Unit 3 : Parametrics

Important Concepts

- Slope of a parametric curve
  - Finding the first derivative of a parametric curve
  - Finding the second derivative of a parametric curve
- Finding the length of a parametric curve
- Combining vectors
- Forms of Vectors
  - Finding the component form of vectors
  - Finding the magnitude of vectors
  - Finding unit vectors
  - Finding tangent and normal vectors
  - Linear combination of vectors (standard vectors)
  - Direction of Motion
- Integration
  - Initial Value Problem
- Position, Velocity, and Acceleration
- Projectile Motion
- Using derivatives to find point-slope form
- Finding the points at which the tangent to the curve is vertical and horizontal

Definitions:

- Parametric Equations: equations that use time (with x and y being dependent on time) to tell us more about position than a typical xy-equation; answering "where" and "when"
- Vector: a directed line that shows quantities that have direction and magnitude
- "Ideal" Projectile Motion: projectile object moving in a vertical plane and the only force is gravity
- Unit Vector: a vector whose magnitude is one
- Zero Vector: a vector whose horizontal and vertical components are zero and therefore its magnitude is zero
- Horizontal Component: x-component of a vector

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- Vertical Component: y-component of a vector
- Tangent : parallel
- Normal: perpendicular
- Position Vector: position of object at time t, relative to the origin
- Velocity Vector: direction and speed of object at time t; perpendicular to position
- Acceleration Vector: how speed and direction are changing at time t; perpendicular to velocity and opposite to position
- Standard Vector: vector that starts at the origin
- Speed: the magnitude of velocity; has no direction
- Direction of Motion: an unit vector that indicates direction but not magnitude
- Newton's 2nd Law of Motion (F = ma): shows the relationship between a force acting on an object, the mass of an object, and the acceleration of an object

Example Problems

1. For the following curve, find the value of the slope at the point (0,3).
   $x = 2\cos(t)$, $y = 3\sin(t)$
2. Find the value of d^2y/dx^2 at the indicated value:
   $x = t - \sin(t)$, $y = 1 - \cos(t)$ at $t = \pi/2$
3. Write, but do not solve, the integral to find the length of the curve:
   $X = 3\sin(3t)$, $y = 4\cos(2t)$ for $-2 \le t \le 4$
4. Find the unit vectors (four in all) that are tangent <u>and</u> normal to the curve at the given point.
   $x = 1 + e^t$, $y = t - e^{-2t}$, $t = 0$
5. The vector PQ, where P = (1,3) and Q = (2,-1)
6. The sum of AB and CD, where A = (1,-1), B = (2,0), C = (-1,3), and D = (-2,2)
7. Evaluate $\int_1^2 [(6-6t)i + 3(\sqrt{t})j]\,dt$
8. $d^2y/dx^2 = -32j$, $r(0) = 100i$, $v(0) = 8i + 8j$
9. $r(t) = (3\cos(t))i + (3\sin(t))j$

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Find velocity and acceleration vectors

10. A baseball is hit from a height of 3 feet with an initial speed of 120 ft/sec at an angle of 30 degrees. Find the horizontal and vertical components of the position function.

11. $x = t^2 + 1$, $y = t^3 - 4t$
    a. Find the equation of the tangent line at t=3.
    b. Find the value(s) of t where the tangent line is horizontal.
    c. Find the point(s) where the tangent line is vertical.

12. $\mathbf{u} = \{3, -2\}$ and $\mathbf{v} = \{-2, 5\}$

Find the component form and magnitude form for: $\frac{3}{5}\mathbf{u} + \frac{4}{5}\mathbf{v}$