

What to do and not to do

Scrapbook/Portfolio

Limit Definitions of Derivatives

There are two limit definitions for derivatives:

Definition One:

$$f'(x) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

This definition uses the algebraic formula for slope as a limit to find the algebraic slope of the function as close as is possible to the point 'x' at which the derivative is being taken. As x approaches a, it essentially becomes a and expresses the value of the derivative as accurately as possible.

Definition Two:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

This definition is the same as the first, though it replaces the difference between each point 'x' and the desired point with a changing variable 'h' representing the distance away from the desired point 'x'. As h approaches zero, the resulting slope is essentially that of the function at the point 'x'.

Note: The two definitions are nearly identical, and both are equally valid definitions of derivatives.

Example Problem:

Given: $f(x) = x^2 + 5x + 10$, find the derivative using the second limit definition of derivatives

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{[(x+h)^2 + 5(x+h) + 10] - [x^2 + 5x + 10]}{h}$$

$$\lim_{h \rightarrow 0} \frac{[x^2 + 2xh + h^2] + (5x + 5h) + 10 - [x^2 + 5x + 10]}{h}$$

$$\lim_{h \rightarrow 0} \frac{h^2 + 2xh + 5h}{h} = 2x + 5$$

Avoid a TEXTBOOK like presentation

Derivative Rules

Product Rule: if $f(x) = u(x) \cdot v(x)$ then

$$f'(x) = u(x)v'(x) + v(x)u'(x)$$

Song to remember it "Hi Ho, Hi Ho, the product rule I know. Its Hi d Ho plus Ho d Hi and then you simplify"

Power rule: $\frac{d}{dx} x^n = nx^{n-1}$

Quotient Rule: if $f(x) = \frac{u(x)}{v(x)}$ then

$$f'(x) = \frac{v(x)u'(x) - u(x)v'(x)}{[v(x)]^2}$$

Song to remember it: "Hi Lo, Hi Lo, the quotient rule I know. It's Lo d Hi minus Hi d Lo divided by Lo squared"

Inverse Trig Derivatives:

$$\frac{d}{dx} \arcsin u = \frac{1}{\sqrt{1-u^2}} \cdot u'$$

$$\frac{d}{dx} \arccos u = \frac{-1}{\sqrt{1-u^2}} \cdot u'$$

$$\frac{d}{dx} \arctan u = \frac{1}{1+u^2} \cdot u'$$

$$\frac{d}{dx} \operatorname{arc cot} u = \frac{-1}{1+u^2} \cdot u'$$

$$\frac{d}{dx} \operatorname{arc sec} u = \frac{1}{|u|\sqrt{u^2-1}} \cdot u'$$

$$\frac{d}{dx} \operatorname{arc csc} u = \frac{-1}{|u|\sqrt{u^2-1}} \cdot u'$$

Trig derivatives:

$$\frac{d}{dx} \sin u = \cos u$$

$$\frac{d}{dx} \cos u = -\sin u$$

$$\frac{d}{dx} \tan u = \sec^2 u$$

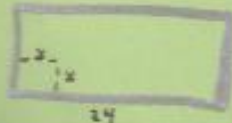
$$\frac{d}{dx} \cot u = -\csc^2 u$$

$$\frac{d}{dx} \sec u = \sec u \tan u$$

$$\frac{d}{dx} \csc u = -\csc u \cot u$$

Pretty paper but still textbook like

Max wants to make a box with no lid from a rectangular sheet of cardboard that is 18 inches by 24 inches. The box is made by cutting a square of side x from each corner and folding up the sides. Find the value of x that maximizes the volume.



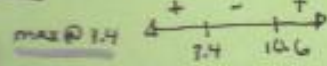
width: $18-2x$
length: $24-2x$
depth: x

$$V = x(18-2x)(24-2x) = 4x^3 - 84x^2 + 432x$$

$$\frac{dV}{dx} = 12x^2 - 168x + 432 \quad 12x^2 - 168x + 432 = 0$$

$$12(x^2 - 14x + 36) = 0$$

$$x = \frac{14 \pm \sqrt{196 - 144}}{2} = 7 \pm \sqrt{11} = 3.4, 10.6$$



O P T

A poster is to contain 100 square inches of picture surrounded by a 4-inch margin at the top and bottom and a 2-inch margin on each side. What dimensions will minimize the area of the poster?



