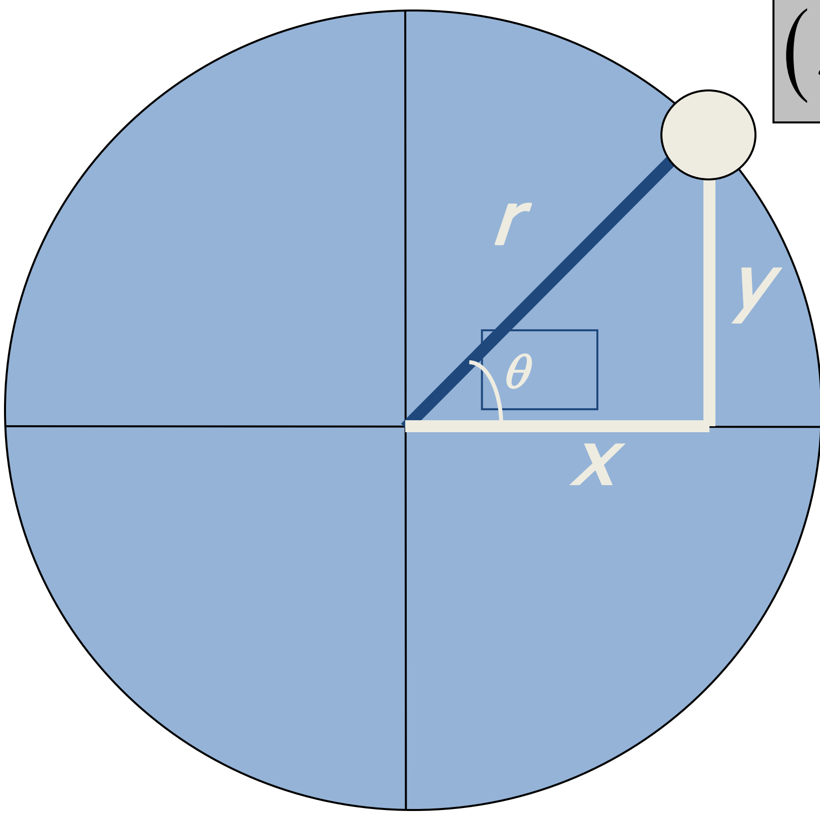
POLAR Review

Polar-Cartesian Relationships



$$(r, \theta)$$

$$(x, y)$$

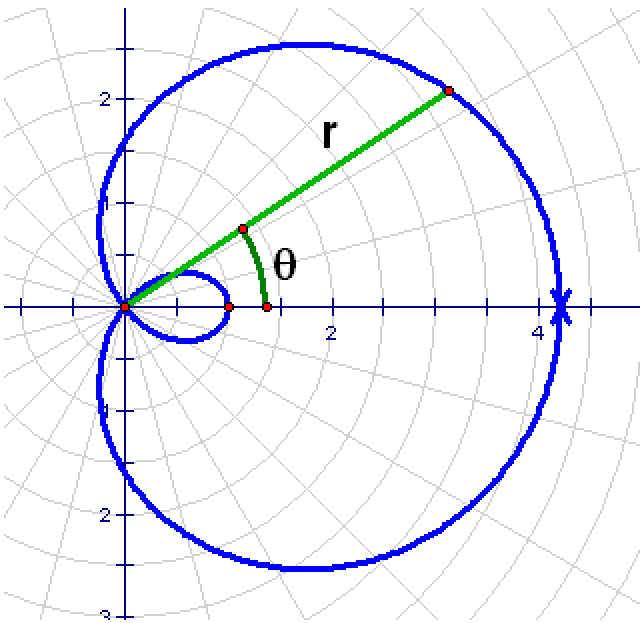
$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

