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Unit 8 – Polar Packet Name: Key

Day 4 – Introduction to the Polar World ☺

Warmup

Convert into radians or degrees.

a) $45°$ $\pi/4$

b) $\frac{7\pi}{6}$ $210°$

c) $-210°$ $\frac{-7\pi}{6} = \frac{5\pi}{6}$

Find the exact value of:

a) $\cos(30°)$ $\sqrt{3}/2$

b) $\sin\left(\frac{4\pi}{3}\right)$ $-1/2$

c) $\cot(300°)$ $-\sqrt{3}/3$

Solve for θ.

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$ $\theta = \pi/6$

$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$ $\theta = \pi/4$

$\theta = \tan^{-1}(1)$ $\theta = \pi/4$

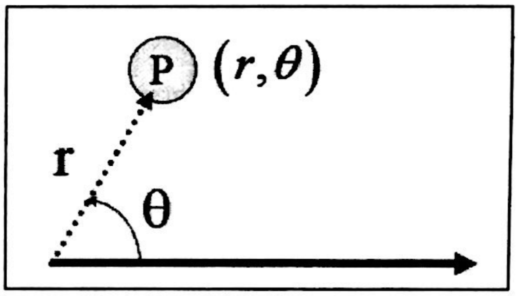
Solve for θ, where $0 \le \theta \le 2\pi$

$\sin\theta = \frac{1}{2}$ $\theta = \pi/6, 5\pi/6$

$\cos\theta = \frac{\sqrt{2}}{2}$ $\theta = \pi/4, 7\pi/4$

$\tan\theta = 1$ $\theta = \pi/4, 5\pi/4$

The Polar Express



First we start with the pole or origin. Then we extend a ray to the right. This is called the polar axis.

Getting to the point…

The location of a point "P" is specified by: (r, θ)

1. Distance from the pole: $\boxed{r}$
2. Angle measure relative to the polar axis: $\boxed{\theta}$

Compare

Cartesian

(x,y)

x = how far to go left or right

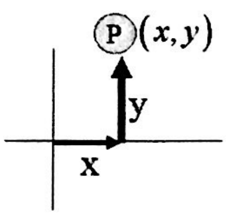
y = how far to go up or down.

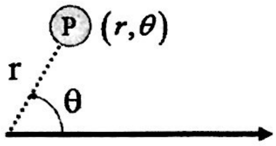
Polar

(r, θ)

r = distance from pole (radius)

θ = the angle the radius makes with the polar axis

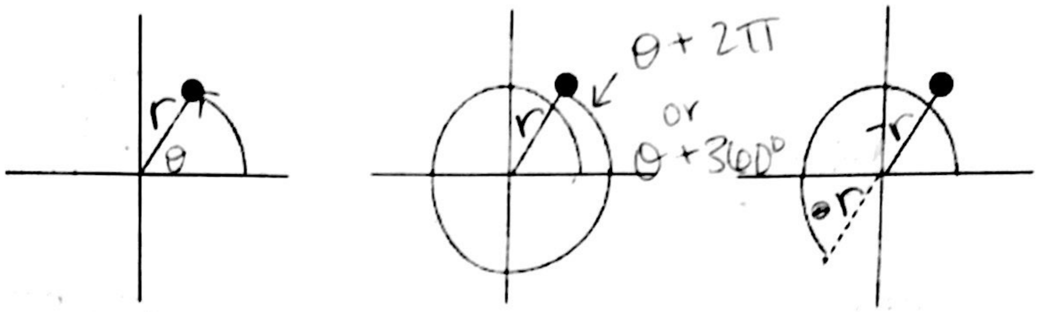




Cartesian vs Polar

<u>Cartesian Coordinate System</u>
A point is named by a unique ordered pair (x,y)

<u>Polar Coordinate System</u>
A point can be named by many different ordered pairs (r, q). Polar points are NOT unique!



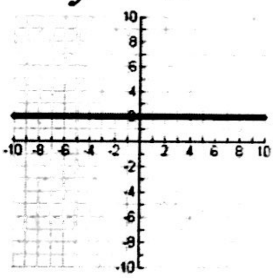
(r, θ) $(r, \theta + 2\pi n)$ $(-5, \theta \pm \pi)$

Let's turn to the last pages of our packet to practice plotting polar points!

