Day 6 – **GENERAL SEQUENCES, ARITHMETIC SEQUENCES, AND RECURSIVE FUNCTIONS**

Sequences are all about PATTERNS!

DEFINITION OF SEQUENCE: function with domain = all positive INTEGERS (natural numbers).

whole #'s > 0

A "normal" function has a curve that connects all the points together. The curve represents all the decimal values that work in the function. A sequence is just a list of points at the integers. NO connecting curve.

SEQUENCE NOTATION: $\{a_n\} = \{\quad\}$

The {} represent the fact that you have a sequence (A LIST). a_n, b_n, c_n, and so on are NAMES (just like we use f(x), g(x), h(x), and so on to NAME "normal" functions). On the right hand side, you will find the rule for the function. We use "n" instead of x because n = natural numbers (1, 2, 3, 4, ...)

We do not have a zero term!

Ex 1: What are the first 5 terms of the sequence: $\{b_n\} = \left\{\left(\frac{1}{2}\right)^n\right\}$?

n=1,2,3,4,5

$(1/2)^1$ $(1/2)^2$ $(1/2)^3$ $(1/2)^4$ $(1/2)^5$

$1/2$, $1/4$, $1/8$, $1/16$, $1/32$

Ex 2: Find the first 6 terms of $\{a_n\} = \left\{(-1)^{n-1}\left(\frac{2}{n}\right)\right\}$.

$a_1 = (-1)^{1-1}\left(\frac{2}{1}\right) = 1(2) = \boxed{2}$

$a_2 = (-1)^{2-1}\left(\frac{2}{2}\right) = (-1)(1) = \boxed{-1}$

$a_3 = (-1)^{3-1}\left(\frac{2}{3}\right) = 1(2/3) = \boxed{2/3}$

$a_4 = (-1)^{4-1}\left(\frac{2}{4}\right) = (-1)(1/2) = \boxed{-1/2}$

$a_5 = (-1)^{5-1}\left(\frac{2}{5}\right) = 1(2/5) = \boxed{2/5}$

$a_6 = (-1)^{6-1}\left(\frac{2}{6}\right) = (-1)(1/3) = \boxed{-1/3}$

In calc.:

2nd STAT

OPS 5

seq((-1)^(x-1)

variable (2/X), X,

start 1, 6) End

MATH ENTER

x2

Ex 3: Find terms 50 through 55 of $\{b_n\} = \{(-1)^{n+1}n^2\}$

seq((-1)^(x+1)x², x, 50, 55)

** What will the $(-1)^{n+1}$ do?

Alternate signs!

-2500	2601	-2704	2809	-2916	3025
n=50	n=51	n=52	n=53	n=54	n=55

FINDING YOUR OWN RULE:

Ex 4: **Find the sequence that will produce the terms 1, 1/3, 1/9, 1/27, ...**

You need to decide what the terms have in common with each other (which is easier said than done). I see that we have a 3, 9, and 27... 3 is 3^1, 9 is 3^2, and 27 is 3^3.

SO, how could we rewrite 1 as a fraction like the others? $1/3^0 = 1$

So our terms are: $1/3^0$, $1/3^1$, $1/3^2$, $1/3^3$, ... The first term MUST result from plugging in 1. The second, from plugging in 2, etc.

$\boxed{\text{Solution: } \{a_n\} = \{1/3^{(n-1)}\}}$

* Changing $\frac{1}{3^n}$ to $\frac{1}{3^{n-1}}$ allows us to start with $\frac{1}{3^0} = 1$. *

Ex 5: **Find the sequence that will produce the terms 1, -1/2, 1/3, -1/4, 1/5,...**

n	a_n
1	1
2	-1/2
3	1/3
4	-1/4
5	1/5

Alternating signs! → $(-1)^{n\pm1}$

$\{a_n\} = \{(-1)^{n+1} \cdot \frac{1}{n}\}$

* Check with your Calculator *

ARITHMETIC SEQUENCES

* These are special sequences where the pattern is found by ADDING the same number each time.

i.e. 5, 7, 9, 11, ...

Ex 6: When training, Johnny B. Goode does 15 pushups/day during week one. During week 2, he does 20/day. During week 3, he does 25/day and so on. How many pushups/day will he have to do in week 500?

Week	Pushups/Day
1	15
2	20
3	25

$> +5$ $> +5$

Add 5 per week: "Common Difference" $d=5$

Add 5 to 15, 499 times:

$15 + 5(499) = \boxed{2510 \text{ pushups/day}}$

$\boxed{\text{FORMULA: } \{a_n\} = \{a_1 + d(n-1)\}}$

1st term, common difference

Ex 7: **Find the formula for the sequence 5, 7, 9, 11, ...**

$a_1 = 5$ $d=2$

$\{a_n\} = \{5+2(n-1)\} = \{5+2n-2\} = \boxed{\{2n+3\}}$

Ex 8: Fredricka weighs 275 pounds on the 30^{th} day of her diet. On day 67, she weighs 130 pounds. What was her original weight, and what was her average daily weight loss?

a_1 d

n	a_n
30	275
67	130

$-\; 130 = a_1 + d(67-1)$

$275 = a_1 + d(30-1)$

$145 = 30d - \cancel{d} - \cancel{67d} + \cancel{d}$

$145 = -37d \rightarrow d = -3.92$ ← daily weight loss (lbs/day)

