Day 4 Notes – Permutations and Combinations

Warmup: Mrs. Pace is having a tough time ranking her four favorite movies ("Furious 7", "Horrible Bosses", "My Best Friend's Girl", and "The Shack"). In how many ways can she arrange her four favorite movies?

24 Total options

1st choice: F7 | HB | MBFG | TS

2nd choice: HB, MBFG, TS

3rd choice: MBFG, TS | HB, TS | HB, MBFG

4th choice: TS | MBFG | TS | HB | MBFG | HB

6 options

HB 6 | MBFG 6 | TS 6

MULTIPLICATION PRINCIPLE OF COUNTING

If a task consists of a sequence of choices in which there are p selections for the first choice, q selections for the second choice, r selections for the third choice, and so on, then the total number of selections possible can be determined by:

$p \cdot q \cdot r \ldots$

Example 1: David has 6 pairs of pants, 10 shirts, and 5 ties. How many different outfits can he wear to work today?

$\underset{\text{pants}}{6} \cdot \underset{\text{shirts}}{10} \cdot \underset{\text{ties}}{5} = 300$ different outfits

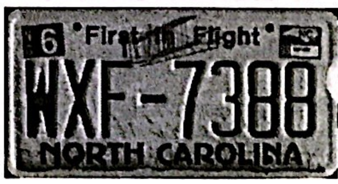
Example 2: Imagine that Mrs. Pace added "Enough" and "Mulan" to her favorite movies collection. In how many ways could she arrange them all?

$\underset{1st}{6} \cdot \underset{2nd}{5} \cdot \underset{3rd}{4} \cdot \underset{4th}{3} \cdot \underset{5th}{2} \cdot \underset{6th}{1} = 720$ ways

Example 3: You are at a math party (because you are super cool!!) and would like to take a bunch of trig selfies to post on Instagram. How many selfies will you need to take if you wish to have a selfie that includes you and each of: 4 friends, 3 backgrounds, and a trigonometric function?

$\underset{\text{friends}}{4} \cdot \underset{\text{BG}}{3} \cdot \underset{\text{Trig}}{6} = 72$ selfies !!

Example 4: The standard NC License Plates have 3 letters followed by 4 digits. How many license plate options are there?



$\underset{A-Z}{26} \cdot \underset{A-Z}{26} \cdot \underset{A-Z}{26} \cdot \underset{0-9}{10} \cdot \underset{0-9}{10} \cdot \underset{0-9}{10} \cdot \underset{0-9}{10}$

Repeats Allowed!! 175,760,000 possibilities

In the license plate example, letters and numbers can be repeated. In some situations, objects cannot be repeated....

PERMUTATIONS

order MATTERS

If a set has n objects, then there are $\underline{n!}$ (read: "factorial") ways of arranging them in order, where no item is repeated. The ordered arrangement is called a **permutation**.

EX #2: $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{720}$ ✓

Example 5: Count the number of ways that the letters in DRAGONFLY can be arranged.

$9! = \boxed{362880}$ In Calc.: $\boxed{\text{MATH}}$ → PRB

Example 6: How many 6-letter "words" can be formed from the letters in the word FRIDAY?

$6! = \boxed{720}$

Example 7: Count the number of ways that the letters in BUTTERFLY can be arranged.

Divide out repeats → $\dfrac{9!}{2!} = \dfrac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{1}} = \boxed{181{,}440}$

Example 8: Count the number of different ways that the letters in BUMBLEBEE can be arranged.

repeat B's → $\dfrac{9!}{3!\,3!}$ ← repeat E's $= \dfrac{9 \cdot 8 \cdot 7 \cdot \cancel{6} \cdot 5 \cdot 4 \cdot \cancel{3 \cdot 2 \cdot 1}}{\cancel{3 \cdot 2 \cdot 1} \cdot \cancel{3 \cdot 2 \cdot 1}} = \boxed{10{,}080}$

Example 9: Four people are chosen from a group of 10 to buy tickets to a concert. The first person chosen gets first choice of seating, second person gets to choose next, and so on. In how many ways can this be done?

$\underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} = \boxed{5040}$ ways

PERMUTATION COUNTING FORMULA

Notation: P(n, r) or ${}_nP_r$. This is read: "n objects taken r at a time" "n choose r"

P = Permutation n = number of objects to choose from r = number of "slots" to fill.

Formula for Permutation:

$${}_nP_r = \frac{n!}{(n-r)!}$$ ← Divide out the spots you don't need to fill.

