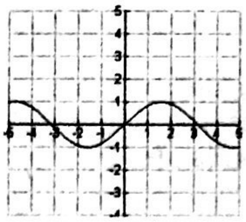
Trig Derivative Introduction... NAME:

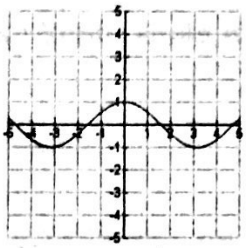
Match the six trig functions on the right with their correct derivative on the left. Do this by paying attention to where slope is positive, negative, or 0. If slope is positive, y ' will have positive y-values (ABOVE the axis). If slope is negative, y ' will have negative y-values (BELOW the axis). If slope is 0, the y ' graph will hit the axis there.

1. $y = \sin(x)$



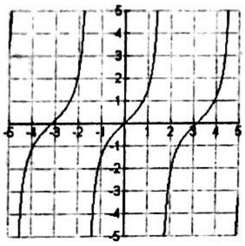
D

2. $y = \cos(x)$



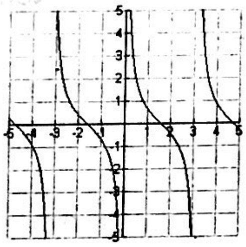
C

3. $y = \tan(x)$



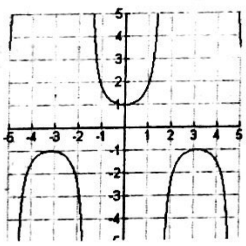
A

4. $y = \cot(x)$



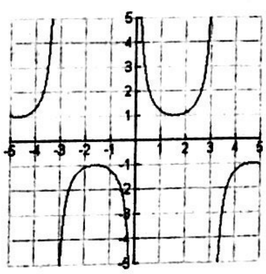
F

5. $y = \sec(x)$



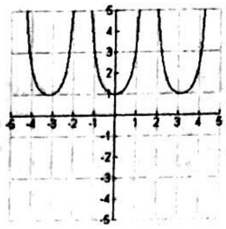
B

6. $y = \csc(x)$

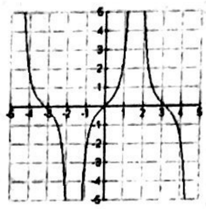


E

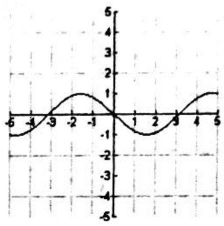
A)



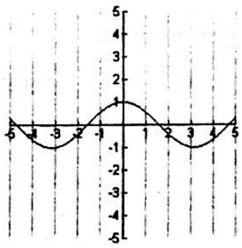
B)



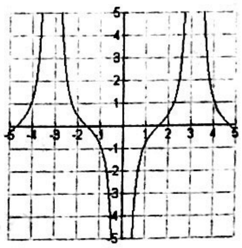
C)



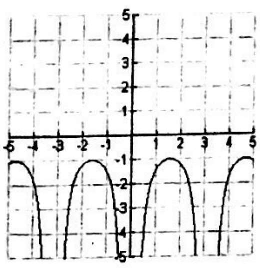
D)



E)



F)



Implicit Differentiation & Inverse Trig

Find y' by implicit differentiation.