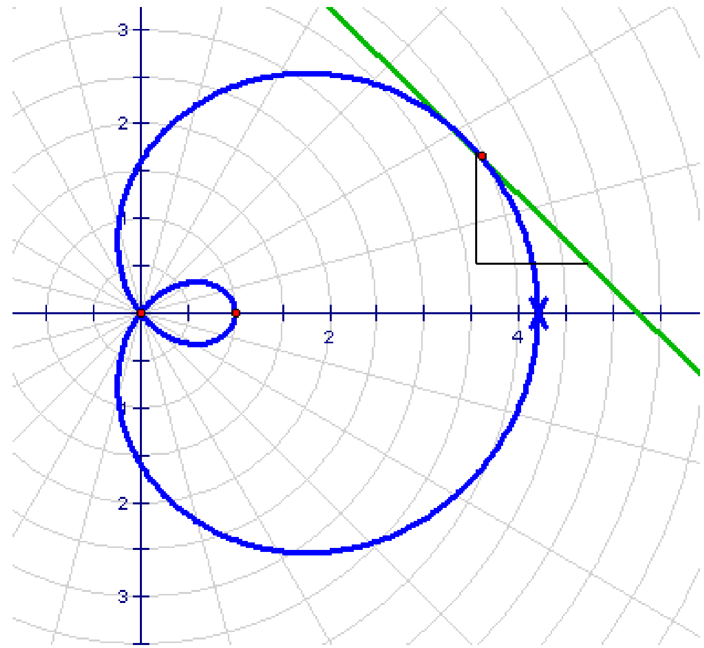
$$x = r\cos\theta$$

$$y = r\sin\theta$$

Compare and Contrast



$$\frac{dr}{d\theta} =$$ Rate of change in radius (relative to the pole) as theta changes.



$$\frac{dy}{dx} =$$ Slope of the tangent

(a.k.a. Slope of Polar Curve)

Other Rates of Change to Think About

- $\frac{dy}{dt}$ is the change in vertical position with respect to "t"

- $\frac{dy}{d\theta}$ is the change in vertical position with respect to "theta"

Likewise for horizontal position

- $\frac{dr}{dt}$ is the change in "radius" (relative to the pole) with respect to "t"

$$y = r\sin\theta$$

$$x = r\cos\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \boxed{\frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}}$$

Product Rule!

Know how to get this instead of memorizing it as a formula

Horizontal tangents → $\frac{dy}{d\theta} = 0$

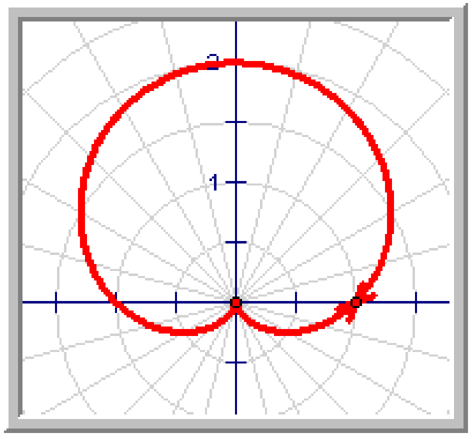
Vertical tangents → $\frac{dx}{d\theta} = 0$

For the curve, $r = 1 + \sin\theta$

A) Find the **equation** for the slopes of the tangent lines

B) Find the slope when $\theta = 0$

C) Find where there are horizontal tangents



$$r = 1 + \sin\theta \quad \therefore\ y = (1+\sin\theta)(\sin\theta)$$

$$x = (1+\sin\theta)(\cos\theta)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\cos\theta\sin\theta + \left(1+\sin\theta\right)\cos\theta}{\cos\theta\cos\theta - \left(1+\sin\theta\right)\sin\theta}$$

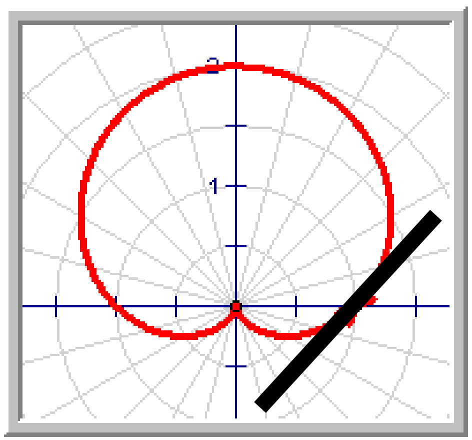
Simplify to $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{\cos\theta + 2\sin\theta\cos\theta}{-\sin\theta - \sin^2\theta + \cos^2\theta}$

For the curve, $r = 1 + \sin\theta$

A) Find the equation for the slopes of the tangent lines

B) Find the slope when $\theta = 0$

C) Find where there are horizontal tangents



$B)$ $$\frac{dy}{dx} = \frac{\cos 0 + 2\sin 0 \cos 0}{-\sin 0 - \sin^2 0 + \cos^2 0} = \frac{1+0}{-0-0+1} = 1$$

