Task 4: Graphing Polar Equations

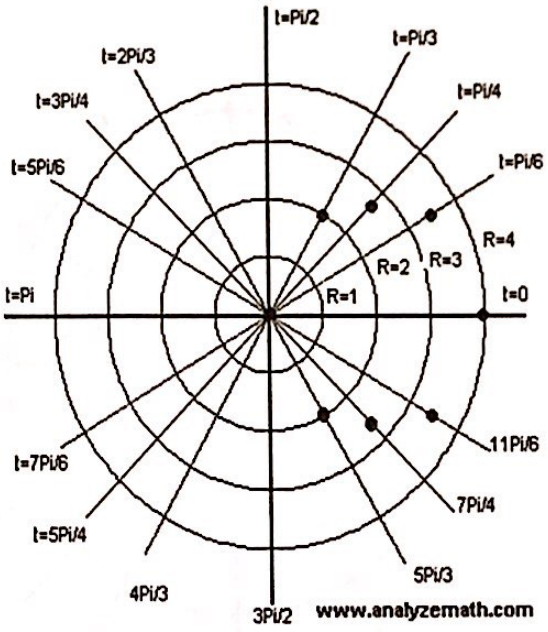
This is a tutorial on graphing polar equations by hand, or sketching, to help you gain a deeper understanding. Points in polar coordinates are represented by (r, θ) where r is the distance from the pole and θ is the direction angle. The method of point-by-point is used here.

Example 1: Graph the polar equation given by $r = 4 \cos \theta$ and identify the graph.

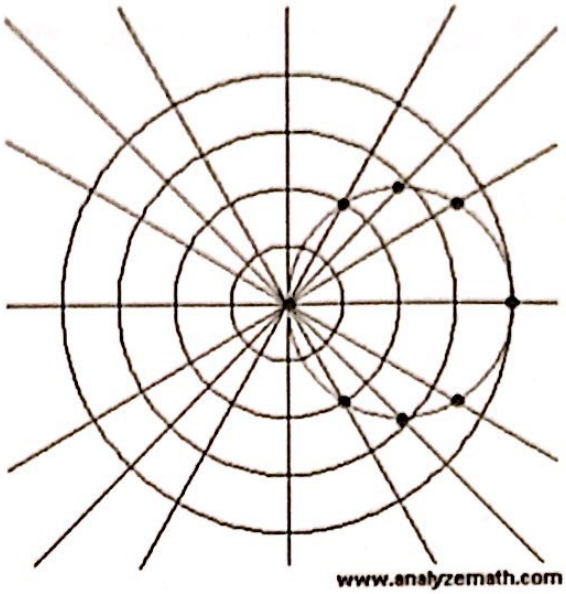
- We first construct a table of values using the special angles and their multiples. Substitute the "main" values of θ into the equation to find out what r will equal.

θ	r
0	
Pi / 6	
Pi / 4	
Pi / 3	
Pi / 2	
2 Pi / 3	
3 Pi / 4	
5 Pi / 6	
Pi	

- We stop the calculations at θ = Pi because the values will repeat.
- We now plot the above points on a grid of polar coordinates, as shown to the right. You need to pay special attention to points with a negative value of r. (t = θ)



- We now join the points drawing a smooth curve as seen on the next page.

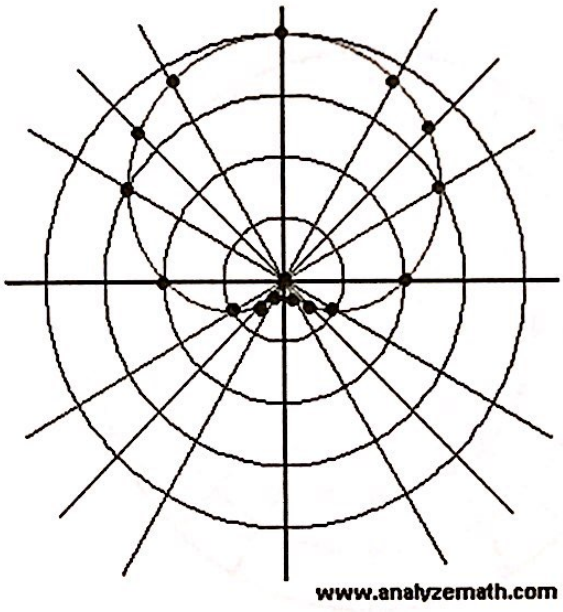


Example 2: Graph the polar equation given by $r = 2 + 2 \sin \theta$ and identify the graph.

- We first construct a table of values using the special angles and their multiples.
  FILL OUT THE TABLE BELOW.

θ	r
0	
Pi / 6	
Pi / 4	
Pi / 3	
Pi / 2	
2 Pi / 3	
3 Pi / 4	
5 Pi / 6	
Pi	
7 Pi / 6	
5 Pi / 4	
4 Pi / 3	
3 Pi / 2	
5 Pi / 3	
7 Pi / 4	
11 Pi / 6	

- We plot the points in the table then join them by a smooth curve. The points and the graph of the given polar equation are shown below.

