Unit 3 Test Review
Pre-Calculus Honors

Name: Key

You need to be able to do the following WITHOUT a calculator:

- Identify polynomials
- Find the possible number of extrema given the degree
- State end behavior using proper limit notation
- Operate with complex numbers
- Find intercepts and asymptotes of rational functions
- State roots and their multiplicity
- Use completing the square to find the vertex and axis of symmetry
- Use Remainder and Factor Theorems
- Write roots as linear factors
- Use factors to write polynomials in standard form
- Solve rational equations
- Check for extraneous solutions of rational equations
- Answer problems such as the ones that follow....

1. Write the equation of a fourth degree polynomial in expanded form with roots 3, -2, and -3 + *i*. $-3-i$

$(x-3)(x+2)(x+3-i)(x+3+i)$

$(x^2-x-6)(x^2+6x+10) \rightarrow \boxed{f(x)=x^4+5x^3-2x^2-46x-10}$

2. Use synthetic division to find the value of k so that the remainder for $(x^3-5x^2+2x+k)\div(x-1)$ is 10.

1	1	-5	2	k
	↓	1	-4	-2
	1	-4	-2	10

$k-2=10$

$\boxed{k=12}$

Simplify.

3. i^{14}

$i^2=-1$, $i^4=1$, $i^6=-1$, $i^8=1$, $i^{10}=-1$, $i^{12}=1$, $i^{14}=\boxed{-1}$

4. $(3-7i)(2+4i)$

$\boxed{34-2i}$

5. $\frac{(5-2i)}{(3+2i)}\cdot\frac{(3-2i)}{(3-2i)}$

$=\boxed{\frac{11-16i}{13}}$

6. $\left(12+\sqrt{-18}\right)\left(-4+4\sqrt{-2}\right)$

$=(12+3i\sqrt{2})(-4+4i\sqrt{2})$

$=\boxed{-24+36i\sqrt{2}}$

7. Given $f(x) = (x+6)^2(x-4)^5$.

   a. State the roots and their multiplicity. $\boxed{x=-6}$ (Mult. of 2) $\boxed{x=4}$ (Mult. of 5)

   b. What is the degree? $2+5=\boxed{7}$

   c. A polynomial with the same degree as f(x) could have $\boxed{6}$ extrema,

   d. Without graphing, state the end behavior of f(x). Use limits!

$\lim_{x\to\infty} f(x)=\boxed{\infty}$ $\lim_{x\to-\infty} f(x)=\boxed{-\infty}$

8. Write a polynomial in expanded form with roots 4 and $-3+2i$. → $-3-2i$

$f(x) = (x-4)(x+3+2i)(x+3-2i)$

$= (x-4)(x^2+6x+13) = \boxed{x^3+2x^2-11x-52}$

9. Write $y = 2x^2 + 36x - 8$ in vertex form, then state the vertex and axis of symmetry.

$y = 2[x^2+18x-4]$

$y = 2[x^2+18x+81-4-81]$

$y = 2[(x+9)^2-85]$ → $y = 2(x+9)^2 - 170$

Vertex: $(-9, -170)$

$x = -9$

10. Find the x-intercepts and the vertical asymptotes of $f(x) = \dfrac{(7x-5)}{(5x-4)(x+6)}$

$7x-5=0$

$x = 5/7$

$(5/7, 0)$

$5x-4=0$ $x+6=0$

$x = 4/5$ $x = -6$

11. Find all the asymptotes and intercepts for $f(x) = \dfrac{3x^4 - 2x^2 + 7x - 1}{x+2}$

$3x^3-6x^2+10x-13$

$x+2 \overline{)\,3x^4+0x^3-2x^2+7x-1}$

$-3x^4-6x^3$

$-6x^3-2x^2+7x-1$

$+6x^3+12x^2$

$10x^2+7x-1$

$-10x^2-20x$

$-13x-1$

$+13x+26$

25

V.A.: $x = -2$

E.B.A.: $y = 3x^3-6x^2+10x-13$

y-int.: $(0, -1/2)$

Solve the rational equations and state all undefined values.

12. $\dfrac{1}{m^2+6m} = \dfrac{5}{m^2+6m} + \dfrac{1}{m}$ LCD: $m(m+6)$

$m(m+6)$ $m(m+6)$ $m(m+6)$

$1 = 5 + m + 6$

$1 = m + 11$

$\boxed{m = -10}$

13. $\dfrac{3}{x^2+5x+6} = \dfrac{x}{x+3} - \dfrac{5}{x^2+5x+6}$

LCD: $(x+2)(x+3)$

$3 = x(x+2) - 5$

$3 = x^2+2x-5$

$0 = x^2+2x-8$

$0 = (x+4)(x-2)$

$x = 2$

$x = -4$

Calculator Active Questions:

Find ALL roots of the polynomial given. Leave ALL answers exact (NO DECIMALS!). Show work!

