Unit 1: Review :)

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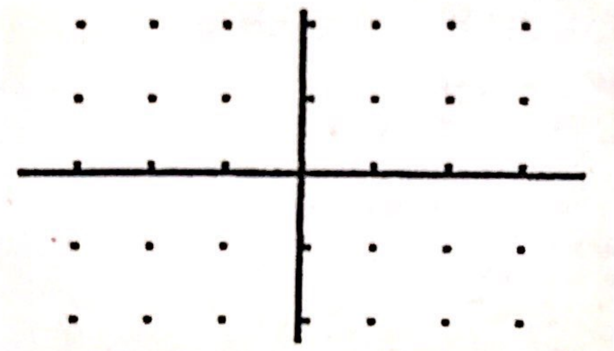
1) Review slope fields and differential Equations

→ Slope fields are just tools to visualize the solution of a differential equation without actually being able to integrate to find the solution.

Ex: $\frac{dy}{dx} = \frac{1}{x}$



Ex: y' = x^2y, (0,2)

2) Euler's Method

→ Point Slope forms form; y-y_1 = m(x-x_1)

→ Error = | $Actual - Approximation$ |

→ Basis of Euler's method

- Patching together a string of small linearizations to approximate the curve of a larger stretch
- Given $\frac{dy}{dx}$ = f'(x,y) and y(x_0)=y_0
  - $y - y_1 = m(x - x_1) \rightarrow y = y_1 + m(x - x_1)$
    - m = slope
    - $(x - x_1)$ = change in x
  - $y_n - y_{n-1} = f'(x_{n-1}, y_{n-1}) \cdot \triangle x$
    - y_n = future y-value
    - y_{n-1} = previous y-value
    - $f'(x_{n-1}, y_{n-1})$ = slope of previous point
    - $\triangle x$ = change in x

Ex: $\frac{dy}{dx}$ = f'(x,y) = x+ y; y(0) = 1; Δx = 0.1 Evaluate at x_n = .2

n	x_n	y_n	(x+y)	$\frac{dy}{dx} \bullet \Delta x$

3) Exponential growth and decay

→ For a function y > 0 that is differentiable function of t and y' = ky

→ y=C e^{kt}

→ C is the initial value of y, k is the proportionality constant

→ for k> 0, we have exponential decay

- $\frac{dy}{dt} = ky$

dy = $ky \cdot dt$

$\frac{dy}{y} = kdt$

$\int \frac{1}{y}\, dt = \int k\, dt$

$ln(y) = kt + c$

$e^{ln(y)} = e^{kt+c}$

$y = e^{kt} \cdot e^c = e^{kt} \cdot c = c = e^{kt}$

Ex: The rate of change of y with respect to t is proportional to the value of y. When t=0, R=300 and when t=1, R=500. Write and solve the differential equation that models this situation.

4) Newtons Law of Cooling

→ Temperature of an object decreases at a rate proportional to the different between its current temperature and the surrounding air

→ $\frac{dT}{dt}$ = -k(T - T_s) where T is current temperature and T's is the surrounding temperature

- Separate and Integrate
- Assume Ts is constant
- T = what we're solving for
- At t=0, temp if T_0

→ $\frac{dT}{dt}$ = -k(T - T_s), (0, T_0)

$$\int \frac{1}{(T - T_s)}\,dt = \int -k\,dt$$

$\text{Ln}\left|T - T_s\right| = -kt + c$

$T - T_s = Ae^{-kt}$

$T_0 - T_s = Ae^{-k(0)}$

$T_0 - T_s = A \rightarrow$ $T - T_s = (T_0 - T_s)e^{-kt}$

Ex: Suppose a corpse at a temperature of 32˚C arrives at a mortuary where the temperature, T, of the corpse, t, hr later is...

A) $\frac{dT}{dt} = -k(T-10)$

B) $\frac{dT}{dt} = k(T-32)$

C) $\frac{dT}{dt} = 32e^{-kt}$

D) $\frac{dT}{dt} = -kt(T-10)$

E) $\frac{dT}{dt} = kT(T-32)$

Ex: CSI Example: Tom the Cat Case file

- Room temp 72˚F
- Toms body was 96˚F when found
- Body temp $\frac{1}{2}$ hr later was 92˚F
- When he was alive, temp was 101˚F

Hint: $T_s =$

$T_{normal} =$

$T_o =$

$T = T_{30} =$

5) Population and logistic growth

Constants: k and M

M = max capacity, upper limit, cary capacity, max. sustainable pop.

	Exponential	Logistical
Diff. Equation	$\frac{dP}{dT} = kp$	$\frac{dP}{dT} = kp(1 - \frac{P}{M})$
Equation:	$P = Pe^{kt}$	$P = \frac{M}{1 + Ae^{-kt}}$

$\frac{M}{M}\frac{1}{P(1-\frac{P}{M})}dp = kdt \cdot \frac{M}{M} \rightarrow P = m;\ m = mB;\ B = 1 \qquad P = 0;\ m = Am;\ A = 1$

$\downarrow$

$\int \frac{m}{P(m-P)}\,dp = \int k\,dt \rightarrow \int \frac{2}{x(2-x)}dx \rightarrow 2\int \frac{1}{x(2-x)}dx$

$\downarrow$

$\frac{m}{P(m-P)} = \frac{A}{P} + \frac{B}{m-p} \rightarrow$ m = A(m-P) = BP

$\int \frac{1}{P} + \frac{1}{m-P}\,dP = \int k\,dt$

$\downarrow$

$ln\,|P| - ln\,|m-P| = kt + c \rightarrow (ln\left|\frac{P}{m-P}\right| = kt + c) \cdot (-1)$

$\downarrow$

$\frac{m-P}{P} = Ae^{-kt} \leftarrow \left|\frac{m-P}{P}\right| = Ae^{-kt} \leftarrow ln\left|\frac{m-P}{P}\right| = -kt - c$

$\downarrow$

$m - P = P(Ae^{-kt}) \rightarrow m = P(Ae^{-k(t)} + P) \rightarrow m = P(Ae^{-k(0)} + 1) \rightarrow P = \frac{m}{Ae^{-k(0)}+1}$

Ex: The max bear population in a park is 100. The population @ t=0 is 10 (t is measured in years.) The constant of proportionality is k=0.1

a) Write a differential equation to model the population.
b) Solve the differential equation.

Ex: Suppose that the growth of a population $y = y(t)$ is given by the logistic equation $\frac{60}{5+7e^{-t}}$

a) What is the population at time t=0.
b) What is the carrying capacity L? (Hint: It is not 60. Carry capacity = $\lim_{t\rightarrow\infty}$ ...
c) What is the constant k?