Cartesian Coordinates

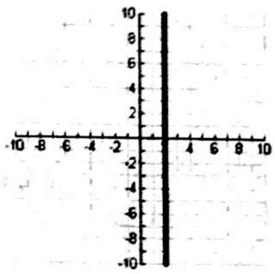
Single variable equations are special cases

$y = 2$



Horizontal line

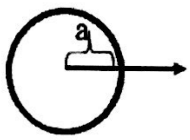
$x = 2$



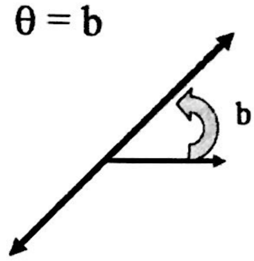
Vertical line

What happens when either r or θ is a constant?

r = a

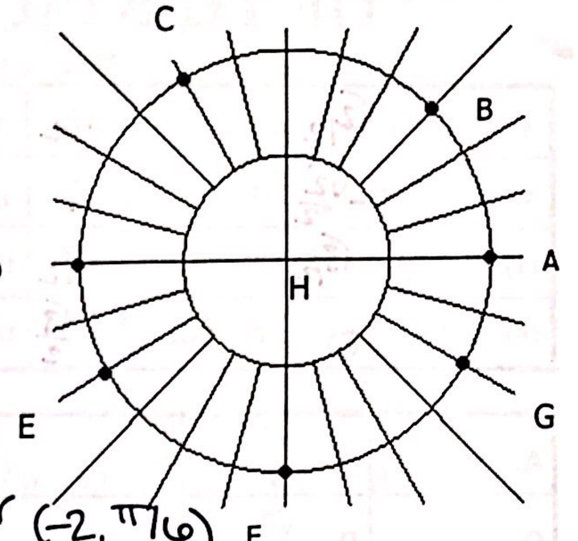


θ = b



Unit Circle Review/Polar Coordinates (Day 4 Homework)

Use the circle to answer the following.



1. At which point(s) shown is the cosine negative?

C, D, E

2. What is the positive radian measure of $\angle AHC$?

$2\pi/3$

3. What is the sine of the angle that would be at point G?

$-1/2$

4. Give the polar coordinates of E. $(2, 7\pi/6)$ or $(-2, \pi/6)$

5. What is the counterclockwise angle measure, in radians, of $\angle AHG$? What is the clockwise measure of the same angle?

$11\pi/6$ $-\pi/6$

6. At which labeled point does sine=cosine? Give the degree and radian measure of this angle.

B, $\pi/4$, $45°$

7. Which point labeled on the circle corresponds to the angle of $-\frac{5\pi}{6}$? $= 7\pi/6$ E

8. Which point labeled on the circle corresponds to the angle $\frac{7\pi}{2}$? F

9. On the graph, plot the following and label:

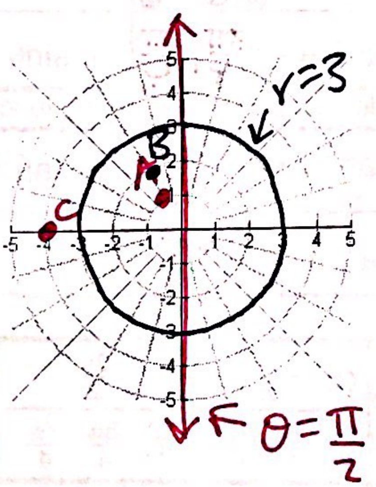
A) $\left(-1, -\frac{\pi}{3}\right)$

B) $\left(2, \frac{2\pi}{3}\right)$

C) $(4, 3\pi)$

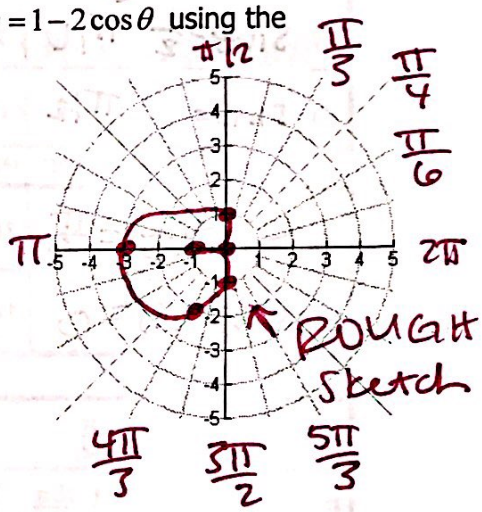
D) r = 3

E) $\theta = \frac{\pi}{2}$



10. On the graph, plot $r = 1 - 2\cos\theta$ using the following points:

r	θ
(−1,	0)
(0,	$\frac{\pi}{3}$)
(1,	$\frac{\pi}{2}$)
(3,	π)
(2,	$\frac{4\pi}{3}$)
(1,	$\frac{3\pi}{2}$)
(0,	$\frac{5\pi}{3}$)
(−1,	2π)



Values.

A. [illegible]	E. $\frac{\sqrt{3}}{3}$	G. $-\frac{1}{2}$	I. $\frac{1}{2}$	L. $-\frac{\sqrt{2}}{2}$	M. -1	N. 1
O. $-$[illegible]	P. $\frac{\sqrt{3}}{2}$	Q. $\sqrt{3}$	R. undefined	S. 0	U. $\frac{\sqrt{2}}{2}$	±. $-\sqrt{3}$

P	L	U	S
11	6	20	10

O	R
17	14

M	I	N	U	S
15	12	5	1	19

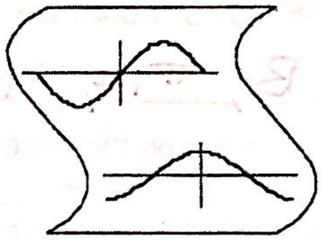
S	I	N	E
16	2	8	3

(

±	S	I	G	N
18	4	9	13	7

)

WHAT IS THE TITLE OF THIS PICTURE?