1. $g(x) = x^4 + x^3 - 19x^2 + 32x - 12$

Degree: 4

# of zeros: 4

# of possible extrema: 3

Actual # of extrema: 3

End behavior: $\lim_{x \to \pm\infty} g(x) = \infty$

Linear *Factor*ization: $(x-2)^2(x-0.54)(x+5.54)$

Roots:

$x = 2$ (mult. of 2)

$x = \dfrac{-5 \pm \sqrt{37}}{2}$

2. $h(x) = x^4 + 4x^3 + 4x^2 - 4x - 5$

Degree: 4

# of zeros: 4

# of possible extrema: 3

Actual # of extrema: 1

End behavior: $\lim_{x \to \pm\infty} h(x) = \infty$

Linear *Factor*ization: $(x+1)(x-1)(x+2+i)(x+2-i)$

Roots:

$x = \pm 1$ $x = -2 \pm i$

3. Find the vertex form of the parabola, the vertex, and the axis of symmetry: $y = 3x^2 - 5x + 10$. SHOW WORK

$y = 3[x^2 - 5/3\,x + 25/36 + 10/3 - 25/36]$

$y = 3[(x - 5/6)^2 + 95/26]$

$y = 3(x - 5/6)^2 + 95/12$

Vertex: $(5/6, 95/12)$

$x = 5/6$

4. One root of $x^3 - x^2 + 4x - 4 = 0$ is $2i$. Find the others.

$x = \pm 2i, 1$

	x	$-2i$
x	x^2	
$2i$		4

$x^2 + 4 \;\overline{)\; x^3 - x^2 + 4x - 4}$ → $x - 1$

$-x^3 + 4x$

$-x^2 - 4$

$+x^2 + 4$

0

Find the End Behavior Asymptotes for:

5. $f(x) = \dfrac{3x^3 - 5x^2 + 6x - 7}{x + 2}$

$-2 \;|\; 3 \;\; -5 \;\; 6 \;\; -7$

$\downarrow \;\; -6 \;\; 22 \;\; -56$

$3 \;\; -11 \;\; 28 \;\; -63$

$y = 3x^2 - 11x + 28$

6. $g(x) = \dfrac{x^3 + x + 1}{2x^2 - 5x - 3}$.

$y = \frac{1}{2}x + \frac{5}{4} + (11)$

$2x^2 - 5x - 3 \;\overline{)\; x^3 + 0x^2 + x + 1}$

$-x^3 + \frac{5}{2}x^2 + \frac{3}{2}x \;\downarrow$

$\frac{5}{2}x^2 + \frac{5}{2}x + 1$

$-\frac{5}{2}x^2 + \frac{25}{4}x + \frac{15}{4}$

7. Analyze: $g(x) = \dfrac{x^3 + x + 1}{2x^2 - 5x - 3}$. $= \dfrac{x^3 + x + 1}{(x-3)(2x+1)}$

Power: jk!

Constant of Variation: jk!

Type of Variation: NOT!

VA: $x = 3, x = -1/2$

HA: none

EBA: $y = 1/2\,x + 5/4$ (see #6)

D: $(-\infty, -1/2) \cup (-1/2, 3) \cup (3, \infty)$

R: $(-\infty, \infty)$

Inc: $(-\infty, \infty)$

Dec: $(-\infty, -1/2) \cup (1/2, 0.26) \cup (5.9, \infty)$

Boundedness: not bounded

Even/Odd: neither

End Behavior: $\lim_{x \to \infty} g(x) = \infty$

$\lim_{x \to -\infty} g(x) = -\infty$

Challenge! – Sketch:

Solve the following inequalities ***by hand*** (show factoring and number line). Write answers in interval notation.

8. $x^3 - 2x^2 - 15x + 36 > 0$

$(x+4)(x-3)^2 > 0$

$-\infty \leftarrow \ominus\ 0\ \oplus\ 0\ \oplus \rightarrow \infty$ (−4, 3)

$(-4, 3) \cup (3, \infty)$

9. $\frac{x^2 + x - 42}{x - 2} \geq 0$

$-\infty\ \ominus\ 0\ \oplus\ \text{und.}\ \ominus\ 0\ \oplus\ \infty$ (−7, 2, 6)

$\frac{(x+7)(x-6)}{(x-2)} \geq 0$

$[-7, 2) \cup [6, \infty)$

10. $\frac{3x}{x-2} - \frac{5}{x+1} \leq \frac{-x}{x^2 - x - 2}$ LCD: $(x-2)(x+1)$

$3x(x+1) - 5(x-2) \leq -x$

$3x^2 + 3x - 5x + 10 + x \leq 0$

$3x^2 - x + 10 \leq 0$

NO SOLUTION

11. $(x+3)\sqrt{x-1} < 0$

und. und. 0 $\oplus$ (−3, 1)

NO SOLUTION

12. When you swim under water, the pressure in your ears varies directly with the depth at which you swim. At 10 feet, the pressure is about 4.3 pounds per square inch (psi).

a) Write an equation to model the situation, expressing pressure in terms of depth.

$p = kd$ $\quad 4.3 = 10k$ $\quad k = 0.43$ $\quad p = 0.43d$

b) Predict the pressure at 50 feet.

$p = 0.43(50) = 21.5$ psi

c) It is unsafe for amateur divers to swim where the pressure is more than 65 psi. How deep can an amateur diver safely swim?

$p \leq 65$

$0.43d \leq 65$

$d \leq 151.16$ ft

d) Sketch the graph of pressure versus depth