\int_{0}^{\infty}\frac{ds}{s\sqrt{s^2-1}}$ $\frac{\pi}{2}$

$\boxed{2}$

$\operatorname{arcsec}(1)=x$

$1=\sec x$

$1=\frac{1}{\cos x}$

$\cos x=1$

21. $\int_{0}^{\infty}\frac{16\tan^{-1}x}{1+x^2}dx$ $2\pi^2$

$\boxed{1}$

23. $\int_{-\infty}^{0}\theta e^{\theta}d\theta$ -1

25. $\int_{-\infty}^{\infty}e^{-|x|}dx$ 2