$130 = a_1 + 3.92(66)$

$a_1 = 388.65$ lbs ← original weight

SUMS:

Discovered by mathematician Gauss when he was 10 years old, the formula for the sum of an arithmetic sequence:

def.: A series is the sum of all terms in a sequence.

$$\boxed{S_n = \frac{n}{2}(a_1 + a_n)}$$

Ex 9: Rafael makes $28,000 as a first year teacher. He gets a $500 increase each year. After 30 years, how much will he be making? How much TOTAL money did he make over his career?

a_{30}

S_{30}

$a_1 = 28000$

$d = 500$

$a_{30} = 28000 + 500(n-1)$ ($n = 30$)

$= 28000 + 500(29)$

$= \boxed{\$42,500}$

$S_{30} = \frac{30}{2}(28000 + 42500)$

$= \boxed{\$1,057,500}$

SIGMA NOTATION:

$\sum$ *means "add up all the terms of the sequence"*

In general, the sigma will have numbers around it like this:

$$\sum_{k}^{n} \boxed{a_k}$$

ending place

starting place

sequential rule

Ex 10:

$$\sum_{k=1}^{4} k^2$$

k	1	2	3	4
k^2	1 +	4 +	9 +	16 = $\boxed{30}$

In the calculator: Math

2nd STAT 5 2nd STAT OPS 5 → sum(seq(X^2, X, 1, 4))

rule variable

start end

Ex 11:

$$\sum_{k=5}^{21} 2k + 4$$

sum(seq(2X + 4, X, 5, 21)) = $\boxed{510}$

Ex 12: Express in summation notation: 1 + 6 + 11 + 16 + ... + 46.

a_1 +5 +5 +5

$d = 5$

n=? rule=?

$$\sum_{k=1}^{10} 5k - 4$$

$= \boxed{235}$

$46 = 1 + 5(n-1)$

$46 = 1 + 5n - 5$

$46 = 5n - 4$ = rule

$10 = n$

RECURSIVE FUNCTIONS: *Always start with n=1 unless otherwise stated*

These give you two pieces of information:

1) a starting place, and 2) the rule for how to find the NEXT term based on the current (NOW) term.

a_1 NEXT = NOW $\boxed{}$ ↖pattern

Ex 14: Write the first 5 terms of the sequence $a_1 = 2$; $a_{n+1} = 3 + a_n$.

$a_1 = \boxed{2}$

$a_2 = 3+2 = \boxed{5}$

$a_3 = 3+5 = \boxed{8}$

$a_4 = 3+8 = \boxed{11}$

$a_5 = 3+11 = \boxed{14}$

Ex 15: Find the first 5 terms of: $a_1 = 1$, $a_2 = 2$, $a_{n+2} = a_n a_{n+1}$ (product of prev. 2 terms)

$a_1 = \boxed{1}$

$a_2 = \boxed{2}$

$a_3 = 1 \cdot 2 = \boxed{2}$

$a_4 = 2 \cdot 2 = \boxed{4}$

$a_5 = 2 \cdot 4 = \boxed{8}$

You Try:

1. Write out each of the following sums.

(a) $\sum_{n=1}^{6} n^4$

(b) $\sum_{k=3}^{7} \frac{k+1}{k}$

(c) $\sum_{i=2}^{n} (2i-1)$

(d) $\sum_{k=0}^{n} 2^{k+1}x^k$

(e) $\sum_{k=0}^{n} \frac{(-1)^k x^k}{2k+1}$

2. Express each of these sums using sigma notation.

(a) $1+4+9+16+25+36$ ($1^2\ 2^2\ 3^2\ 4^2\ 5^2\ 6^2$) → $\boxed{\sum_{k=1}^{6} k^2}$

(b) $3-5+7-9+11-13+15$

$a_1 = 3$

$d = \pm 2$

Alternating! $\boxed{\sum_{k=1}^{7} (-1)^{k+1}(2k+1)}$

$3+2(n-1)$

$3+2n-2$

$2n+1$

Find the first five terms of these recursively defined sequences

1. $a_1 = 2, a_{n+1} = 4a_n - 3$

$\boxed{2, 5, 17, 65, 257}$

$a_2 = 4(2)-3 = 8-3 = 5$

$a_3 = 4(5)-3 = 20-3 = 17$

$a_4 = 4(17)-3 = 68-3 = 65$

$a_5 = 4(65)-3 = 260-3 = 257$

2. $a_1 = -3, a_{n+1} = 2a_n - 5$

$\boxed{-3, -11, -27, -59, -123}$

$a_2 = 2(-3)-5 = -6-5 = -11$

$a_3 = 2(-11)-5 = -27$

$a_4 = 2(-27)-5 = -59$

$a_5 = 2(-59)-5 = -123$

3. $a_1 = 8, a_{n+1} = \frac{1}{2}a_n + 2$

$\boxed{8, 6, 5, 9/2, 17/4}$

$a_2 = 1/2(8)+2 = 6$

$a_3 = 1/2(6)+2 = 5$

$a_4 = 1/2(5)+2 = 9/2$

$a_5 = 1/2(9/2)+2 = 17/4$

4. $a_1 = 3, a_2 = 3, a_{n+1} = a_n - a_{n-1}$

$\boxed{3, 3, 0, -3, -3}$

$a_3 = 3-3 = 0$

$a_4 = 0-3 = -3$

$a_5 = -3-0 = -3$