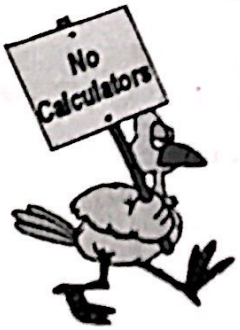
AP Calculus Summer Assignment Name: Key



NEATLY SHOW YOUR WORK ON A SEPARATE SHEET OF PAPER. Please BOX YOUR FINAL ANSWERS.

Evaluate. ***You should be able to do these without a calculator. You will be expected to work similar problems on a quiz or test without a calculator. Your answers should be in radian measures.***

1.) $\arcsin\left(\frac{\sqrt{2}}{2}\right)$ $\pi/4$

2.) $\arctan(-1)$ $-\pi/4$

3.) $\cos^{-1}\left(-\frac{1}{2}\right)$ $2\pi/3$

4.) $\arccos(0)$ $\pi/2$

5.) $\tan^{-1}(0)$ 0

Solve for y:

6.) $-\frac{1}{3y^2}=\frac{1}{5}+4x^3$ $y=\pm\sqrt{\frac{-5}{3+60x^3}}$

Rewrite the following using interval notation

7.) $-7\le x<9$ $[-7,9)$

8.) $x<12$ $(-\infty,12)$

9.) All real numbers $(-\infty,\infty)$

Find $f^{-1}(x)$ for the following functions.

10.) $f(x)=e^x$ $f^{-1}(x)=\ln x$

11.) $f(x)=\frac{1}{x-2}$ $f^{-1}(x)=\frac{1}{x}+2$

12.) $f(x)=\frac{1}{\sec x}$ $f^{-1}(x)=\cos^{-1}x$

13.) $f(x)=x^2+5$ $f^{-1}(x)=\pm\sqrt{x-5}$

For each of the functions find the indicated value.

14.) Given $f(x)=\sin x$; Find $f(\pi)$; $f\left(\frac{2}{3}\pi\right)$ $f(\pi)=0$ $f\left(\frac{2\pi}{3}\right)=\frac{\sqrt{3}}{2}$

15.) Given $f(x)=3x-1$; Find $\frac{f(x)-f(1)}{x-1}$ 3

16.) Given $f(x)=x^3$; Find $\frac{f(a+h)-f(a)}{h}$ $3a^2+3ah+h^2$

17.) $f(x)=\begin{cases}-2x-6, & \text{for } x\le -2\\ 2-x^2, & \text{for } -2<x<2\\ 2x-6, & \text{for } x\ge 2\end{cases}$; $f(-4), f(2)$ $f(-4)=2$, $f(2)=-2$

Simplify.

18.) $\ln e^{x-3}$ $x-3$

19.) $e^{\ln(x-1)}$ $x-1$

Rewrite the following in simplified form.

20.) $\ln\frac{2a}{b}-\ln\frac{a}{b}$ $\ln 2$

21.) $4\log(xy)-3\log(xy)$ $\ln(xy)$

22.) $\frac{1}{3}\ln e^2$ $2/3$

Algebraically determine the x and y intercepts. Name the intercepts as points.

23.) $y^2=x^2-4x$ $(0,0)$, $(4,0)$

24.) $y=\frac{x^2+3x+2}{(3x+1)^2}$ $(-2,0)$, $(-1,0)$, $(0,2)$

Algebraically determine the domain and range of the following functions.
State your answer using interval notation.

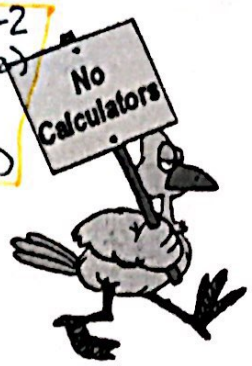
25.) $f(x)=\frac{2x+4}{x^2-4}$ D: $(-\infty,2)\cup(-2,2)\cup(2,\infty)$ R: $(-\infty,0)\cup(0,\infty)$

26.) $f(x)=\ln(2x^2+4x-30)$ D: $(-\infty,5)\cup(3,\infty)$ R: omit

State the EQUATIONS of any horizontal asymptotes and/or vertical asymptotes.
State the domain and range.

