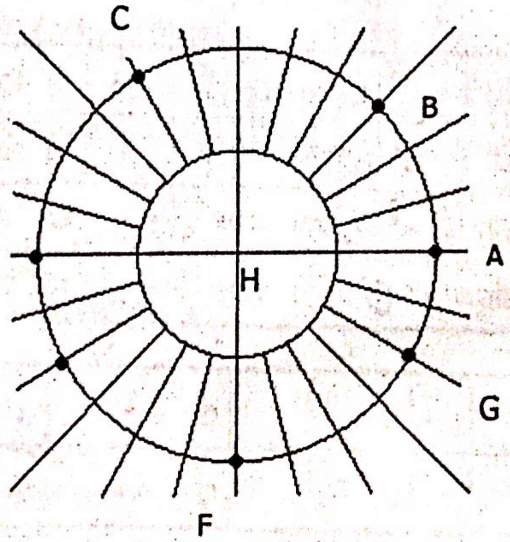
Unit 2 Packet

Name: Key

Unit Circle Review/Polar Coordinates (Homework)

Use the circle to answer the following.



1. At which point(s) shown is the cosine negative?

   C, D, E

2. What is the positive radian measure of $\angle AHC$?

   $2\pi/3$

3. What is the sine of the angle that would be at point G?

   $-1/2$

4. Give the polar coordinates of E. $(2, 7\pi/6)$

5. What is the counterclockwise angle measure, in radians, of $\angle AHG$? What is the clockwise measure of the same angle?

   $11\pi/6$ $-\pi/6$

6. At which labeled point does sine=cosine? Give the degree and radian measure of this angle.

   B, 45°, $\pi/4$

7. Which point labeled on the circle corresponds to the angle of $-\frac{5\pi}{6}$? E

8. Which point labeled on the circle corresponds to the angle $\frac{7\pi}{2}$? F

9. On the graph, plot the following and label:

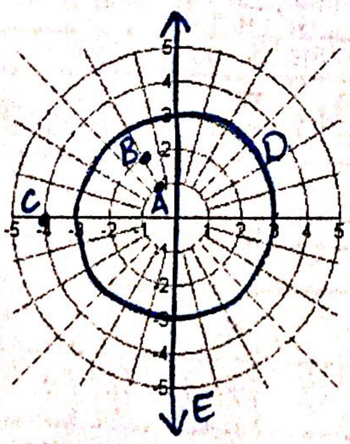
   A) $\left(-1,-\frac{\pi}{3}\right)$

   B) $\left(2,\frac{2\pi}{3}\right)$

   C) $(4,3\pi)$

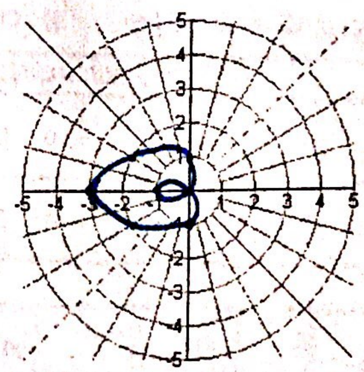
   D) r = 3

   E) $\theta=\frac{\pi}{2}$



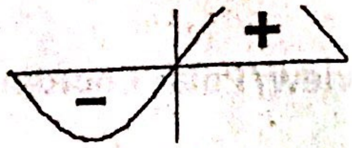
10. On the graph, plot $r=1-2\cos\theta$ using the following points:

r	θ
-1	0
0	$\frac{\pi}{3}$
1	$\frac{\pi}{2}$
3	π
2	$\frac{4\pi}{3}$
1	$\frac{3\pi}{2}$
0	$\frac{5\pi}{3}$
-1	2π



(9)

WHAT IS THE TITLE OF THIS PICTURE?



Match each expression with its value.