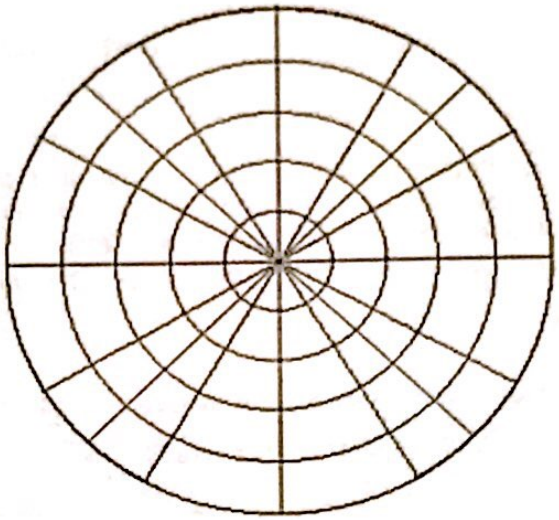


This graph is called a CARDIOID.

Example 3: Graph the polar equation given by $r = 4 \cos 2\theta$. **FILL IN TABLE AND GRAPH.**

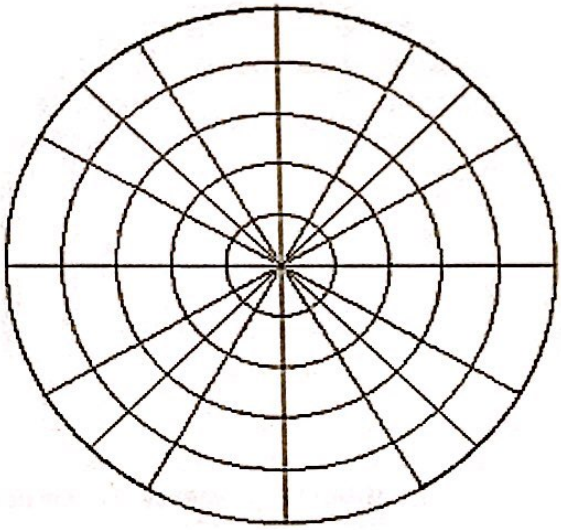
θ	r
0	
Pi / 6	
Pi / 4	
Pi / 3	
Pi / 2	
2 Pi / 3	
3 Pi / 4	
5 Pi / 6	
Pi	
7 Pi / 6	
5 Pi / 4	
4 Pi / 3	
3 Pi / 2	
5 Pi / 3	
7 Pi / 4	
11 Pi / 6	

We first plot the points in the table then join them by a smooth curve. **YOUR GRAPH SHOULD LOOK LIKE A FLOWER!**



Example 4: Graph the polar equation given by $r = 2 + \cos\theta$.

θ	r
0	
Pi / 6	
Pi / 4	
Pi / 3	
Pi / 2	
2 Pi / 3	
3 Pi / 4	
5 Pi / 6	
Pi	
7 Pi / 6	
5 Pi / 4	
4 Pi / 3	
3 Pi / 2	
5 Pi / 3	
7 Pi / 4	
11 Pi / 6	



I. USING YOUR CALCULATOR TO *HELP* YOU:

To graph in POLAR form in the calculator, you must follow these steps:

- MODE
- On the fourth line, select POL
- Also in MODE, make sure you are in RADIANS
- Y = (it should now say r =)
- Type in the equation
- ZOOM TRIG
- Change the window as necessary for each problem
- TBLSET – set TblStart = 0 and ΔTbl = π/6 (this will help you fill out tables on your paper)

II. PRACTICE WITH DIFFERENT TYPES

1. The **Archimedean spiral** (also known as the **arithmetic spiral**) is a spiral named after the 3rd century BC Greek mathematician Archimedes. It is the locus of points corresponding to the locations over time of a point moving away from a fixed point with a constant speed along a line which rotates with constant angular velocity. Equivalently, in polar coordinates (r, ϑ) it can be described by the equation

$$r = a + b\theta$$

with real numbers a and b. Changing the parameter a will turn the spiral, while b controls the distance between successive turnings.

Example: In your calculator, graph r = 1 + θ from 0 to 8π.

WINDOW: Change θmax to 8π.