27.) $y=\frac{x^2-4}{x^2-x-12}$ H.A.: $y=1$ V.A.: $x=-3, x=4$ D: $(-\infty,-3)\cup(-3,4)\cup(4,\infty)$ R: $(-\infty,1)\cup(1,\infty)$

28.) $y=\frac{2x-3}{2x^2+x-6}$ H.A.: $y=0$ V.A.: $x=-2$ D: $(-\infty,-2)\cup(-2,3/2)\cup(3/2,\infty)$ R: $(-\infty,0)\cup(0,\infty)$



Sketch a graph of the function and find the value of the indicated limits. State whether the function is continuous or discontinuous.

29.) $f(x)=\begin{cases}3-x, & x\le 1\\ 2x, & 1<x\end{cases}$; $\lim_{x\to 1^+} f(x)$, $\lim_{x\to 1^-} f(x)$, $\lim_{x\to 1} f(x)$

2 2 2 continuous

30.) $f(x)=\begin{cases}\frac{3}{2}x+\frac{3}{2} & \text{for } 0\le x\le 3\\ x+5 & \text{for } x>3\end{cases}$; $\lim_{x\to 3^+} f(x)$, $\lim_{x\to 3^-} f(x)$, $\lim_{x\to 3} f(x)$

8 6 ONE discontinuous

see notebook for sketches!

Rewrite the following as a piecewise function.

31.) $y=\frac{|x+1|}{x+1}$ $y=\begin{cases}1 & \text{if } x>-1\\ -1 & \text{if } x<-1\end{cases}$

32.) $y=|x-2|+3$ $y=\begin{cases}-x+5 & \text{if } x<2\\ x+1 & \text{if } x\ge 2\end{cases}$

***Graphically* determine if the following is an even, odd or neither function.**

33.) $y=\sin x$ odd

34.) $y=\cos x$ even

Use the $f(-x)$ test to determine whether the following is an even, odd or neither function.
Then state the type of symmetry (y-axis, origin or neither) of the function.

35.) $f(x)=x^3+1$ neither

36.) $f(x)=x^{\frac{5}{3}}$ odd, origin

Use your graphing calculator to find the POINTS of intersection of the graphs.

37.) $x^2+y^2=13$, $2x+y=4$ (0.2, 3.6) (3, -2)

38.) $y=e^x$, $y=\sin(2x)$ (-1.6666, 0.189) (-3.119, 0.044) etc...

True or False

T 39.) If (1, -2) is a point on a graph that is symmetric with respect to the y-axis, then (-1, -2) is also a point on the graph.

F 40.) Given a function *f(x)*, if $f(a)=f(b)$, then $a=b$.

F 41.) If *f* is a function, then *f(ax) = af(x).*

Find the equation of each line. Answer MUST be in POINT-SLOPE FORM!

42.) point (–2, 4); $m=-\frac{3}{5}$

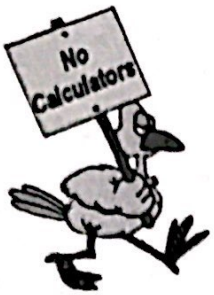
$y-4=\frac{-3}{5}(x+2)$

43.) through (–3, 0) and normal to $2x-y=6$

(You will need to look up the definition of a normal line.)

$y-0=\frac{-1}{2}(x+3)$

Solve. You should be able to do these without a calculator. You will be expected to work similar problems on a quiz or test without a calculator.