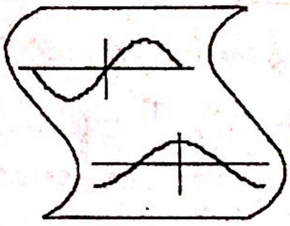
1) $\sin\frac{\pi}{4} = \sqrt{2}/2$	2) $\cos\frac{\pi}{3} = 1/2$	3) $\tan\frac{\pi}{6} = \sqrt{3}/3$	4) $\tan\pi = 0$	5) $\sin\frac{\pi}{2} = 1$
6) $\sin\frac{5\pi}{4} = -\sqrt{2}/2$	7) $\tan\frac{5\pi}{4} = 1$	8) $\tan\frac{\pi}{4} = 1$	9) $\sin\frac{5\pi}{6} = 1/2$	10) $\cos\frac{\pi}{2} = 0$
11) $\sin\frac{2\pi}{3} = \sqrt{3}/2$	12) $\cos\frac{5\pi}{3} = 1/2$	13) $\cos\frac{2\pi}{3} = \frac{-1}{2}$	14) $\tan\frac{3\pi}{2} =$ und	15) $\cos\pi = -1$
16) $\sin\pi = 0$	17) $\cos\frac{7\pi}{6} = \frac{-\sqrt{3}}{2}$	18) $\tan\frac{5\pi}{3} = -\sqrt{3}$	19) $\sin 0 = 0$	20) $\cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$

Values.

A. $-\frac{\sqrt{3}}{3}$	E. $\frac{\sqrt{3}}{3}$	G. $-\frac{1}{2}$	I. $\frac{1}{2}$	L. $-\frac{\sqrt{2}}{2}$	M. -1	N. 1
O. $-\frac{\sqrt{3}}{2}$	P. $\frac{\sqrt{3}}{2}$	Q. $\sqrt{3}$	R. undefined	S. 0	U. $\frac{\sqrt{2}}{2}$	±. $-\sqrt{3}$

P	L	U	S		O	R		M	I	N	U	S		S	I	N	E	(	±	S	I	G	N	)
11	6	20	10		17	14		15	12	5	1	19		16	2	8	3		18	4	9	13	7	

WHAT IS THE TITLE OF THIS PICTURE?



Match each equation with a solution where $0 \le \theta < 2\pi$.

1) $\sin\theta = \frac{1}{2}$ $\frac{\pi}{6}, \frac{5\pi}{6}$	2) $\cos\theta = \frac{\sqrt{2}}{2}$ $\frac{\pi}{4}, \frac{7\pi}{4}$	3) $\tan\theta = \frac{\sqrt{3}}{3}$ $\frac{\pi}{6}, \frac{7\pi}{6}$	4) $\tan\theta = -\sqrt{3}$ $\frac{2\pi}{3}, \frac{5\pi}{3}$
5) $\sin\theta = 1$ $\pi/2$	6) $\cos\theta = -\frac{\sqrt{3}}{2}$ $\frac{5\pi}{6}, \frac{7\pi}{6}$	7) $\tan\theta = -1$ $\frac{3\pi}{4}, \frac{7\pi}{4}$	8) $\sin\theta = -1$ $\frac{3\pi}{2}$

Match each expression with the angle θ as defined by the inverse trig function.

9) $\sin^{-1}\left(\frac{1}{2}\right) = \pi/6$	10) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \pi/4$	11) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$	12) $\tan^{-1}(-\sqrt{3}) = \frac{-\pi}{3}$
13) $\sin^{-1}(1) = \pi/2$	14) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$	15) $\tan^{-1}(-1) = \frac{-\pi}{4}$	16) $\sin^{-1}(-1) = \frac{-\pi}{2}$

Values.

A. $\frac{5\pi}{6}$	E. $\frac{\pi}{2}$	H. $-\frac{\pi}{4}$	I. $\frac{\pi}{4}$	G. $-\frac{\pi}{2}$	N. $\frac{3\pi}{2}$	S. $\frac{\pi}{6}$
E. $-\frac{\pi}{3}$	I. $\frac{\pi}{6}, \frac{5\pi}{6}$	N. $\frac{\pi}{6}, \frac{7\pi}{6}$	P. $\frac{5\pi}{6}, \frac{7\pi}{6}$	S. $\frac{\pi}{4}, \frac{7\pi}{4}$	T. $\frac{3\pi}{4}, \frac{7\pi}{4}$	U. $\frac{2\pi}{3}, \frac{5\pi}{3}$

A		S	I	N	E	(	S	I	G	N	)	U	P		S	H	E	E	T
14		11	1	8	13		9	10	16	3		4	6		2	15	5	12	7

NO CALCULUATOR!!!