Example 9 could be done using the formula:

n = 10 choices

r = 4 slots

$P(10,4) = {}_{10}P_4 = \dfrac{10!}{(10-4)!} = \dfrac{10!}{6!}$

$= \dfrac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = 5040$ ✓

Example 10: A teacher draws names out of a hat to determine the seating chart. If there are 25 students and 25 desks, how many different seating charts can be made?

$n=25$ $\quad 25! = {}_{25}P_{25} = \dfrac{25!}{(25-25)!} = 25!$

$r=25$

COMBINATIONS: ORDER IS IRRELEVAT

If you are picking 4 people out of 10 to serve on a committee, does the order in which they are chosen really matter? Nothing happens if they are chosen first versus chosen fourth. They would still be on the committee. This is a **combination**. ORDER DOES NOT MATTER!!!!!

Notation: C(n, r) or $_nC_r$.

Formula for Combination: $_nC_r = \dfrac{n!}{(n-r)!r!}$ This divides out the repeated possibilities.

Example 11: For Homecoming, the ballot of 30 girls must be narrowed to 10 for the court. In how many ways can this happen? (Order does not matter – you just need to be one of the 10.)

$n=30$ $\quad {}_{30}C_{10} = \dfrac{30!}{20!10!} = 30{,}045{,}015$

$r=10$

Example 12: For the Annual MathRules Party, Mrs. Pace is buying treats! At the store, she finds 7 varieties of soda and 10 varieties of snacks. How many combination of 3 soda options and 4 snack options are possible?

soda: $n=7$, $r=3$, $_7C_3$ $\quad$ snacks: $n=10$, $r=4$, $_{10}C_4$

$\underset{\text{soda}}{35} \cdot \underset{\text{snacks}}{210} = 7350$

Example 13: Papa Joe's offers 10 different toppings on their new specialty pizza. To use a coupon, you must choose 3. How many different pizzas do you have to choose from?

$_{10}C_3 = 120$ pizzas

<u>You decide: Permutation or Combination?</u>

a. A president, vice-president, and secretary are chosen from a 25-member garden club P
b. A cook chooses 5 potatoes from a bag of 12 potatoes to make potato salad C
c. A teacher makes a seating chart for 22 students in a classroom with 30 desks P
d. 30 people running for 4 spots on the Principal's Advisory Board C
e. 30 people running for Student Body Pres/VP/Sect/Treas P
f. "Pick 3" lotto. 3 numbers are chosen from 54 numbered balls (no repeats). C
g. 7 digit telephone number (no repeats). P

Day 4 Homework – Permutations and Combinations

1. There are four candidates for Prom Queen and three for King. How many King-Queen pairs are possible?

$\underset{\text{King}}{\underline{{}_3P_1}} \cdot \underset{\text{Queen}}{\underline{{}_4P_1}} =$ 12 possibilities

2. Excluding J, Q, X, and Z, how many 3-letter crossword entries can be formed that contain no repeated letters?

$\underline{22} \cdot \underline{21} \cdot \underline{20} = 9240$

3. How many different arrangements are there for the letters in the word MISSISSIPPI?

$\frac{11!}{4!\,2!\,4!} = 34650$

4. How many different license plates can be made containing 2 digits followed by 2 letters and then 3 more digits, if there are to be no repeated letters nor digits?

$\underline{10} \cdot \underline{9} \cdot \underline{26} \cdot \underline{25} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6} = \boxed{19{,}656{,}000}$

5. How many different outcome sequences of heads and tails are there if a coin is tossed 10 times?

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^{10} = 1024$

6. How many different 5-card hands are there from a deck of 52 cards?

${}_{52}C_5 = 2{,}598{,}960$

7. How many different 5-card hands include the ace and king of spades?

$\underline{A}$ ✓ $\underline{K}$ ✓ 50 cards left, 3 spots ${}_{50}C_3 = 19{,}600$

8. The head of the personnel department interviews 8 people for 3 identical job openings. How many different groups of 3 can be employed?

__ __ __ ${}_8C_3 = 56$

9. Mrs. Pace gives 20 study questions from which 8 will be chosen to be on the test. How many unique tests can she develop?

${}_{20}C_8 = 125{,}970$

10. How many different answer keys are there for a 10 question True/False test? What about a 10 question multiple choice test (A, B, C, D)?

$4^{10} = 1048576$ $2^{10} = 1024$

11. Luigi sells one size two-topping pizzas, but claims that his selection of toppings allows for "more than 4000 different choices." What is the smallest amount of toppings Luigi could offer?

$2^x = 4000$ $x = 12$