44.) $4\cos^2 x - 4\cos x + 1 = 0$ where $0 \le x \le 2\pi$ $x = \frac{\pi}{3}, \frac{5\pi}{3}$

45.) $\sin^2 x - 2\sin x = 0$ where $0 \le x \le 2\pi$ $x = 0, \pi, 2\pi$

46.) $e^x x^2 + 4xe^x + 3e^x = 0$ $x = -3, -1$

47.) $e^{4x} - 1 = 0$ $x = 0$

48.) $e^x + 5 = 0$ $\emptyset$

49.) Given $f(x) = \frac{1}{x}$; $g(x) = \sqrt{x+2}$;

Find a.) $f(g(x))$ $\frac{1}{\sqrt{x+2}}$ b.) $f(g(14))$ $\frac{1}{4}$ c.) $g(f(x))$ and give domain. $\sqrt{\frac{1}{x}+2}$

D: $[-1/2, 0) \cup (0, \infty)$

50.) Given $f(x) = x^2 - 1$; $g(x) = \cos x$;

Find a.) $f(g(x))$ $\cos^2 x - 1$ b.) $g \circ f(x)$ and give domain. $\cos(x^2 - 1)$ D: $(-\infty, \infty)$

51.) **Find the area of an equilateral triangle whose sides are 6cm each.** $9\sqrt{3}\ cm^2$

52.) **Find the area of a trapezoid with height 12in and bases of 6in and 9in.** $90\ in^2$

53.) **Find the volume of the right circular cylinder with radius 5in and height 14in.** $350\pi\ in^3$

54.) **Find the lateral surface area of the right circular cylinder with radius 5in and height 14in.** $190\pi\ in^2$

Reduce each completely.

55.) $\frac{x^2}{x^{\frac{1}{2}}}$ $x^{3/2}$

56.) $\frac{(x+1)^3(x-2)+3(x+1)}{(x+1)^4}$ $\frac{x^3 - 4x^2 - 3x + 1}{x^3 + 3x^2 + 3x + 1}$

Trig Identities—Match the statement on the left with an equivalent statement from the right.

57.) $\sin^2\theta + \cos^2\theta =$	C	**A.** $\cos^2\theta - \sin^2\theta$
58.) $\sin^2\theta =$	E	**B.** $\sec^2\theta$
59.) $\cos^2\theta =$	F	**C.** 1
60.) $\tan^2\theta + 1 =$	B	**D.** $\sin 2\theta$
61.) $2\sin\theta\cos\theta =$	D	**E.** $\frac{1-\cos 2\theta}{2}$
62.) $\cos 2\theta =$	A	**F.** $\frac{1+\cos 2\theta}{2}$

For $f(x) = kx^a$, where k is a constant, indicate the concavity and increasing/decreasing behavior of the curve in the first quadrant for the following:

63.) if $a < 0$	concave up, decreasing.
64.) if $a > 1$	concave up, increasing
65.) $0 < a < 1$	concave down, increasing

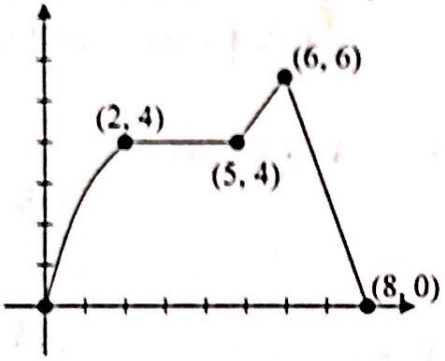
66.) A student who commutes 27 miles to attend college remembers, after driving a few minutes, that he forgot his term paper that is due. Driving faster than usual, the student returns home, picks up the paper, and once again starts toward school. Sketch a possible graph of the student's distance from school as a function of time. Sketch a possible graph of the student's velocity as a function of time.

(See notebook!)

67.) Suppose a pet owner decides to wash her dog in the laundry tub. She fills the tub with water, puts the dog in the tub and shampoos it, removes the dog from the tub to towel it, then pulls the plug to drain the tub. Let t be the time in minutes and $h(t)$ be the water level in the tub at time t. If the total time for filling, washing, and draining is 40 minutes, sketch a possible graph of $h(t)$.