1. $x^2+y^2=25$

$2x+2y\frac{dy}{dx}=0$

$2y\frac{dy}{dx}=-2x$

$\boxed{\frac{dy}{dx}=\frac{-x}{y}}$

2. $\frac{1}{x}+\frac{1}{y}=1$

$-x^{-2}-y^{-2}\frac{dy}{dx}=0$

$-y^{-2}\frac{dy}{dx}=x^{-2}$

$\boxed{\frac{dy}{dx}=\frac{-y^2}{x^2}}$

3. $(7x-1)^3=2y^4$

$3(7x-1)^2(7)=8y^3\frac{dy}{dx}$

$\boxed{\frac{dy}{dx}=\frac{21(7x-1)^2}{8y^3}}$

4. $3x^2-xy+4y^2=90$

$6x-y-x\frac{dy}{dx}+8y\frac{dy}{dx}=0$

$\frac{dy}{dx}(8y-x)=y-6x$

$\boxed{\frac{dy}{dx}=\frac{y-6x}{8y-x}}$

5. $x^2y^3-5xy^2-4y=4$

$2xy^3+3x^2y^2\frac{dy}{dx}-5y^2-10xy\frac{dy}{dx}-4\frac{dy}{dx}=0$

$\frac{dy}{dx}(3x^2y^2-10xy-4)=5y^2-2xy^3$

$\boxed{\frac{dy}{dx}=\frac{5y^2-2xy^3}{3x^2y^2-10xy-4}}$

6. $\frac{x^2}{9}+\frac{y^2}{4}=1$

$\frac{2}{9}x+\frac{1}{2}y\frac{dy}{dx}=0$

$\frac{1}{2}y\frac{dy}{dx}=-\frac{2}{9}x$

$\boxed{\frac{dy}{dx}=\frac{-4x}{9y}}$

Use implicit differentiation to find the slope-intercept equation of the **tangent** line at the indicated point.

1) $(y-x)^2+y^3=xy+7, at\ (1,2)$

$2(y-x)(\frac{dy}{dx}-1)+3y^2\frac{dy}{dx}=y+x\frac{dy}{dx}$

$2y\frac{dy}{dx}-2y-2x\frac{dy}{dx}+2x+3y^2\frac{dy}{dx}=y+x\frac{dy}{dx}$

$\frac{dy}{dx}(2y-2x+3y^2-x)=3y-2x$

$\frac{dy}{dx}=\frac{3y-2x}{2y-2x+3y^2-x}$

$=\frac{3(2)-2(1)}{(2)(2)-2(1)+3(2)^2-(1)}=\frac{4}{13}$

$\boxed{y-2=\frac{4}{13}(x-1)}$

2) $\frac{x^2}{16}+y^2=1\ at\ \left(2,\frac{\sqrt{3}}{2}\right)$

$\frac{x}{8}+2y\frac{dy}{dx}=0$

$\frac{dy}{dx}=\frac{-x}{16y}$

$=\frac{-2}{16(\sqrt{3}/2)}=\frac{-2}{8\sqrt{3}}=\frac{-1}{4\sqrt{3}}$

$\boxed{y-\frac{\sqrt{3}}{2}=\frac{-1}{4\sqrt{3}}(x-2)}$

Use implicit differentiation to find the slope-intercept equation of the **normal** line at the indicated point.

1) $y^3x + 2y = x^2$, $at\ (2,1)$

$3y^2x\frac{dy}{dx} + y^3 + 2\frac{dy}{dx} = 2x$

$\frac{dy}{dx} = \frac{2x - y^3}{-3y^2x - 2}$

$= \frac{2(2) - (1)^3}{-3(1)^2(2) - 2} = \frac{3}{-8}$

$\boxed{y - 1 = \frac{8}{3}(x-2)}$

2) $y\sqrt{x} - x\sqrt{y} = 12$, $at\ (9,16)$

$x^{1/2}\frac{dy}{dx} + \frac{1}{2}yx^{-1/2} - y^{1/2} - \frac{1}{2}xy^{-1/2}\frac{dy}{dx} =$

$\frac{dy}{dx}(x^{1/2} - \frac{1}{2}xy^{-1/2}) = y^{1/2} - \frac{1}{2}yx^{-1/2}$

$\frac{dy}{dx} = \frac{\sqrt{y} - \frac{y}{2\sqrt{x}}}{\sqrt{x} - \frac{x}{2\sqrt{y}}} = \frac{4 - \frac{16}{6}}{3 - \frac{9}{8}} = \frac{\frac{96-64}{24}}{\frac{72-27}{24}}$

$= \frac{32}{45}$

$\boxed{y - 16 = \frac{-45}{32}(x-9)}$

Find the derivative of the function. Simplify where possible.