1. Convert the **RECTANGULAR** coordinate TO a **POLAR** coordinate. $Q(1, \sqrt{3})$

$(2, \pi/3)$ or $(-2, 4\pi/3)$

2. Convert the **POLAR** coordinate TO a **RECTANGULAR** coordinate. P: $\left(4, \frac{3\pi}{4}\right)$

$(-2\sqrt{2}, 2\sqrt{2})$

3. Convert the polar equation to rectangular form. $r + 8\sin\theta = 4\cos\theta$

$x^2 + y^2 = 4x - 8y$

4. Write the polar equation of a circle with radius $\sqrt{10}$ and center $(-1,3)$.

$r^2 + 2r\cos\theta - 6r\sin\theta = 0$

5. Convert the rectangular equation $x = 9$ to a polar equation.

$r\cos\theta = 9$

6. Represent $\left(4, \frac{\pi}{6}\right)$ in at least four different ways.

$\left(4, \frac{13\pi}{6}\right), \left(4, \frac{-11\pi}{6}\right), \left(-4, \frac{7\pi}{6}\right), \left(-4, \frac{-5\pi}{6}\right)$

7. Convert the polar equation to rectangular form. $r = 2\sin\theta$

$x^2 + y^2 = 2y$

8. Convert the polar equation to rectangular form. $r = 3\cos\theta$

$x^2 + y^2 = 3x$

9. Integrate: $\int 4\sin(2\theta)\,d\theta$

$-2\cos(2\theta) + C$

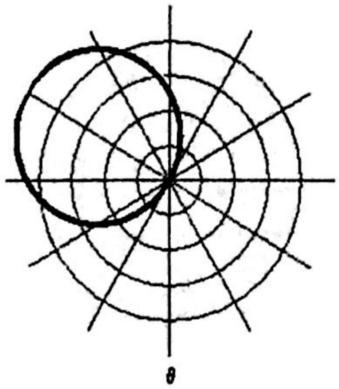
10. Integrate: $\int(\sin^2\theta)\,d\theta$ using the identity: $\cos(2\theta) = 1 - 2\sin^2\theta$

$$\frac{\theta}{2} - \frac{\sin(2\theta)}{4} + C$$

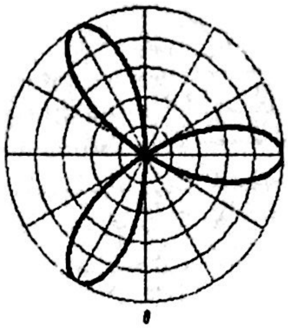
BC Calculus
Polar Lab Wrap Up

Let's see what you have learned...**Without a calculator**, match the following graphs with their equation.

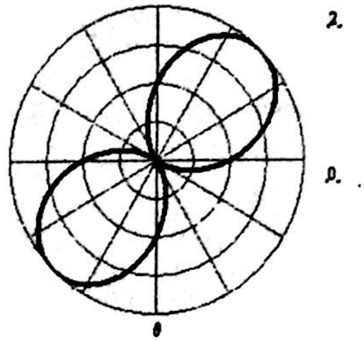
GRAPHS:



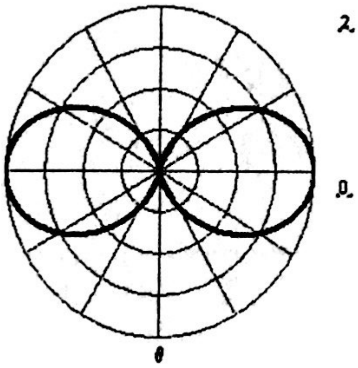
$\sin(\theta-\pi/3)$



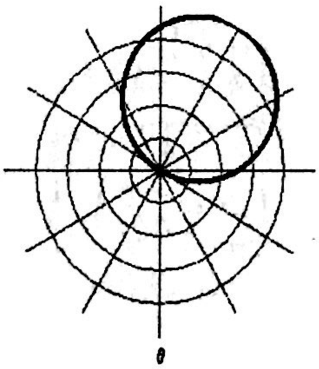
$\cos(3\theta)$



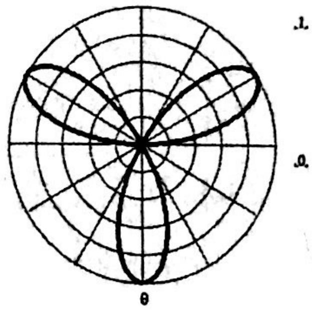
$1+\sin(2\theta)$



$1+\cos(2\theta)$



$\cos(\theta-\pi/3)$



$\sin(3\theta)$

Functions:

A) $\sin\left(\theta-\frac{\pi}{3}\right)$ B) $\cos\left(\theta-\frac{\pi}{3}\right)$ C) $\sin 3\theta$

D) $\cos 3\theta$ E) $1+\sin 2\theta$ F) $1+\cos 2\theta$

AP Calculus BC

Polar Lab Follow Up Practice (ROSE curves)

Complete each problem. When finished, each problem should have an equation, graph, domain (where graph starts repeating) and description of symmetry (relative to the x-axis and y-axis).

1.

Equation:

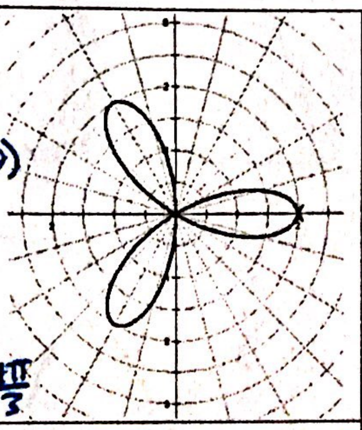
$r = 2\cos(3\theta)$

Domain:

$[0, \pi]$

Symmetry

$\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$



2.

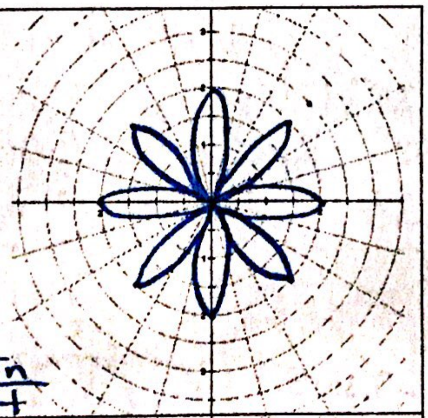
Equation: $r = 2\cos 4\theta$

Domain:

$[0, 2\pi]$

Symmetry

x+y axes, $\theta = \frac{\pi n}{4}$



3.

Equation:

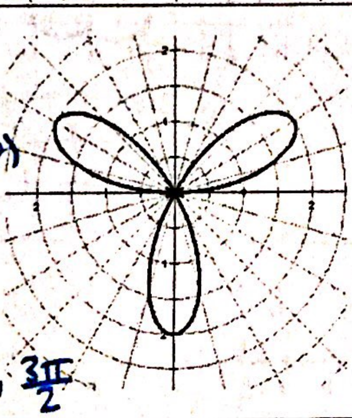
$r = 2\sin(3\theta)$

Domain:

$[0, \pi]$

Symmetry

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$



4.

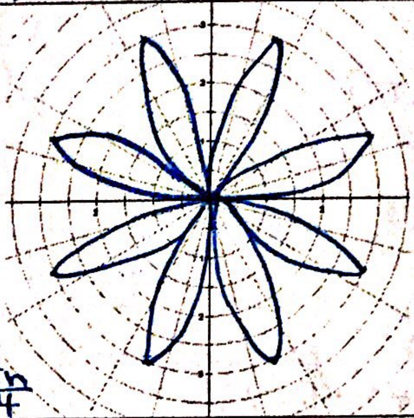
Equation: $r = 3\sin 4\theta$

Domain:

$[0, 2\pi]$

Symmetry

x+y axes, $\theta = \frac{\pi n}{4}$



5.

Equation:

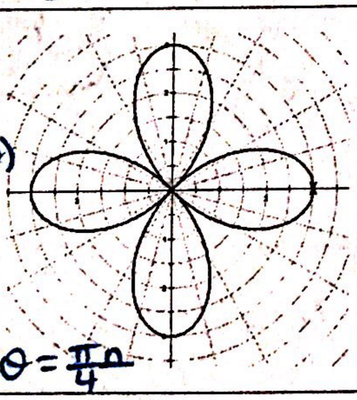
$r = 3\cos(2\theta)$

Domain:

$[0, 2\pi]$

Symmetry

x+y axes, $\theta = \frac{\pi n}{4}$



6.