(See notebook!)

Use the graph below to graph the transformations.



68.) $f(x)+2$

69.) $2f(x)$

70.) $f(x-1)$

71.) $2f(x)-2$

See notebook for sketches

Graph each function. Clearly indicate units. (see notebook !!)

72.) $f(x)=x$ 73.) $f(x)=x^2$ 74.) $f(x)=x^3$ 75.) $f(x)=|x|$

76.) $f(x)=\sin x$ 77.) $f(x)=\cos x$ 78.) $f(x)=\tan x$ 79.) $f(x)=\sin^{-1} x$

80.) $f(x)=\cos^{-1} x$ 81.) $f(x)=\tan^{-1} x$ 82.) $f(x)=e^x$ 83.) $f(x)=\ln x$

84.) $f(x)=\sqrt{x}$ 85.) $f(x)=\frac{1}{x^2}$ 86.) $f(x)=\frac{1}{x}$ 87.) $f(x)=\sqrt{a^2-x^2}$

Limit Problems. Find the following limits.

88.) $\lim_{x\to 0}\frac{x+\sin x}{x}$ 2

89.) $\lim_{x\to\infty}\frac{x^3-1}{2-x-x^2}$ $-\infty$

90.) $\lim_{x\to 1^+}\frac{x^3-1}{2-x-x^2}$ -1

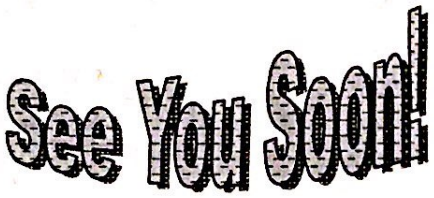
91.) $\lim_{x\to 1^-}\frac{x^3-1}{2-x-x^2}$ -1

92.) $\lim_{x\to 1}\frac{x^3-1}{2-x-x^2}$ -1

Define the following. You might have to do some research.

93.) Average rate of change
94.) Instantaneous rate of change
95.) Extreme value of a function
96.) Critical point of a function
97.) Point of inflection of a function

see notebook pages!



Congrats! You finished ☺ Almost… 98.) Go get some ice cream and do something FUN outside!!

Summer Assignment - KEY

1. $\sin^{-1}(\sqrt{2}/2) = \boxed{\pi/4}$
2. $\tan^{-1}(-1) = \boxed{-\pi/4}$
3. $\cos^{-1}(-1/2) = \boxed{2\pi/3}$
4. $\cos^{-1}(0) = \boxed{\pi/2}$
5. $\tan^{-1}(0) = \boxed{0}$
6. $\frac{-1}{3y^2} = \frac{1}{5} + 4x^3 \rightarrow \frac{-1}{3(1/5 + 4x^3)} = y^2 \rightarrow \boxed{y = \pm\sqrt{\frac{-5}{3 + 60x^3}}}$
7. $\boxed{[-7, 9)}$
8. $\boxed{(-\infty, 12)}$
9. $\boxed{(-\infty, \infty)}$
10. $\boxed{f^{-1}(x) = \ln x}$
11. $x = \frac{1}{y-2} \rightarrow y - 2 = \frac{1}{x} \rightarrow y = \frac{1}{x} + 2 \rightarrow \boxed{f^{-1}(x) = \frac{1}{x} + 2}$
12. $f(x) = \cos x \rightarrow \boxed{f^{-1}(x) = \cos^{-1}(x)}$
13. $x = y^2 + 5 \rightarrow y^2 = x - 5 \rightarrow y = \pm\sqrt{x-5} \rightarrow \boxed{f^{-1}(x) = \pm\sqrt{x-5}}$
14. $f(\pi) = \boxed{0}$ $f(2\pi/3) = \boxed{\sqrt{3}/2}$
15. $\frac{f(x) - f(1)}{x-1} = \frac{3x - 1 - [3-1]}{x-1} = \frac{3x-3}{x-1} = \frac{3(x-1)}{x-1} = \boxed{3}$
16. $\frac{f(a+h) - f(a)}{h} = \frac{(a+h)^3 - a^3}{h} = \frac{a^3 + 3a^2h + 3ah^2 + h^3 - a^3}{h}$
$= \boxed{3a^2 + 3ah + h^2}$
17. $f(-4) = -2(-4) - 6 = 8 - 6 = \boxed{2}$ $f(2) = 2(2) - 6 = 4 - 6 = \boxed{-2}$
18. $\boxed{x - 3}$
19. $\boxed{x - 1}$
20. $\ln\left(\frac{\frac{2a}{b}}{\frac{a}{b}}\right) = \ln\left(\frac{2a}{b} \cdot \frac{b}{a}\right) = \boxed{\ln(2)}$
21. $\boxed{\log(xy)}$
22. $\ln(e^2)^{1/3} = \ln e^{2/3} = \boxed{2/3}$