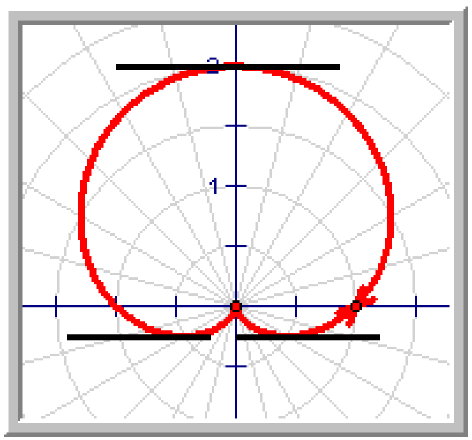
For the curve, $r = 1 + \sin\theta$

A) Find the equation for the slopes of the tangent lines

B) Find the slope when

C) Find where there are horizontal tangents

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\cos\theta + 2\sin\theta\cos\theta}{-\sin\theta - \sin^2\theta + \cos^2\theta} = 0$$



$$\therefore \frac{dy}{d\theta} = 0 \Rightarrow \cos\theta\left(1 + 2\sin\theta\right) = 0$$

$$\cos\theta = 0 \qquad\qquad 1 + 2\sin\theta = 0$$

$$\theta = \frac{\pi}{2}, \cancel{\frac{3\pi}{2}} \qquad\qquad \theta = \frac{11\pi}{6}, \frac{7\pi}{6}$$

$but\ldots\frac{3\pi}{2}$ gives a zero for $\frac{dx}{d\theta}$

So we discard it.

Area bounded by a polar graph:

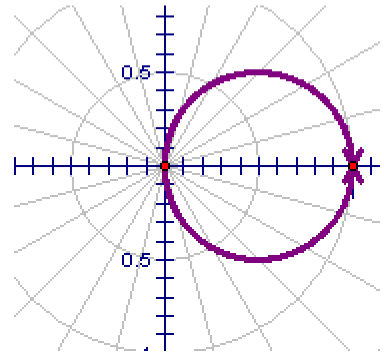
$$A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta$$

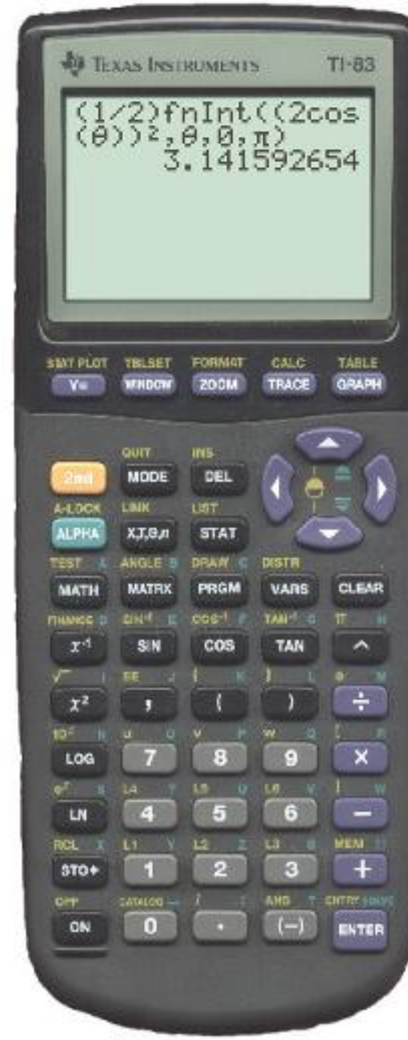
Finding the limits…

- Normally, you can use the limits of $0 \ and \ 2\pi$ unless that will cause the graph to draw over itself.
- Sketch it by plotting some points and check!
- You can also use symmetry to simplify the integral

