# Unit 0.5: More Integration Techniques

Integration by Parts:

What is Integration by Parts? When is it used?

- Integration by parts is the process of taking the integral of the product of a set of functions in terms of the integral of their derivative and antiderivative. It is usually used to take the integral of an integral that can't be solved through other methods, and when it is the product of 2 things and neither is the derivative of the other.

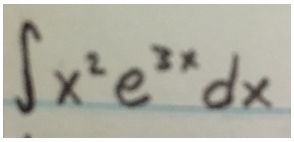
Equation

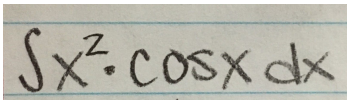
- The equation for integration by parts comes from the derivative product rule. The equation for Integration by parts is:

$$\int udv = uv - \int vdu$$

Deciding on a U

- When choosing U, there is an acronym that makes it easier to remember the order of choosing U:
  - L - Logarithmic
  - I - Inverse Trig
  - A - Algebraic (polynomial)
  - T - Trig
  - E - Exponential
- Keep in mind, the integral should become easier.

Ex1. 

Ex3. 

Partial Fraction Decomposition:

What is Partial Fraction Decomposition?

- PFD is an operation that consists of expression the fraction as a sum of a polynomial and one or several fractions with a simpler denominator.

How is it done?

- Usually, Partial Fraction Decomposition is used when the denominator of a function within an integral is able to be factors. You take the factors of the denominator and set up the equation as: f(x) = A / Factor #1 + B /

Factor #2. You then multiple by the bottom of the f(x) and solve for A and B.

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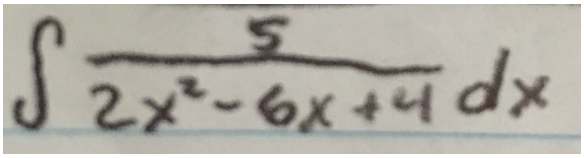
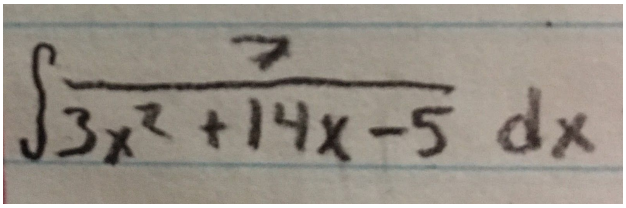
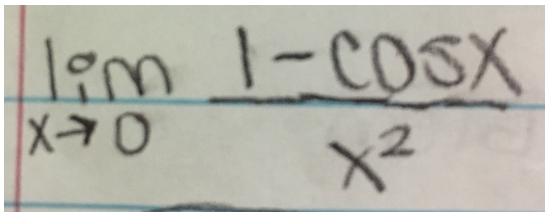
Ex2. 

Ex4. 

## Unit 4: Improper Integrals

L'Hopital's Rule:

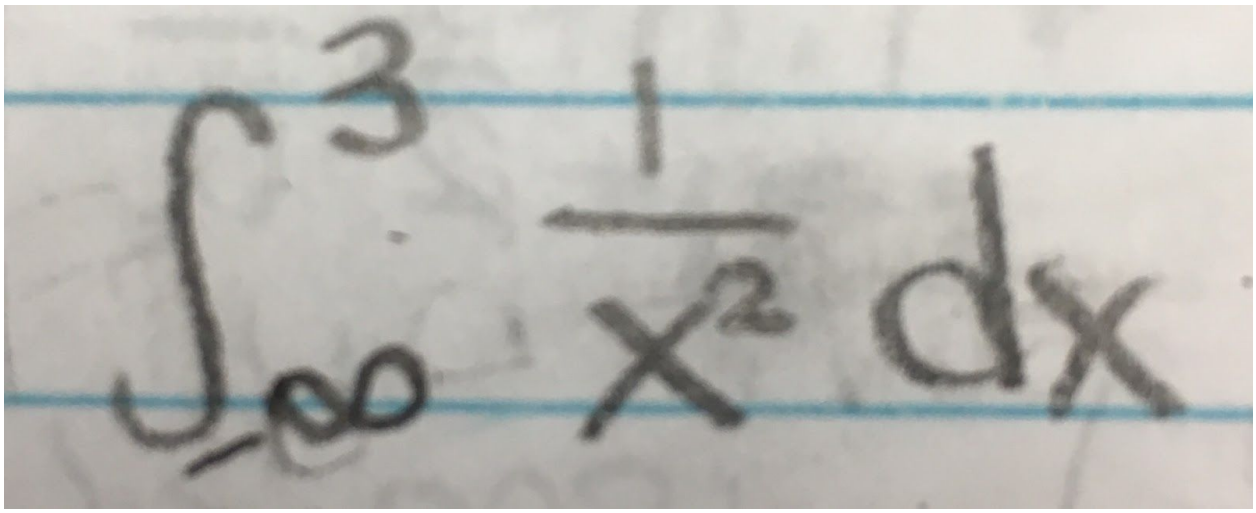
- When do we apply L'Hopital's Rule? We use this rule when direct substitution gives an indeterminate form.
    - Indeterminate form can look like….
        - 0/0
        - $\pm\infty/\pm\infty$
        - $0 \cdot \infty$
        - $\infty - \infty$
        - $1^{\infty}$
        - $0^{0}$
        - $\infty^{0}$
- L'Hopital's rule is that when direct substitution gives an indeterminate form we can take the derivative of the numerator and denominator separately.
    - After that we can try direct substitution again or we might require to use L'Hopital's again.
        - Repeat this process until direct substitution gives an answer or you give up.
- L'Hopital's Formula: $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)} = \frac{f'(a)}{g'(a)}$

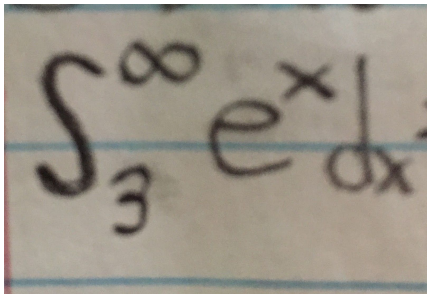
Ex1. 


Improper Integrals:

- When do we use the improper integrals method? When a integral isn't continuous over every point of the interval of integration.
    - When presented with one of these problems you should find at what point the integral is discontinuous (Graphing the function helps).
        - Once you know the point at which the integral is discontinuous you can make a limit statement of the integral evaluated at that point.
            - If the point of discontinuity is in between the bounds then you must separate the integral at that point and make limit statements of both.
            - If the discontinuity is at the upper bound then the limit approaches from the left.
            - If the discontinuity is at the lower bound then the limit approaches from the right.

Ex2.



Ex3. 


Ex4.