23 x-int.: $0 = x(x-4)$ y-int.: $y^2 = 0^2 - 4(0)$

$x = 0, 4$ $y^2 = 0 \rightarrow y = 0$

$(0,0), (4,0)$ $(0,0)$

24 x-int.: $0 = x^2 + 3x + 2$ y-int.: $y = \frac{2}{1} = 2$

$x = -2, -1$ $(0,2)$

$(-2,0), (-1,0)$

25 D.: $x^2 - 4 \neq 0$ R.: H.A.: $y = 0$

$(x+2)(x-2) \neq 0$ $(-\infty, 0) \cup (0, \infty)$

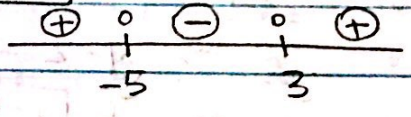
$x \neq \pm 2$

$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

26 D.: $2x^2 + 4x - 30 > 0$

$x^2 + 2x - 15 > 0$

$(x+5)(x-3) > 0$



$(-\infty, -5) \cup (3, \infty)$

R.: OMIT ☺

27 $y = \frac{(x+2)(x-2)}{(x-4)(x+3)}$ V.A.: $x = -3, x = 4$

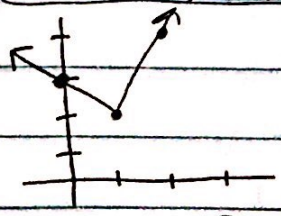
H.A.: $y = 1$

D.: $(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$ R.: $(-\infty, 1) \cup (1, \infty)$

28 $y = \frac{2x-3}{(x+2)(2x-3)}$ Hole @ $x = 3/2$ V.A.: $x = -2$

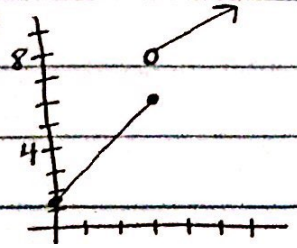
H.A.: $y = 0$

D.: $(-\infty, -2) \cup (-2, 3/2) \cup (3/2, \infty)$ R.: $(-\infty, 0) \cup (0, \infty)$

29 

$\lim_{x \to 1+} f(x) = \lim_{x \to 1-} f(x) = \lim_{x \to 1} f(x) = 2$

continuous

30 

$\lim_{x \to 3+} f(x) = 8$

$\lim_{x \to 3-} f(x) = 6$

$\lim_{x \to 3} f(x) = \text{DNE}$

discontinuous

31 $y = \begin{cases} 1 & \text{if } x > -1 \\ -1 & \text{if } x < -1 \end{cases}$

32 $y = \pm(x-2)+3 \rightarrow y = x-2+3$ and $y = -x+2+3$

$y = x+1$ $\quad y = -x+5$

$y = \begin{cases} -x+5 & \text{if } x < 2 \\ x+1 & \text{if } x \geq 2 \end{cases}$ ← Note: Equal bar can go w/ either 2!

