

Day 5 Notes - Binomial Theorem

Warmup: Expand  $(x + y)^3 = (x + y)(x + y)(x + y)$

$$\begin{matrix} & x & y \\ x & \begin{matrix} x^2 & xy \end{matrix} \\ y & \begin{matrix} xy & y^2 \end{matrix} \end{matrix} = (x^2 + 2xy + y^2)(x + y)$$

$$\begin{matrix} & x^2 & 2xy & y^2 \\ x & \begin{matrix} x^3 & 2x^2y & xy^2 \end{matrix} \\ y & \begin{matrix} x^2y & 2xy^2 & y^3 \end{matrix} \end{matrix} = x^3 + 3x^2y + 3xy^2 + y^3$$

Let's look at the expansion of  $(x + y)^n$

$(x + y)^0 = 1$

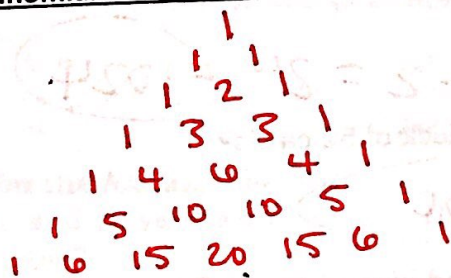
$(x + y)^1 = x + y$

$(x + y)^2 = x^2 + 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

Expanding a Binomial Using Pascal's Triangle



EX 1: Expand  $(x + 3)^4$ .

Fourth Row Coefficients: 1 4 6 4 1

These numbers are the same numbers that are the coefficients of the binomial expansion:

$$1x^43^0 + 4x^33^1 + 6x^23^2 + 4x^13^3 + 1x^03^4$$

$$= x^4 + 12x^3 + 54x^2 + 108x + 81$$

The expansion of  $(a + b)^4$  is:  
 $1a^4b^0 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1a^0b^4$   
 Notice that the exponents always add up to 4  
 with the a's going in descending order  
 and the b's in ascending order.  
 Now, just substitute x in for "a" and 3 in for "b".

EX 2: Expand  $(x - 2y)^4$ .

This time substitute  $x$  in for "a" and  $-2y$  for "b". Use  $( )$ .

$$1x^4(-2y)^0 + 4x^3(-2y)^1 + 6x^2(-2y)^2 + 4x^1(-2y)^3 + 1x^0(-2y)^4$$

$$= x^4 - 8x^3y + 24x^2y^2 - 32xy^3 + 16y^4$$

### The Binomial Theorem

In the expansion of  $(x + y)^n$ ...

$$(x + y)^n = x^n + nx^{n-1}y + \dots + {}_n C_m x^{n-m}y^m + \dots + nxy^{n-1} + y^n$$

The coefficient of  $x^{n-m}y^m$  is given by:

Exponent  
on  $x$

$${}_n C_m = \frac{n!}{(n-m)!m!}$$

Exponent  
on  $y$

EX 3: Find the following binomial coefficients.

$${}_8 C_2 = 28$$

$${}_{10} C_3 = 120$$

$${}_7 C_3 = 35$$

$${}_7 C_4 = 35$$

EX 4: Find the 6<sup>th</sup> term in the expansion of  $(3a + 2b)^{12}$ .

(Using the Binomial Theorem, let  $x = 3a$  and  $y = 2b$ , and note that in the 6<sup>th</sup> term, the exponent of  $y$  is

$$m = 5 \text{ and the exponent of } x \text{ is } n - m = 12 - 5 = 7.)$$

Consequently, the 6<sup>th</sup> term of the expansion is:

$${}_{12} C_5 x^7 y^5 = 792(3a)^7(2b)^5$$

$$= 55,427,328a^7b^5$$

## Binomial Theorem

Find each coefficient described.

1) Coefficient of  $vu^2$  in expansion of  $(v - 5u)^3$

-75

2) Coefficient of  $m^2$  in expansion of  $(4 - m)^3$

12

3) Coefficient of  $x^2y^2$  in expansion of  $(x - 2y)^4$

24

4) Coefficient of  $v$  in expansion of  $(3v - 1)^4$

-12

Find each term described.

5) 2nd term in expansion of  $(v + 4u)^4$

$16v^3u$

6) 4th term in expansion of  $(n + 5)^3$

125

Expand completely.

7)  $(u - v)^4$

$u^4 - 4u^3v + 6u^2v^2 - 4uv^3 + v^4$

8)  $(4y - x)^3$

$64y^3 - 48y^2x + 12yx^2 - x^3$

9)  $(x - 3)^4$

$x^4 - 12x^3 + 54x^2 - 108x + 81$

10)  $(4v + 1)^3$

$64v^3 + 48v^2 + 12v + 1$