Day 7 – **GEOMETRIC SEQUENCES AND SERIES**

GEOMETRIC SEQUENCE: A pattern where the RATIO of consecutive terms is always the same (something very similar to this was "Geometric Mean" in Math III).

2, 6, 18, 54, 162, ...

$2\cdot 3^0 \quad 2\cdot 3^1 \quad 2\cdot 3^2 \quad \ldots$

If you divide the second term by the first, the third term by the second, and so on, what do you get? $\boxed{3}$

3 is called the COMMON RATIO, denoted $\boxed{r}$.

In general terms, $\boxed{r = \frac{a_{n+1}}{a_n}}$ 2nd Term / 1st Term

Also, generally speaking, a geometric sequence can be written as: $\{a_1, a_1 r, a_1 r^2, \ldots\}$

Ex 1: Show that $\{a_n\} = \{2^{-n}\}$ is geometric.

$$r = \frac{2^{-(n+1)}}{2^{-n}} = 2^{-n-1-(-n)} = 2^{-1} = \frac{1}{2}$$

Since r is a constant, this $\boxed{\text{is}}$ Geometric.

Ex 2: Is $\{b_n\} = \left\{-3\left(\frac{1}{4}\right)^n\right\}$ a geometric sequence?

$$r = \frac{\cancel{-3}(1/4)^{n+1}}{\cancel{-3}(1/4)^n} = (1/4)^{n+1-n} = (1/4)^1$$

$\boxed{\text{yes}}$

Ex 3: Is $\{a_n\} = \{2n\}$ a geometric sequence?

$$r = \frac{2(n+1)}{2n} = \frac{2n+2}{2n} = \frac{n+1}{n}$$

Since r is NOT a constant, this is $\boxed{\text{not}}$ Geometric.

FINDING A RULE FOR A GEOMETRIC SEQUENCE:

Formula: $\boxed{\{a_n\} = \{a_1 \cdot r^{n-1}\}}$

Ex 4: Find the rule for the sequence 2, 6, 18, 54, 162, ...

$\boxed{\{a_n\} = \{2\cdot 3^{n-1}\}}$

Ex 5: Find the 17th term of the geometric sequence 2, 2/3, 2/9, 2/27, ...

$$\{a_n\} = \{2\cdot(1/3)^{n-1}\} \qquad r = \frac{2/3}{2} = 1/3$$

$$a_{17} = 2(1/3)^{17-1} = 2(1/3)^{16} = \boxed{\frac{2}{43046721}}$$

Ex 6: Given $a_1 = 2$, $r = 3$, find the 15th term.

$$\{a_n\} = \{2(3)^{n-1}\}$$

$$a_{15} = 2(3)^{15-1} = 2(3)^{14} = \boxed{9565938}$$

Adding the First N Terms of a Geometric Sequence:

Finite Sequences

Formula: $\boxed{S_n = a_1 \frac{(1-r^n)}{(1-r)}}$

Ex 7: Find the general sum of $\frac{1}{4}+\frac{2}{4}+\frac{2^2}{4}+\frac{2^3}{4}+\cdots+\frac{2^{n-1}}{4}$.

$r = \frac{2/4}{1/4} = 2$ $\quad S_n = \frac{1}{4}\left(\frac{1-2^n}{1-2}\right) = \frac{1\cdot(1-2^n)}{4\cdot(-1)} = \boxed{\frac{1-2^n}{-4}}$

Ex 8: Find the sum of $\frac{1}{4}+\frac{2}{4}+\frac{2^2}{4}+\frac{2^3}{4}+\cdots+\frac{2^{14}}{4}$.

$a_1 = 1/4$

$r = \frac{2/4}{1/4} = 2$

$\frac{2^{14}}{4} = \frac{1}{4}\cdot(2)^{n-1}$

$n = 15$

$S_{15} = \frac{1}{4}\left(\frac{1-2^{15}}{1-2}\right)$

or sum(seq((1/4)(2)^(X-1), X, 1, 15) = ~~[illegible]~~ $\boxed{8191.75}$

Geometric Series:

A geometric series is the $\boxed{\text{SUM}}$ of a geometric sequence that gets infinitely smaller (so $0 < r < 1$).

convergent

Notation:

$$\sum_{k=1}^{\infty} a_1 r^{n-1}$$

INFINITE

If we use the sum formula listed above, $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$ and consider that $0 < r < 1$ and look at what happens as n gets closer to infinity, we get that: $\boxed{S_{\infty} = \frac{a_1}{1-r}}$

INFINITE GEOMETRIC SERIES

Ex 9: Find $1+\frac{1}{3}+\frac{1}{9}+\ \ldots$

$$\sum_{k=1}^{\infty} 1(1/3)^{n-1} = \frac{1}{1-1/3} = \frac{1}{2/3} = \boxed{3/2}$$

* NO "n" because is NO END *

Ex 10: Find 8 + 4 + 2 + ...

$a_1 = 8$

$r = \frac{1}{2}$

$$\sum_{k=1}^{\infty} 8(1/2)^{k-1} = \frac{8}{1-1/2} = \frac{8}{1/2} = \boxed{16}$$

Ex 11:

$$\sum_{k=1}^{\infty} 2\left(\frac{2}{3}\right)^{k-1} = \frac{2}{1-2/3} = \frac{2}{1/3} = \boxed{6}$$