Draw a sketch of your Archimedean Spiral:

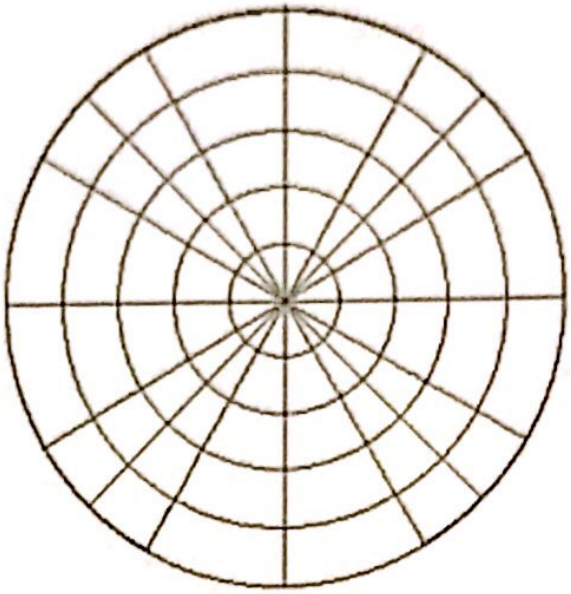
2. **Limacons with an Inner Loop**

Example: Use the TABLE of your calculator to help you graph the following BY HAND:

$$r = \frac{1}{2} + \cos t$$

Fill out the table on the next page and graph.

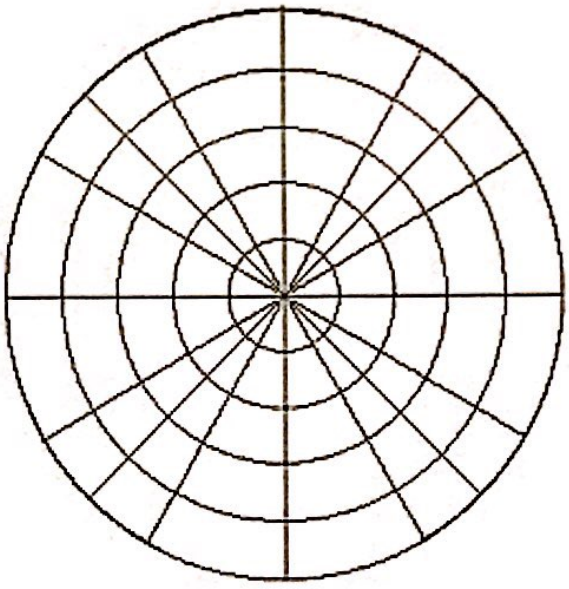
t	r
0	
Pi / 6	
Pi / 4	
Pi / 3	
Pi / 2	
2 Pi / 3	
3 Pi / 4	
5 Pi / 6	
Pi	
7 Pi / 6	
5 Pi / 4	
4 Pi / 3	
3 Pi / 2	
5 Pi / 3	
7 Pi / 4	
11 Pi / 6	



3. A Rose Within A Rose

Example: Use the TABLE of your calculator to help you graph the following BY HAND:
$r = 1 - 2 \sin(3\theta)$

θ	r
0	
Pi / 6	
Pi / 4	
Pi / 3	
Pi / 2	
2 Pi / 3	
3 Pi / 4	
5 Pi / 6	
Pi	
7 Pi / 6	
5 Pi / 4	
4 Pi / 3	
3 Pi / 2	
5 Pi / 3	
7 Pi / 4	
11 Pi / 6	



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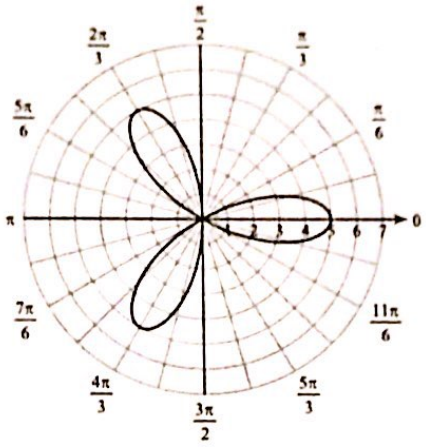
Graphs of Polar Equations

Date_____ Period____

Consider each polar graph. Classify the curve; and determine if the graph is symmetric with respect to the origin, polar axis, and line $\theta=\pi/2$.

1)



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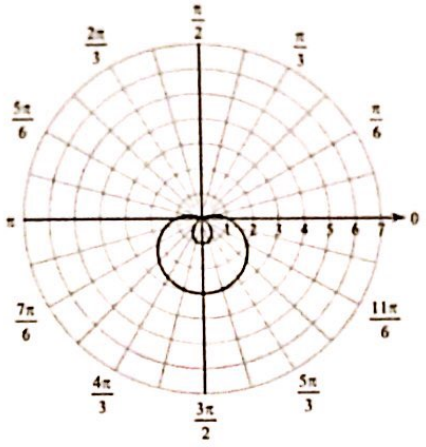

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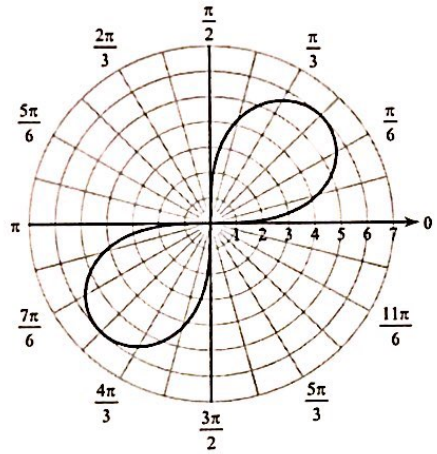

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