Unit 10 Day 2

Warm Up:

Parameterizations of an Equation:

Eliminate the parameter for the following equations:

1. $x = 4t - 3$ and $y = 2t + 1$

$+3 \quad +3$

$\frac{x+3}{4} = \frac{4t}{4}$

$t = \frac{x+3}{4}$

$y = 2\left(\frac{x+3}{4}\right) + 1$

$y = \frac{x+3}{2} + 1$

$\boxed{y = \frac{1x}{2} + \frac{5}{2}}$

2. $x = t^2$ and $y = t + 1$

$-1 \quad -1$

$y - 1 = t$

$x = (y-1)^2$

$\boxed{x = y^2 - 2y + 1}$

or

$\boxed{y = \pm\sqrt{x} + 1}$

3. $x = t - 3$ and $y = 2/t$

$-5 \le t \le 5$

$ty = 2$

$t = \frac{2}{y}$

$x = (2/y) - 3$

$x + 3 = \frac{2}{y}$

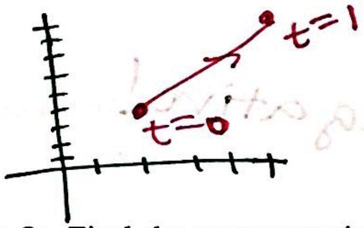
$\boxed{y = \frac{2}{x+3}}$

Notes:

Parameterization of a line is the equation of the line in parametric form. In other words, we are going to separate the x and y coordinates and describe how each of them change in a function of a parameter t, usually time.

start x   start y

Example 1: Find the parameterization of the line segment with endpoints (2, 4) and (5, 9).



$t=1$

$t=0$

$\boxed{\begin{aligned} x &= 2 + 3t \\ y &= 4 + 5t \end{aligned}}$

$\Delta x = 5 - 2 = 3$

$\Delta y = 9 - 4 = 5$

Example 2: Find the parameterization of the line through the point (-2, 4) and (1, 2)

$\boxed{\begin{aligned} x &= -2 + 3t \\ y &= 4 - 2t \end{aligned}}$

$\Delta x = 1 - -2 = 3$

$\Delta y = 2 - 4 = -2$

CIRCLES (These aren't functions, but now can be graphed! Woohoo!!)

Thinking of the Ferris Wheel, we can come up with a parameterization for a circle:

Center = (0, 0)

~~x=cos t~~   ~~y=sin t~~ $0 \le t < 2\pi$

$x = r\cos t \quad y = r\sin t$

a. Now, make a circle with radius 5 $\boxed{x = 5\cos t \quad y = 5\sin t}$
b. Make Part a. go around 2x $\boxed{x = 5\cos(2t) \quad y = 5\sin(2t)}$
c. Make Part a. go clockwise $\boxed{x = 5\cos t \quad y = -5\sin t}$
d. Make Part a. start at (0,5) $\boxed{\text{d. set } \pi/2 \le t \le 5\pi/2}$
e. Make the center of Part a. be (4,-2)

h   k

Center (h, k):

$x = r\cos t + h$

$y = r\sin t + k$

$\boxed{\begin{aligned} x &= 5\cos t + 4 \\ y &= 5\sin t - 2 \end{aligned}}$

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Ex) Parameterize a circle with r=11 and center (0,3).

$$\boxed{\begin{aligned} x &= 11\cos t \\ y &= 11\sin t + 3 \end{aligned}}$$

ELLIPSES (This is the same idea as with circles with $\cos^2 t + \sin^2 t = 1$… just one side is longer.)

Ex) How can we represent an ellipse with center (0, 0) with vertex (0, 4) and b=3 starting at (0, 4) and rotating counterclockwise?

$a=4$ Major Axis $= y$

Minor Axis $= x$

$$\boxed{\begin{aligned} x &= 3\cos t \\ y &= 4\sin t \end{aligned} \qquad \frac{\pi}{2} \le t \le \frac{5\pi}{2}}$$

What happens if you increase the interval $0 \le t < 4\pi$?

2 full rotations / cycle

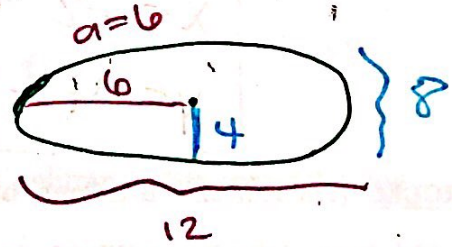
How would you go the opposite way around the ellipse?

