Day 5 Notes – Binomial Theorem

Warmup: Expand $(x + y)^3$. $=(x+y)(x+y)(x+y)$

	x	y
x	x^2	xy
y	xy	y^2

$= (x^2+2xy+y^2)(x+y)$

	x^2	$2xy$	y^2
x	x^3	$2x^2y$	xy^2
y	x^2y	$2xy^2$	y^3

$=\boxed{x^3+3x^2y+3xy^2+y^3}$

Let's look at the expansion of $(x + y)^n$

$(x + y)^0 = 1$

$(x + y)^1 = x + y$

$(x + y)^2 = x^2 + 2xy + y^2$

$(x + y)^3 = x^3+3x^2y+3xy^2+y^3$

$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

Expanding a Binomial Using Pascal's Triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1 . . .

EX 1: Expand $(x + 3)^4$.

Fourth Row Coefficients: 1 4 6 4 1

These numbers are the same numbers that are the coefficients of the binomial expansion:

The expansion of $(a + b)^4$ is:

$1a^4b^0 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1a^0b^4$

Notice that the exponents always add up to 4 with the a's going in descending order and the b's in ascending order.

Now, just substitute x in for "a" and 3 in for "b".

$1x^4 3^0 + 4x^3 3^1 + 6x^2 3^2 + 4x^1 3^3 + 1x^0 3^4$

$=\boxed{x^4+12x^3+54x^2+108x+81}$

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EX 2: Expand $(x-2y)^4$.

This time substitute x in for "a" and -2y for "b". Use ( ).

$$1x^4(-2y)^0 + 4x^3(-2y)^1 + 6x^2(-2y)^2 + 4x^1(-2y)^3 + 1x^0(-2y)^4$$

$$= \boxed{x^4 - 8x^3y + 24x^2y^2 - 32xy^3 + 16y^4}$$

The Binomial Theorem

In the expansion of $(x+y)^n$...

$$(x+y)^n = x^n + nx^{n-1}y + \ldots + {}_nC_m x^{n-m}y^m + \ldots + nxy^{n-1} + y^n$$

The coefficient of $x^{n-m}y^m$ is given by:

Exponent on x

$$\boxed{{}_nC_m = \frac{n!}{(n-m)!m!}}$$

Exponent on y

EX 3: Find the following binomial coefficients.

${}_8C_2 =$ 28

${}_{10}C_3 =$ 120

${}_7C_3 =$ 35

${}_7C_4 =$ 35

EX 4: Find the 6th term in the expansion of $(3a+2b)^{12}$.

(Using the Binomial Theorem, let $x = 3a$ and $y = 2b$, and note that in the 6th term, the exponent of y is $m = 5$ and the exponent of x is $n - m = 12 - 5 = 7$.)

Consequently, the 6th term of the expansion is:

$${}_{12}C_5 x^7 y^5 = 792(3a)^7(2b)^5$$

$$= \boxed{55{,}427{,}328a^7b^5}$$

Day 5 Homework

Binomial Theorem

Find each coefficient described.

1) Coefficient of vu^2 in expansion of $(v - 5u)^3$

75

2) Coefficient of m^2 in expansion of $(4 - m)^3$

12

3) Coefficient of x^2y^2 in expansion of $(x - 2y)^4$

24

4) Coefficient of v in expansion of $(3v - 1)^4$

-12

Find each term described.

5) 2nd term in expansion of $(v + 4u)^4$

$16v^3u$

6) 4th term in expansion of $(n + 5)^3$

125

Expand completely.

7) $(u - v)^4$

$u^4 - 4u^3v + 6u^2v^2 - 4uv^3 + v^4$

8) $(4y - x)^3$

$64y^3 - 48y^2x + 12yx^2 - x^3$

9) $(x - 3)^4$

$x^4 - 12x^3 + 54x^2 - 108x + 81$

10) $(4v + 1)^3$

$64v^3 + 48v^2 + 12v + 1$