 Name: Key

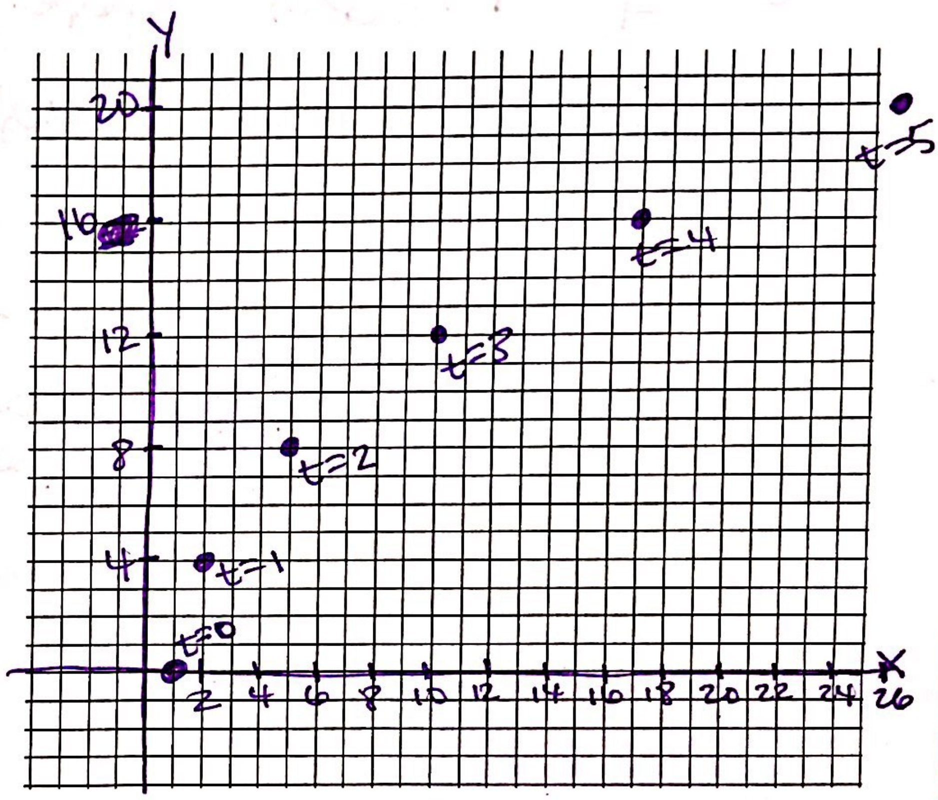
INTRODUCTION TO PARAMETRICS

Suppose t represents time in seconds and the position (x, y) of a particle at time t is... $x = t^2 + 1$ and $y = 4t$ with $t \geq 0$

a) Find the position (x,y) at these times:

t	x	y
0	1	0
1	2	4
2	5	8
3	10	12
4	17	16
5	26	20

b) Plot the path of the particle on the graph



c) Find the rectangular equation of the path. (Hint: Solve an equation for t and substitute.)

$$x = t^2 + 1$$

$$y = 4t$$

$$\frac{y}{4} = \frac{4t}{4}$$

$$\frac{y}{4} = t$$

$$x = (y/4)^2 + 1$$

$$x = \frac{y^2}{16} + 1$$

$$16(x-1) = \frac{y^2}{\cancel{16}} \cdot \cancel{16}$$

$$\pm\sqrt{16x - 16} = \sqrt{y^2}$$

$$\boxed{y = \pm\sqrt{16x - 16}}$$

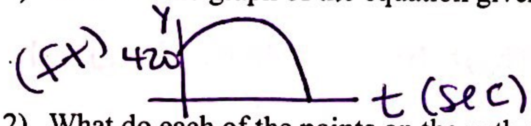
Day 1 of Parametric Equations

Suppose a rock is dropped from a 420 foot tower. The rock's height y in feet above the ground t seconds after it has been dropped (ignoring air resistance) is modeled by $\boxed{y=-16t^2+420.}$

y-int: (0,420)

Thinking ahead ....

1) Describe the graph of the equation given above. (Shape, key features, etc.)



Quadratic Function
Shape: parabola (opens down)

2) What do each of the points on the path of the curve represent?

X: time (in seconds) Y: height (in feet)

3) Calculate the position of the rock after 1, 2, 3, 4, and 5 seconds. (Make it easy on yourself ... remember how to have your calculator do this on the graph....)

t	y
1	404
2	356
3	276
4	164
5	20

$Y_1=-16(x)^2+420$

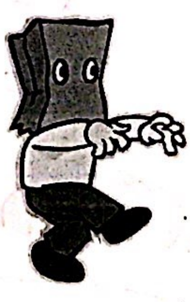
looked @ Table!

4) If the rock is dropped, what path will it take on the way down? (Remember, no air resistance...also, don't think about what the graph looks like – picture yourself dropping a rock off a building, what happens?)

A vertical Line!