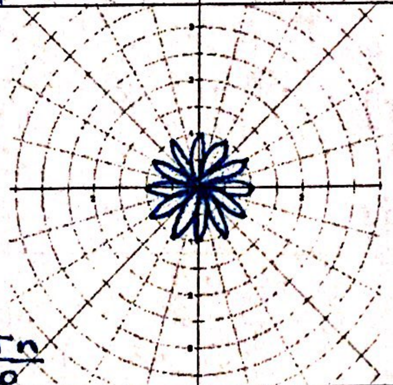
Equation: $r = \cos 6\theta$

Domain:

$[0, 2\pi]$

Symmetry

x+y axes, $\theta = \frac{\pi n}{6}$



7.

Equation:

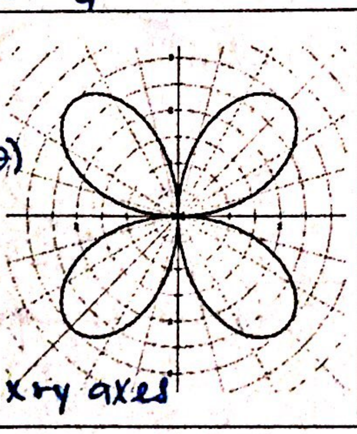
$r = 3\sin(2\theta)$

Domain:

$[0, 2\pi]$

Symmetry

$\theta = \frac{\pi n}{4}$, x+y axes



8.

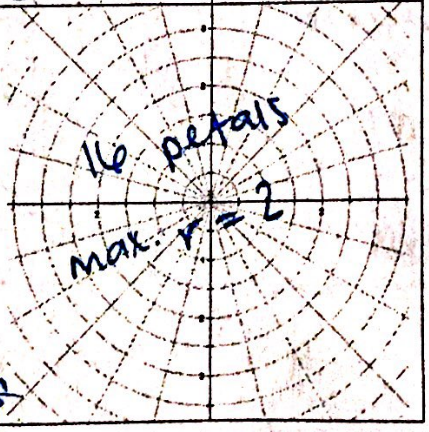
Equation: $r = 2\sin 8\theta$

Domain:

$[0, 2\pi]$

Symmetry

x+y axes, $\theta = \frac{\pi n}{8}$



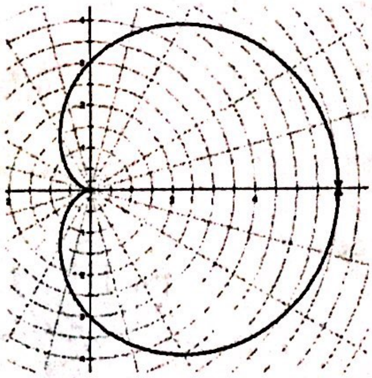
LIMACON curves

9.

Equation:
$r = 3 + 3\cos\theta$

Domain:
$[0, 2\pi]$

Symmetry
x-axis

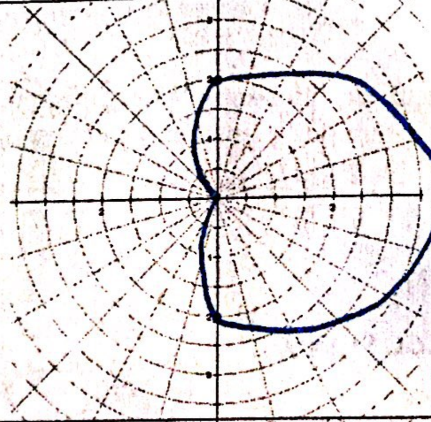


10.

Equation:
$r = 2 + 2\cos\theta$

Domain:
$[0, 2\pi]$

Symmetry
x-axis

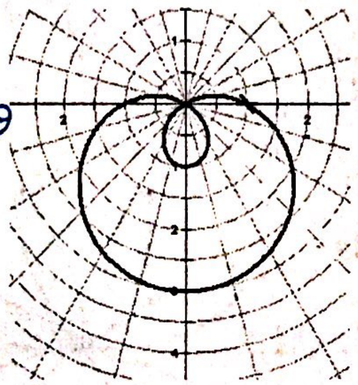


11.