Area = xy
 $xy = 100 \quad y = \frac{100}{x}$
 $A = (x+4)(y+8)$
 $A = xy + 4y + 8x + 32$
 $A = 100 + \frac{400}{x} + 8x$

$$\frac{dA}{dx} = 8 - \frac{400}{x^2} = 0 \quad x = \sqrt{50} \quad y = 2\sqrt{50}$$

minimum dimensions = $4 + \sqrt{50}$ and $8 + 2\sqrt{50}$ inches.

An open-top box with a square bottom and rectangular sides is to have a volume of 256 cubic inches. Find the dimensions to minimize the amount of material.



$SA = x^2 + 4xy \quad V = x^2y = 256 \quad y = \frac{256}{x^2}$
 $SA = x^2 + 4x \frac{256}{x^2} = x^2 + \frac{1024}{x}$

$$\frac{dS}{dx} = 2x - \frac{1024}{x^2} = 0 \quad x^3 = 512 \quad x = 8 \quad y = 4$$

dimensions = $8 \times 8 \times 4$

Rectangular field, bounded by a building on one side, is to be fenced in the other 3 sides. If 3,000 ft of fence is to be used, find the dimensions of the largest field that could be fenced in.

$P = 2x + y = 3000$
 $y = 3000 - 2x$
 $A = x(3000 - 2x)$
 $A = 3000x - 2x^2$
 $\frac{dA}{dx} = 3000 - 4x = 0 \quad x = 750$
 $y = 1500$



largest field measures 750 ft by 1,500 ft

N O

- Steps:**
1. Write an equation demonstrating the max or min.
 2. Multiply all terms.
 3. Take the derivative.
 4. Set the derivative equal to zero and solve for x .
 5. Make a sign chart to determine maximums and minimums.
 6. Draw a picture and use substitution!!!

A lot of work but still textbook like

One more. . . .
Neatly done BUT still
textbook like

Can you tell that this
is something to
avoid?

Asymptotes

Asymptotes are lines that a graph gets closer and closer to, but never touches or crosses.

- The $y = b$ is a horizontal asymptote of the graph of a function $y = f(x)$ if either...
- The line $x = a$ is a vertical asymptote of the graph of a function $y = f(x)$ if either...

Find the asymptotes of the following function:

$$y = \frac{x^2 + 3x + 1}{4x^2 - 9}$$

The vertical asymptotes (and any restrictions on the domain) come from the zeroes of the denominator, so set the denominator equal to zero and solve to find any vertical asymptotes.

$$\begin{aligned}4x^2 - 9 &= 0 \\4x^2 &= 9 \\x^2 &= \frac{9}{4} \\x &= \pm \frac{3}{2}\end{aligned}$$

Since the degrees of the numerator and the denominator are the same, then this function has a non-zero (non-x-axis) horizontal asymptote. The horizontal asymptote is found by dividing the leading terms:

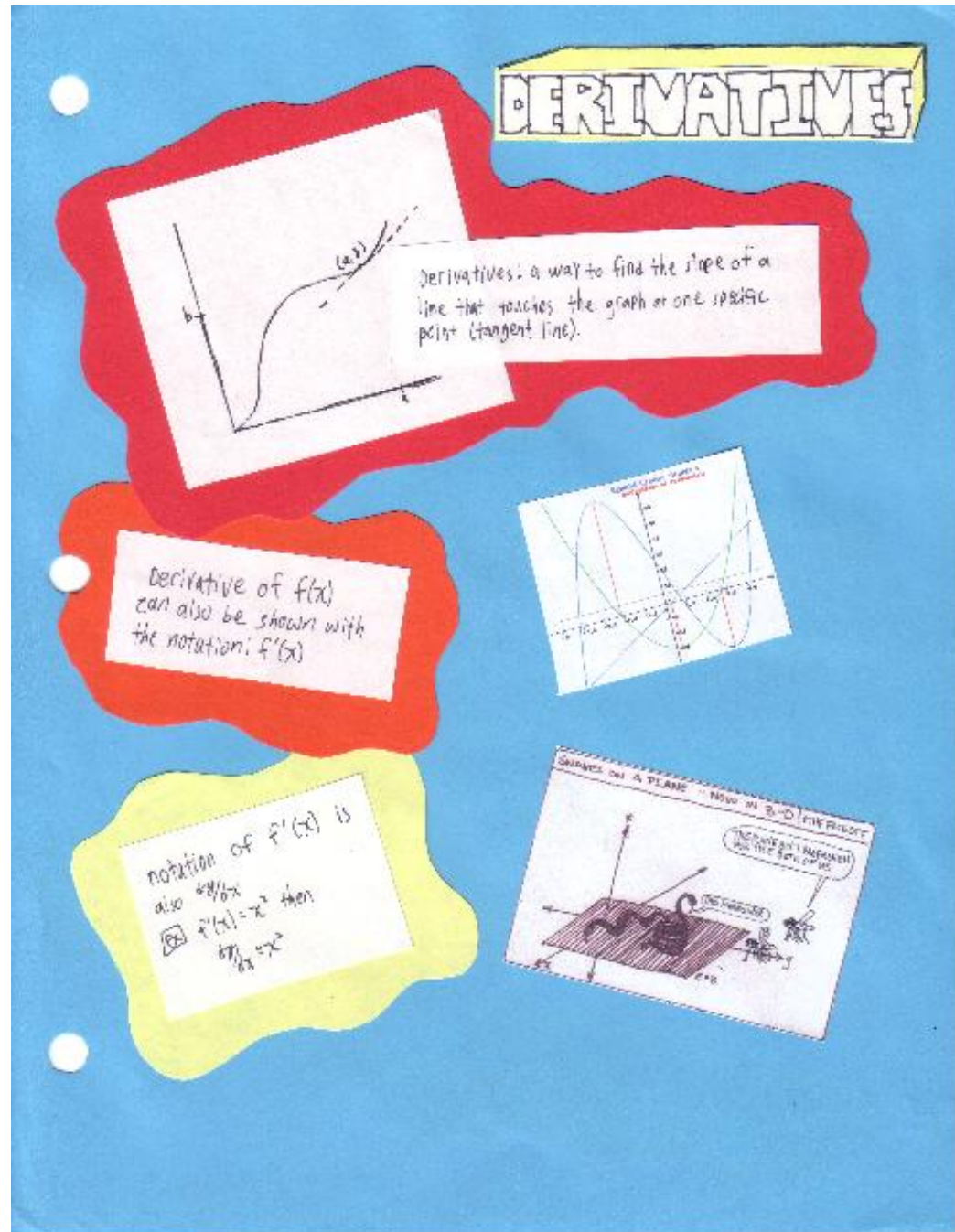
$$y = \frac{x^2}{4x^2} = \frac{1}{4}$$

The vertical asymptote is at $x = \pm 3/2$

The horizontal asymptote is at $y = 1/4$

Limit Statement is
1/4 of 6/6

This is better



DERIVATIVES

Diagrams are required

Derivatives: a way to find the slope of a line that touches the graph at one specific point (tangent line).

Right Riemann Sum.

$S(t)$
 $V(t)$
 $a(t)$

Maximum: when the first derivative goes from positive to negative, or function goes from increasing to decreasing. $f'(x): + \text{ to } -$
Minimum: when the first derivative goes from negative to positive, or function goes from decreasing to increasing. $f'(x): - \text{ to } +$
 A point where the first derivative changes like this is called a critical point.

$f(x) = x^2$ $f'(x) = 2x$
 $0 = 2x$
 $x = 0$

Concave up: when the second derivative is positive. $f''(x) > 0$
Concave down: when the second derivative is negative. $f''(x) < 0$

$f(x) = x^3$
 $f'(x) = 3x^2$
 $f''(x) = 6x$
 $0 = 6x$
 $x = 0$

A point where the concavity changes is called a point of inflection.

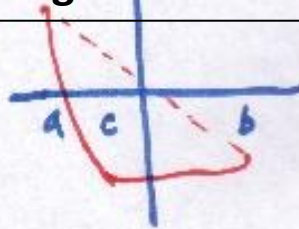
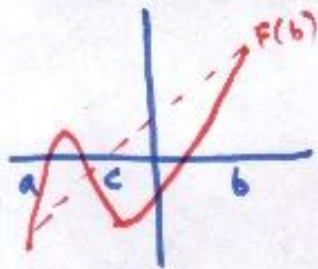
M E A N V A L U E T H E O R E M

If f is continuous on $[a, b]$ and differentiable on (a, b) then there exists a number c , in (a, b) such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

