DAY 5 POST-QUIZ NOTES

TYPES OF TRANSFORMATIONS:

Vertical/Horizontal Shifts

$y = f(x)$

$f(x \boxed{-c})$ MOVE Right c units $f(x \boxed{+c})$ ~~MOVE~~ LEFT c units

$f(x) \boxed{-k}$ MOVE Down k units $f(x) \boxed{+k}$ MOVE UP k units

Reflections

outside!

$\boxed{-}f(x)$ Reflect over x-axis

inside!

$f(\boxed{-x})$ Reflect over y-axis

(Recall even symmetry)

Stretches/Shrinks

Horizontal: $f\!\left(\boxed{\dfrac{x}{c}}\right)$

Vertical: $\boxed{c}f(x)$

Stretch by c if: $\underline{c > 1}$

Shrink by c if: $\underline{c < 1}$

Parent

call it $g(x)$!!

Vertical shrink ← up 2

Ex. 1. Let $\boxed{f(x) = x^3 - 2}$. Transform $f(x)$ into $\boxed{\tfrac{1}{2}}\,f(\boxed{x-1})\,\boxed{+2}$.

Right 1

start: $f(x) = x^3 - 2$

input $(x-1)$: $f(x-1) = (x-1)^3 - 2$

Mult. $\tfrac{1}{2}$: $\tfrac{1}{2} f(x-1) = \tfrac{1}{2}\left((x-1)^3 - 2\right)$

$\tfrac{1}{2} f(x-1) = \tfrac{1}{2}(x-1)^3 - 1$

Add 2: $\tfrac{1}{2} f(x-1) + 2 = \tfrac{1}{2}(x-1)^3 - 1 + 2$

$\boxed{g(x) = \tfrac{1}{2}(x-1)^3 + 1}$

Ex. 2. Let $\boxed{f(x) = 2x + 1}$. Transform $f(x)$ into $\boxed{3}\,f(\boxed{-x})\,\boxed{-4}$.
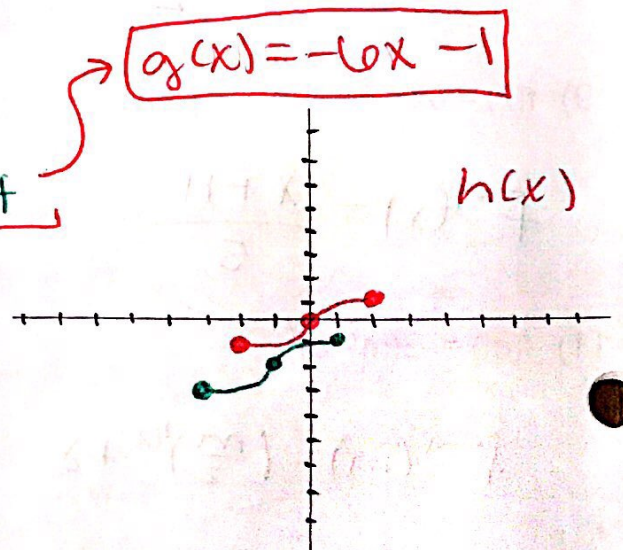
Input $(-x)$: $f(-x) = 2(-x) + 1$

$f(-x) = -2x + 1$

Mult. 3: $3f(-x) = 3(-2x + 1)$

$3f(-x) = -6x + 3$

Subtr. 4: $3f(-x) - 4 = -6x + 3 - 4$

$\boxed{g(x) = -6x - 1}$

Ex. 3. Graph $h(x+1) - 2$.

Left 1   Down 2

$h(x)$

Example 1: Describe how the graph of $y = |x|$ can be transformed to the graph of the given equation.

a) $y = |x| - 4$

Move down 4 units

b) $y = |x + 2|$

Move Left 2 units

Example 2: Find an equation for the reflection of $f(x) = \dfrac{5x-9}{x^2+3}$ across each axis.

y-axis
$$f(-x) = \frac{-5x-9}{x^2+3}$$

x-axis
$$-f(x) = \frac{-5x+9}{x^2+3} \quad \text{or} \quad \frac{5x-9}{-x^2-3}$$

Example 3: Let C_1 be the curve defined by $y_1 = f(x) = x^3 - 16x$. Find the following equations:

a) C_2 is a vertical stretch of C_1 by a factor of 3.

$$C_2 = 3x^3 - 48x$$

b) C_3 is a horizontal shrink of C_1 by a factor of $1/2$.

$$C_3 = \frac{1}{2}(x^3 - 16x)$$

Example 4: Find the equation of the graph of $y = x^2$ if it undergoes the following transformations in order.

Does the order matter? yes

1. ● A horizontal shift 2 units to the right
2. ● A vertical stretch by a factor of 3
3. ● A vertical translation 5 units up

① $y = (x - 2)^2$

② $y = 3(x - 2)^2$

③ $y = 3(x - 2)^2 + 5$

Example 5: Sketch the graph of $y = 1 + 2f(x - 1)$.

vertical stretch by 2
right 1
up 1