33 $y = \sin x$ has symmetry across the origin.

odd

34 $y = \cos x$ is symmetric across the y-axis.

even

35 $f(-x) = -x^3+1$ $\quad f(x) \neq f(-x) \neq -f(x)$

$-f(x) = -x^3-1$ $\quad$ neither

36 $f(-x) = \sqrt[3]{(-x)^5} = -\sqrt[3]{x^5}$ $\quad f(-x) = -f(x)$

$-f(x) = -\sqrt[3]{x^5}$ $\quad$ odd, origin

37 $y_1 = \sqrt{13-x^2}$ [2nd] [TRACE] [5] . . .

$y_2 = -y_1$

$y_3 = 4-2x$ $\quad$ (0.2, 3.6) and (3, −2)

38 * Radian Mode! There are infinitely many solutions. Ex: (−1.666, 0.189), (−3.119, 0.044)

39 True $\quad$ 40. False $\quad$ 41. False

42 $y-4 = -3/5(x+2)$

43 $y = 2x-6$ $\quad m = 2$ $\quad m_{normal} = -1/2$

$y-0 = -1/2(x+3)$

44 $4x^2-4x+1=0$ $\rightarrow \cos x = 1/2$

$(x-1/2)^2 = 0$ $\quad x = \pi/3, 5\pi/3$

$x-1/2 = 0$

$x = 1/2$

45 $y^2-2y=0$

$y(y-2)=0$

$y=0,2$

$\sin x=0$ $\quad x=0,\pi,2\pi$

$\sin x=2$ $\quad \emptyset$

46 $e^x(x^2+4x+3)=0$

$e^x(x+1)(x+3)=0$

$e^x=0$ $\quad \emptyset$

$x+1=0$ $\quad x=-1$

$x+3=0$ $\quad x=-3$

47 $e^{4x}-1=0$

$e^{4x}=1$

$4x=\ln(1)$

$4x=0$

$x=0$

48. $e^x+5=0$

$e^x=-5$

$\emptyset$

49 a. $f(g(x))=\frac{1}{\sqrt{x+2}}$

b. $f(g(14))=\frac{1}{\sqrt{16}}=\frac{1}{4}$

c. $g(f(x))=\sqrt{\frac{1}{x}+2}$

D.: $x\neq 0$ $\quad \frac{1}{x}+2\geq 0$

$\frac{1}{x}\geq -2$

D: $[-1/2,0)\cup(0,\infty)$ $\quad x\geq -1/2$

50 a. $f(g(x))=\cos^2 x-1$

b. $g(f(x))=\cos(x^2-1)$ $\quad$ D.: $\mathbb{R}$, $(-\infty,\infty)$

51 $A=\sqrt{3}/4\cdot(6)^2=9\sqrt{3}\ cm^2$

52 $A=1/2\cdot(12)\cdot(6+9)=90\ in^2$

53 $V=\pi(5)^2(14)=350\pi\ in^3$

54 $A=2\pi(5)(14)+2\pi(5)^2=2\pi(95)=190\pi\ in^2$

55 $x^{2-1/2}=x^{3/2}$

56 $\frac{(x+1)[(x+1)^2(x-2)+3]}{(x+1)\cdot(x+1)^3}=\frac{(x^2+2x+1)(x-2)+3}{(x+1)^3}$

$=\frac{x^3-4x^2-3x+1}{x^3+3x^2+3x+1}$

57 [C] 58. [E] 59. [F]