1) $y = \sin^{-1}(x^2)$

$\boxed{y' = \frac{2x}{\sqrt{1-x^4}}}$

2) $y = (\sin^{-1}x)^2$

$\boxed{y' = \frac{2\sin^{-1}x}{\sqrt{1-x^2}}}$

3) $f(x) = \tan^{-1}(5x)$

$\boxed{f'(x) = \frac{5}{1+25x^2}}$

4) $f(x) = x^2 \arctan x$

$\boxed{f'(x) = 2x\tan^{-1}x + \frac{x^2}{1+x^2}}$

5) $f(x) = x\arcsin(1-x^2)$

$f'(x) = \sin^{-1}(1-x^2) - \frac{2x^2}{\sqrt{1-(1-2x^2+x^4)}}$

$= \sin^{-1}(1-x^2) - \frac{2x^2}{\sqrt{2x^2 - x^4}}$

$= \boxed{\sin^{-1}(1-x^2) - \frac{2x}{\sqrt{2-x^2}}}$

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Free Response Practice 1 (no calculator)

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

(a)

$y^2 + 2xy\frac{dy}{dx} - 3x^2y - x^3\frac{dy}{dx} = 0$ ①

+2

$\frac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$

① $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$

(b)

$(1)y^2 - (1)^3y = 6$ ①

$y^2 - y - 6 = 0$

+4

$(y-3)(y+2) = 0$

$y = -2, 3$ ①

$(1,-2)$

$\frac{dy}{dx} = \frac{3(1)^2(-2) - (-2)^2}{2(1)(-2) - (1)^3} = \frac{-6-4}{-4-1} = \frac{-10}{-5} = 2$

① $y + 2 = 2(x-1)$

$(1,3)$

$\frac{dy}{dx} = \frac{3(1)^2(3) - (3)^2}{2(1)(3) - (1)^3} = \frac{9-9}{6-1} = 0$

① $y = 3$

* Note: $\frac{0}{4}$ pts. if not solving an equation of the form $y^2 - y = K$

(c)

+3

① $2xy - x^3 = 0$

$2xy = x^3$

$y = \frac{x^2}{2}$

or

$\cancel{x=0}$ ← DNE

① $x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6$

$\frac{x^5}{4} - \frac{x^5}{2} = 6$

$-\frac{x^5}{4} = 6$

① $x = \sqrt[5]{-24}$

Free Response Practice 2 (no calculator)

Let f be a function given by $f(x) = \ln\left(\dfrac{x}{x-1}\right)$.

(a) What is the domain of f?
(b) Write an expression for the general derivative of $f(x)$.
(c) Write an equation for the line tangent to $f(x)$ at $x = -1$.

(a)

+3

$f(x)$ has zero @ $x = 0$

$f(x)$ discontinuous @ $x = 1$

notation ( ) 1

$\frac{-}{-} = \oplus \quad \frac{+}{-} = \ominus \quad \frac{+}{+} = \oplus$

0 1

↑ $\ln(x)$ must be > 1 always!

$\boxed{x \in (-\infty, 0) \cup (1, \infty)}$

1 1

(b)

+3

$$f'(x) = \left(\frac{1(x-1) - x(1)}{(x-1)^2}\right) \cdot \left(\frac{x-1}{x}\right) = \boxed{\frac{-1}{x^2 - x}}$$

1 1 1

(c)

+3

1

$$f(-1) = \ln\left(\frac{-1}{-1-1}\right) = \ln(1/2) = -\ln(2)$$

$$f'(-1) = \frac{-1}{(-1)^2 - (-1)} = \frac{-1}{1+1} = \frac{-1}{2} \quad 1$$

$$\boxed{y + \ln(2) = \frac{-1}{2}(x+1)}$$

1