Equation:
$r = 1 - 2\sin\theta$

Domain:
$[0, 2\pi]$

Symmetry
y-axis

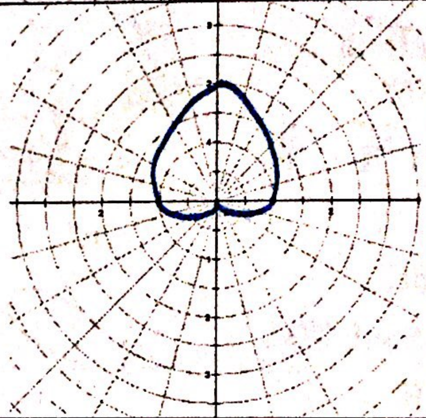


12.

Equation:
$r = 1 + \sin\theta$

Domain:
$[0, 2\pi]$

Symmetry
y-axis

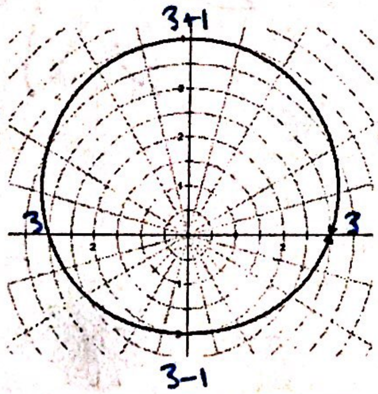


13.

Equation:
$r = 3 + \sin\theta$

Domain:
$[0, 2\pi]$

Symmetry
y-axis

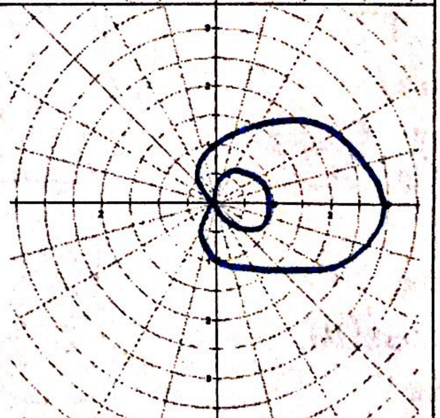


14.

Equation:
$r = 1 + 2\cos\theta$

Domain:
$[0, 2\pi]$

Symmetry
x-axis

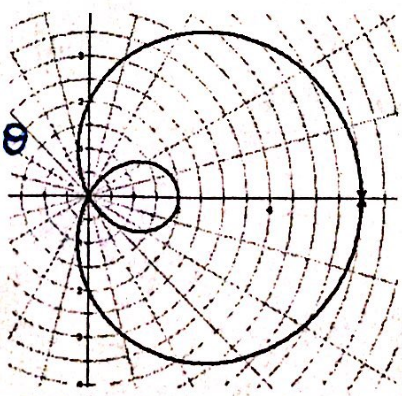


15.

Equation:
$r = 2 + 4\cos\theta$

Domain:
$[0, 2\pi]$

Symmetry
x-axis

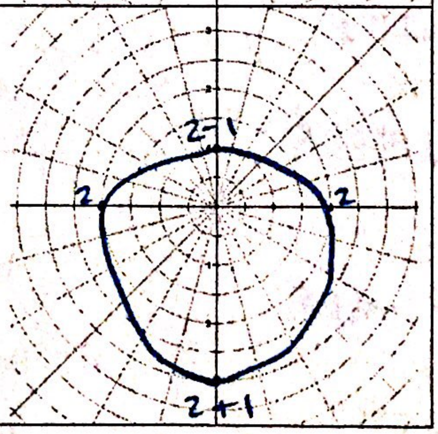


16.

Equation:
$r = 2 - \sin\theta$

Domain:
$[0, 2\pi]$

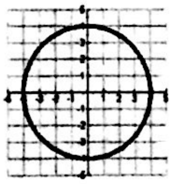
Symmetry
y-axis



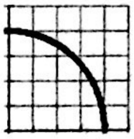
More circular!

Self-Study Lengths of Polar Curves

What is the circumference ("length around") the circle at the right? 8π

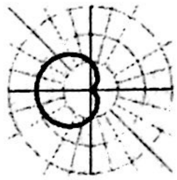


What is the length of the arc in quadrant I? 2π



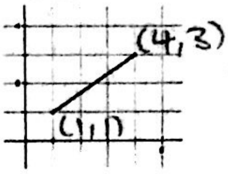
This is TOO EASY!!