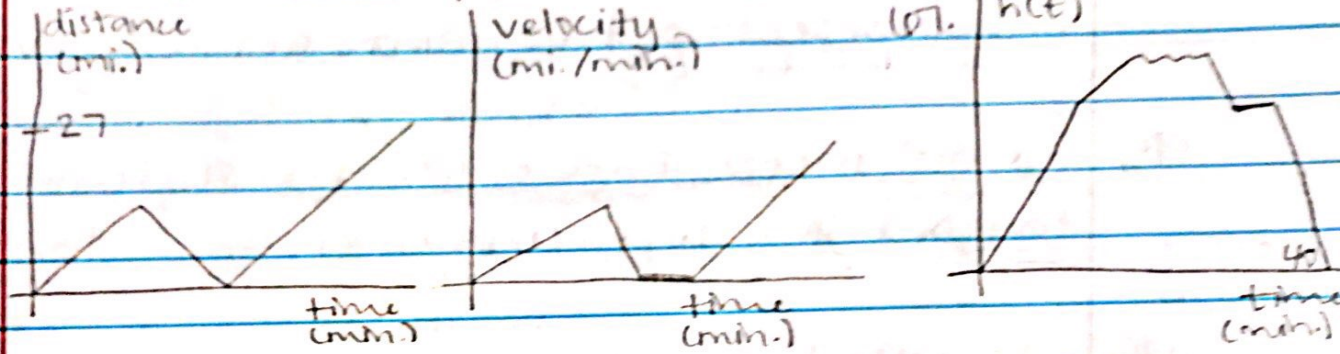
60 [B] 61. [D] 62. [A]

63 concave [up], [decreasing] ← ex: $f(x) = x^{-2}$

64 concave [up], [increasing] ← ex: $f(x) = x^2$

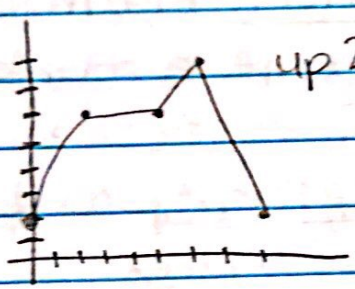
65 concave [down], [increasing] ← ex: $f(x) = \sqrt{x}$

66



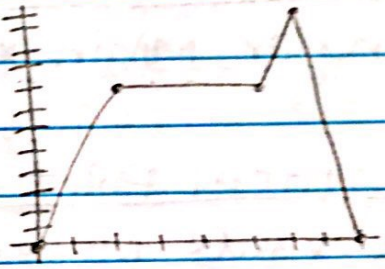
67.

68



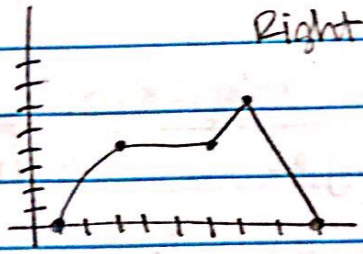
up 2

69.



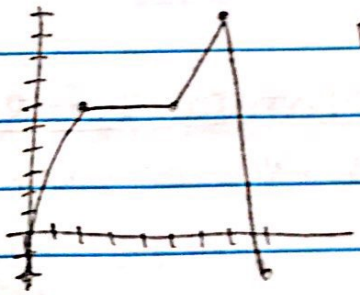
dilation by factor of 2

70



Right 1

71.



Double & Down 2

72-87 (see "Important Functions to Know")

88 $\lim_{x \to 0} 1 + \frac{\sin x}{x} = \boxed{2}$

89 $\lim_{x \to \infty} \frac{x^3 - 1}{-x^2 - x + 2} = \boxed{-\infty}$

90 $\lim_{x \to 1^+} \frac{(x-1)(x^2+x+1)}{-(x-1)(x+2)} = \frac{1^2+1+1}{-(1+2)} = \boxed{-1}$

91-92 $\boxed{-1}$ * Note: $\lim_{x \to 1} f(x)$ exists even though $f(1)$ does not!

93 Average Rate of Change: slope of the secant line intersecting two points on a curve

94 Instantaneous Rate of Change: slope of the tangent line to a curve at a particular point

95 Extreme Value of a function: Maximum and/or Minimum y-value of a function

96 Critical Point of a function: possible x-value of an extreme value

97 Point of Inflection of a function: point where a curve changes concavity