Directions: Set up your calculator in the following manner:

a) MODE: Highlight and press enter on *Parametric (par), Degree and Simultaneous (simul)*
b) Go to "y=" and let $X_{1T}=2.5$ and $Y_{1T}=-16T^2+420$.
c) Go to "window" and set the following dimensions:
   a. Tmin = 0; Tmax = 5; Tstep = .2
   b. Xmin = 0; Xmax = 5; Xscl = 0
   c. Ymin = -10; Ymax = 420; Yscl = 0
d) Go back to "y=", arrow ***all the way to the left*** in the X_{1T} equation line (past X_{1T}) and continue pressing "enter" until you get a symbol that looks like: ─o
e) Now press graph and watch the magic unfold ☺...



1) What does the graph represent?

The height of the rock as it is dropped IN REAL TIME!

2) Trace along the graph and notice the different points. What do each of the values given represent?

T: TIME (sec)

X: HORIZONTAL distance (constant in this ex.)

Y: VERTICAL height (ft)

3) The two equations that you entered are called "parametric equations with parameter t". In your own words describe what that means.

Both x and y are dependent on "t", NOT dependent on each other!

4) What is the "t-step" value? (It's under "window" on your calculator.) Change it's value a few times (make it bigger or smaller – but remember what your Tmax and Tmin are set at) and then press "graph" each time you change it. What happens?

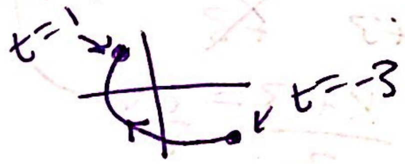
"t-step" affects how fast the graph ~~[illegible]~~ will graph

"t-step" bigger → faster, smaller → slower!

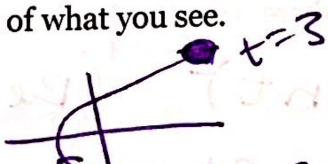
A few more parametric equations to graph...

For the given parameter interval, graph the parametric equations: $x = t^2 - 2$, and $y = 3t$, (set your window to X:[-10,10] and Y:[-10,10]
min max min max

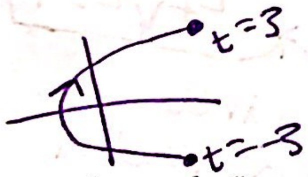
a) $-3 \le t \le 1$ (use that to set your Tmin and Tmax on your calculator. Sketch a graph of what you see.
min max



b) $-2 \le t \le 3$ Sketch a graph of what you see.



c) $-3 \le t \le 3$



Notice that as you change the "parameter" ← t – the element that defines both the "x" and the "y" value of the graph, the picture of the graph changes.

GET IT? A parametric equation is an equation that defines **both elements** x and y in terms of a 3rd element, the parameter, t. Often, "t" represents time.

Notice that we were able to describe the same situation in two different ways. One was in function form $\boxed{y=f(x)}$ and the other was in parametric form. Sometimes it is helpful to define an equation in a different format than the way it is given to us. When we convert from parametric form to function form, it is called "Eliminating the Parameter". In other words, your goal is to get the equation in terms of x's and y's.

① Solve one eqn. for t.
② Subst. into other eqn.
③ Solve for y.

Eliminating the Parameter

1) Use the equations from the previous example and eliminate the parameter.

$x = t^2 - 2$

$y = 3t$

$\frac{y}{3} = \frac{3t}{3}$

$\frac{y}{3} = t$

$x = (y/3)^2 - 2$

$x = \frac{y^2}{9} - 2$

$9(x+2) = \frac{y^2}{9} \cdot 9$

$\sqrt{9x+18} = \sqrt{y^2}$

$$\boxed{y = \pm\sqrt{9x+18}}$$

2) For the equations $\boxed{x = 1-2t}$ and $\boxed{y = 2-t}$, where $-\infty < t < \infty$ eliminate the parameter and identify the graph of the parametric curve (i.e. line, slope? Parabola, opens? etc.)

$y = 2 - t$
$+t \quad +t$

$y + t = 2$
$-y \quad -y$

$t = 2 - y$

$x = 1 - 2t$

$x = 1 - 2(2-y)$

$x = 1 - 4 + 2y$
$+3 \quad +3$

$\frac{x+3}{2} = \frac{2y}{2}$

$$\boxed{y = \frac{x}{2} + \frac{3}{2}}$$

Linear Function

slope $= \frac{1}{2}$, $(0, 3/2)$

3) For the equations $x = 2\cos t$ and $y = 2\sin t$ where $0 \le t \le 360°$, eliminate the parameter and identify the parametric curve.

$(x = 2\cos t)^2 \quad (y = 2\sin t)^2$

(Recall: $r^2 = x^2 + y^2$)

$x^2 = 4\cos^2 t \quad y^2 = 4\sin^2 t$

* ADD these equations!! *

$x^2 + y^2 = 4\cos^2 t + 4\sin^2 t$

$x^2 + y^2 = 4(\cos^2 t + \sin^2 t)$

$= 1$!! ☺

$\boxed{x^2 + y^2 = 4}$

$(r^2 = 4)$

$r = 2$

circle w/ radius of 2!