Today, you will find out how to find the length around other curves like:



First revisit the formula for finding the distance between two points:

Find the length of this line segment:



$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$d=\sqrt{3^2+2^2}$
$=\sqrt{13}$

Think of the cardioid as a set of really small line segments with lengths that could be determined by the distance formula and then add all of these tiny lengths together to find the length of the curve. Imagine making the segments smaller and smaller so that we have more and more of them. Sound like a similar calculus topic? But of course. . . . . Riemann Sums and Integration!

Now, account for the fact that we are using polar coordinates instead of rectangular coordinates and we have...

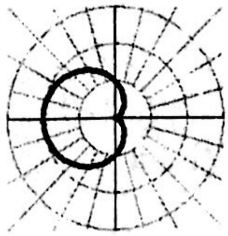
If $r=f(\theta)$ has a continuous first derivative for $\alpha \le \theta \le \beta$ and if the point $P(r,\theta)$ traces the curve $r=f(\theta)$ exactly once as θ runs from α to β, then the length of the curve is

$$L=\int_\alpha^\beta \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$$

Example: Find the length of $r=1-\cos\theta$.

$$\frac{dr}{d\theta}=\sin\theta$$

$$r^2+\left(\frac{dr}{d\theta}\right)^2=(1-\cos\theta)^2+(\sin\theta)^2$$
$$=1-2\cos\theta+\cos^2\theta+\sin^2\theta$$
$$=1-2\cos\theta+1$$
$$=2-2\cos\theta$$



Now, we need to know what the boundaries of integration will be. Think back to your discover lab and to our discussions and what is the smallest θ-interval (domain) that we can use to get the entire cardioid curve?

Domain is $[0,2\pi]$

Use your calculator to do the integration. $L=\int_0^{2\pi}\sqrt{2-2\cos\theta}\,d\theta=$ 8 So, ...

fnInt(r, θ, 0, 2π)

Over for Practice

Your Turn to Try: (YOU can use your calculator to do the integrating!) $L=\int_{\alpha}^{\beta}\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$

1. Find the length of the curve: $r=\cos 2\theta$. Answer: $\boxed{L=9.688}$ ✓

$$L=\int\sqrt{\cos^2(2\theta)+4\sin^2(2\theta)}\,d\theta$$

$$=\int_0^{2\pi}\left(\cos^2(2\theta)+4\sin^2(2\theta)\right)^{1/2}d\theta$$

$r=\cos(2\theta)$

$\frac{dr}{d\theta}=-2\sin(2\theta)$

2. Find the length of the specified portion of the polar curve: Answer: $\boxed{L=9.182}$ ✓

$r=\frac{4}{1+\sin\theta}, 0\le\theta\le\pi$

$$L=\int_0^{\pi}\sqrt{\left(\frac{4}{1+\sin\theta}\right)^2+\left(\frac{-4\cos\theta}{(1+\sin\theta)^2}\right)^2}\,d\theta$$

$$=\int_0^{\pi}\sqrt{\frac{16}{1+2\sin\theta+\sin^2\theta}+\frac{16}{(1+\sin\theta)^4}}\,d\theta$$

$r=\frac{4}{1+\sin\theta}$

$=4(1+\sin\theta)^{-1}$

$\frac{dr}{d\theta}=\frac{-4\cos\theta}{(1+\sin\theta)^2}$

3. Find the length of the curve, $r=a\sin\theta, 0\le\theta\le\pi$, where a is an unknown constant. Answer: $\boxed{L=a\pi}$ ✓

* could be calc. inactive! *

$r=a\sin\theta$

$\frac{dr}{d\theta}=a\cos\theta$

$$L=\int_0^{\pi}\sqrt{(a\sin\theta)^2+(a\cos\theta)^2}\,d\theta$$

$$=\int_0^{\pi}\sqrt{a^2(\sin^2\theta+\cos^2\theta)}\,d\theta$$

$$=\int_0^{\pi}a\,d\theta$$

$$=a\int_0^{\pi}1\,d\theta=a(\pi-0)=\boxed{a\pi}$$

10