Make y negative!

Ex) Parameterize an ellipse with a center of (-3,5), a horizontal major axis length of 12, and a vertical minor axis length of 8.

$\leftrightarrow x$

$\updownarrow y$ $b=4$



$$\boxed{\begin{aligned} x &= 6\cos t - 3 \\ y &= 4\sin t + 5 \end{aligned}}$$

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Word Problems

Example 1: Wayne and Garth are in a Foot Race. Wayne can sprint at a rate of 20 ft/sec. Garth can sprint at a rate of 18 ft/sec. Wayne gives Garth a 4ft head start. The parametric equations to model the race are:

x: ft   t: sec   y: "lane"

G $\boxed{X_1 = 18t \quad y_1 = 3}$
W $\boxed{X_2 = 20t - 4 \quad y_2 = 5}$ (This number doesn't matter, we can pretend that this is the lane #)

a) Find a viewing window to simulate a 100 yard dash. Mode: simul. T = 0, Tstep = .05…

=300 ft

Let x be at least 300.

b) Who is ahead after 3 seconds and by how much? (We can use TRACE to do this.)

Garth: $t=3 \rightarrow x=56$ ft

Wayne: $t=3 \rightarrow x=54$ ft

$\boxed{\text{Garth is ahead by 2 ft.}}$

Example 2: Grayson Allen and Theo Pinson are sprinting a race. Theo can sprint at the rate of 24 ft/sec and Grayson can sprint at 20 ft/sec. Theo knows that Grayson loves to trip people so he gives him a 10 ft head start. The race they are running is the 100 yard dash.

a) Find the parametric equations to model Theo and Grayson's race.

~~Garth~~ Grayson: $x_1 = 20t$ $y_1 = 3$

Theo: $x_2 = 24t - 10$ $y_2 = 5$

irrelevant

b) Who wins?

$\boxed{\text{Theo}}$ reaches x = 300 ft first!

Parabolas

The trigonometric functions can also be used with parabolas if we think of the initial velocity in terms of the vertical and horizontal components of the velocity vector.

initial speed $= V_0$

$v_0 \sin\theta$: vertical component

$V_0 \cos\theta$: horizontal component

Recall: $\boxed{\textbf{Distance = rate x time}}$

As a result, Horizontal motion can be defined as: $x = (V_0 \cos\theta)t$

The Vertical motion is: $$y = -16t^2 + (v_0 \sin\theta)\,t + h$$

(h = initial height at t = 0, height is in feet)

$h=3$ $V_0 = 150$

Example 1: Kevin hits a baseball at 3 feet above the ground with an initial speed of 150 ft/sec at an angle of 20° with the horizontal. If the outfield wall is 20 feet high and 400 feet away from Kevin, will he hit a homerun?

$\theta = 20°$ $x = (150\cos(20°))t$ *Use $\boxed{\text{TRACE}}$ to find max: $x = 225.5$ ft, $y = 44.1$ ft

$y = -16t^2 + (150\sin(20°))t + 3$

Set: $0 \le t < 4$, xmin=0, Xmax=500, xscl=25, ymin=0, ymax=100, yscl=20

How long will the ball be in the air?

(Use Trace)

Between 3.2 + 3.3 sec.

Look at Table. ($\boxed{\text{2nd}}$ $\boxed{\text{WINDOW}}$ Indep. $\boxed{\text{ASK}}$)

Between $2.83 < t < 2.84$, we see when $x > 400$, $y < 20$. NO

What angle will be necessary to hit the ball over the wall?

Class Discussion...

Example 2: Les is riding on a Ferris Wheel with a radius of 30 feet. The bottom of the wheel is 10 feet off the ground. The wheel is turning counterclockwise at a rate of 1 revolution every 10 seconds. Les is sitting at 0°. Find the parametric equations to model Les's ride. Find his position 22 seconds into the ride.

$x = 30\cos t$ $y = 30\sin t + 10$

$\text{Freq} = \frac{1}{10} = \frac{b}{2\pi}$

$10b = 2\pi$

$b = \frac{\pi}{5}$

$$\boxed{x = 30\cos\frac{\pi}{5}t \qquad y = 30\sin\frac{\pi}{5}t + 10}$$

When $T = 22$, $x = 9.27$ ft (horizontal)

and $y = 38.5$ ft (vertical).