Find the area within the curve:

$$r = 2\cos\theta$$



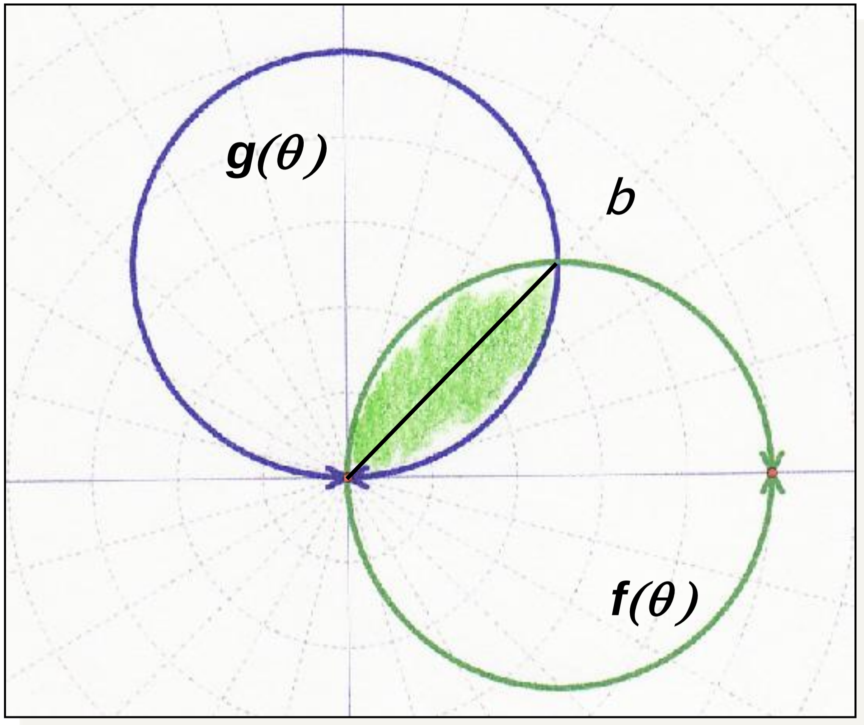


$$\frac{1}{2}\int_0^{\pi}\left(2\cos\theta\right)^2 d\theta$$

$$\text{or } (2)\frac{1}{2}\int_0^{\pi/2}\left(2\cos\theta\right)^2 d\theta$$

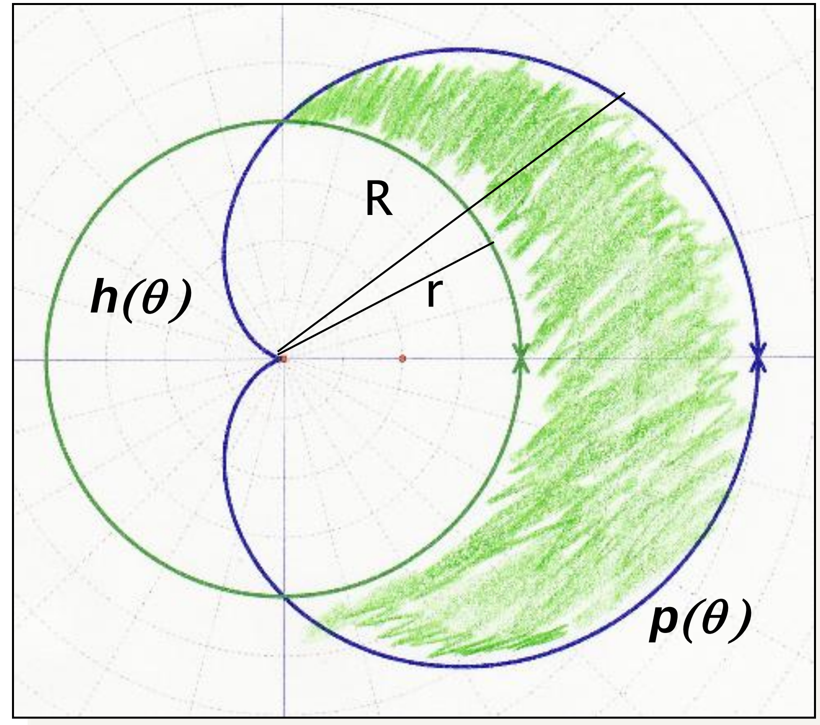
Let's try this one on the calculator...

How would you find the shaded areas?



Piecewise

$$\frac{1}{2}\int_0^b g^2(\theta)\,d\theta+\frac{1}{2}\int_b^{\frac{\pi}{2}} f^2(\theta)\,d\theta$$



$$R^2-r^2$$

$$\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} p^2(\theta)-h^2(\theta)\,d\theta$$

Length of Polar Curve

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$