draw in tang. line to show slopes are equal

Incomplete Diagram



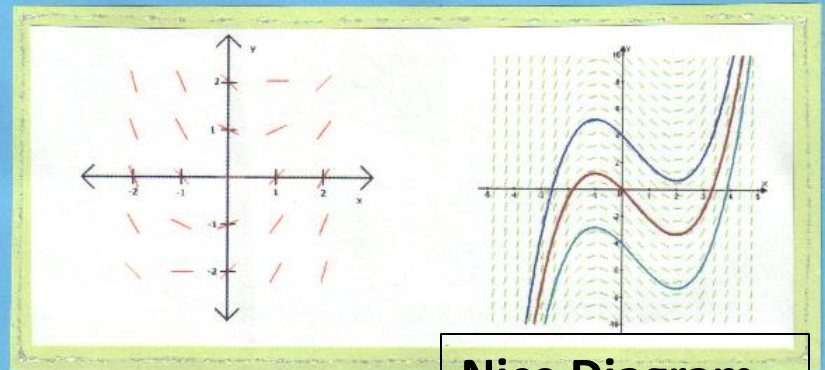
$y = x^3 - 1$ on $[-1, 2]$



$y(-1) = -2$
 $y(2) = 7$
 $y' = 3x^2 = 3$
 $x = \pm 1$

Mean Value

Complete diagrams help for better understanding.



Nice Diagram

A better diagram would have made the relevance of this example more obvious.

Does the order in which you present your topics make sense?

What do you see wrong with this presentation?

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Don't forget the table of contents AND don't just throw it in the project at the last minute!!!

These are good and were definitely in the book before arriving to class on the due date!

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Asymptotes

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lim + stabs +
1/4 off

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The vertical asymptote is at $x = \pm 3/2$

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Make sure that topics that were introduced in precalculus are presented from the calculus perspective.

Formulas.

Growth = $y = y_0 e^{kt}$ $k = \frac{\ln 2}{\text{half life}}$ always decay

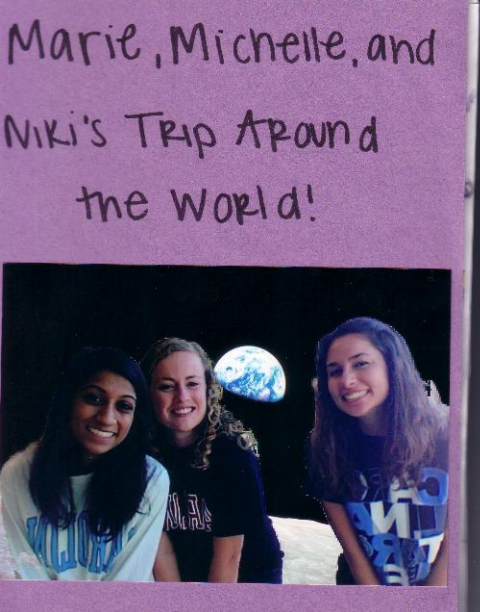
Decay = $y = y_0 e^{-kt}$

Ex. write an equation for the amount Q of a radioactive substance with a half-life of 30 days, if 10 grams are present when $t=0$.

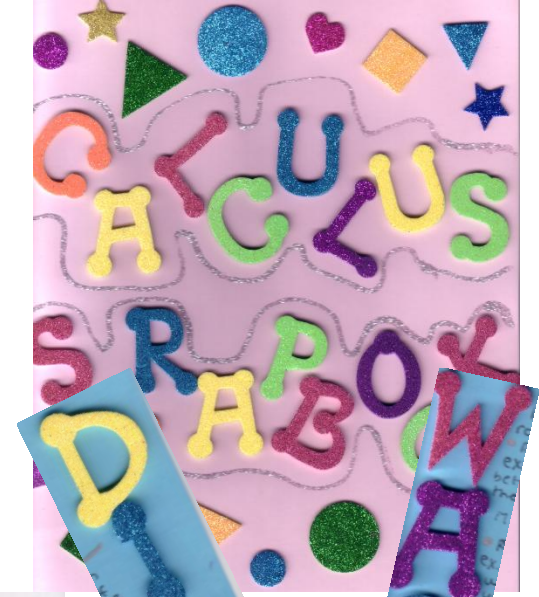
$$H = \frac{\ln 2}{k} \quad 30 = \frac{\ln 2}{k} \quad k = \frac{\ln 2}{30}$$

$Q = 10e^{-0.0231t}$

horizontal asymptote: a horizontal line that the function approaches as x goes to ∞ or $-\infty$ ex: $\lim_{x \rightarrow \infty} f(x) = 1$



Have a Theme
Story
Journey
Consistent Page Formats



to find the area enclosed by a curve, we use the formula:

$$\int_a^b \frac{1}{2} r^2 d\theta$$

where α & β are the radians where the curve is enclosed.

we used area of polar curves to find the area of a snowflake in Antarctica.

$$r = 2 + 2\cos\theta$$
$$\int_0^{2\pi} \frac{1}{2} (2 + 2\cos\theta)^2 d\theta = 10\pi$$