Match each equation with a solution where $0 \le \theta < 2\pi$.

1) $\sin\theta = \frac{1}{2}$ $\pi/6$ $5\pi/6$	2) $\cos\theta = \frac{\sqrt{2}}{2}$ $\pi/4$ $5\pi/4$	3) $\tan\theta = \frac{\sqrt{3}}{3}$ $\frac{\pi}{6}, \frac{5\pi}{6}$	4) $\tan\theta = -\sqrt{3}$ $\frac{2\pi}{3}, \frac{5\pi}{5}$
5) $\sin\theta = 1$ $\pi/2$	6) $\cos\theta = -\frac{\sqrt{3}}{2}$ $\frac{5\pi}{6}, \frac{7\pi}{6}$	7) $\tan\theta = -1$ $\frac{3\pi}{4}, \frac{5\pi}{4}$	8) $\sin\theta = -1$ $3\pi/2$

Match each expression with the angle θ as defined by the inverse trig function.

9) $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$	10) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \pi/4$	11) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$	12) $\tan^{-1}(-\sqrt{3}) = \frac{-\pi}{3}$
13) $\sin^{-1}(1) = 0$	14) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \pi/6$	15) $\tan^{-1}(-1) = \frac{-\pi}{4}$	16) $\sin^{-1}(-1) = \frac{-\pi}{2}$

Values.

A.	E. $\frac{\pi}{2}$	H. $-\frac{\pi}{4}$	I. $\frac{\pi}{4}$	G. $-\frac{\pi}{2}$	N. $\frac{3\pi}{2}$	S. $\frac{\pi}{6}$
E. [illegible], $\frac{5\pi}{6}$		N. $\frac{\pi}{6}, \frac{7\pi}{6}$	P. $\frac{5\pi}{6}, \frac{7\pi}{6}$	S. $\frac{\pi}{4}, \frac{7\pi}{4}$	T. $\frac{3\pi}{4}, \frac{7\pi}{4}$	U. $\frac{2\pi}{3}, \frac{5\pi}{3}$

11	1	8	13

(

9	10	16	3

)

4	6

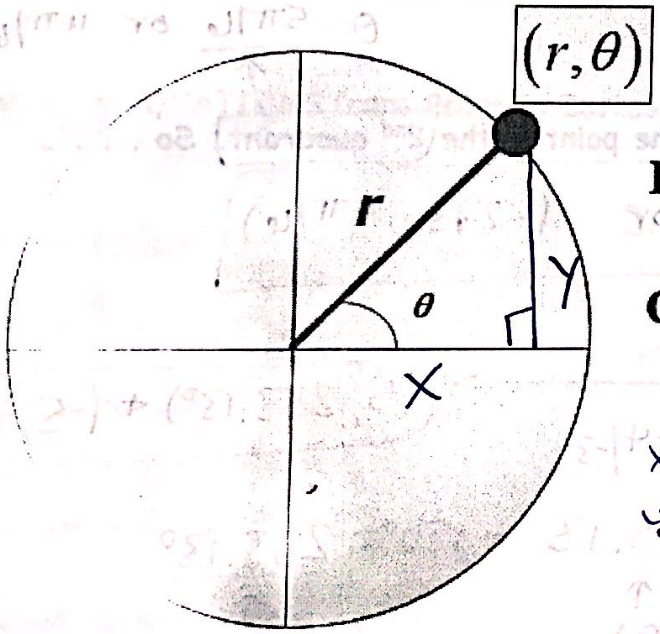
2	15	5	12	7

It is g[illegible] [illegible]ve a Cartesian coordinate system AND a Polar coordinate system because some [illegible] [illegible]re easier in Cartesian and some are easier in Polar:

Polar	*Cartesian*
$r\cos\theta=2$	$x=2$
$r^2\cos\theta\sin\theta=4$	$xy=4$
$r^2\cos^2\theta-r^2\sin^2\theta=1$	$x^2-y^2=1$
$r=1+2r\cos\theta$	$y^2-3x^2-4x-1=0$
$r=1-\cos\theta$	$x^4+y^4+2x^2y^2+2x^3+2xy^2-y^2=0$

Conv[illegible] [illegible] [illegible]lar to Rectangular:

$$\boxed{\begin{aligned} x&=r\cos\theta \\ y&=r\sin\theta \end{aligned}}$$



Example:

Convert $\left(\overset{r}{4},\overset{\theta}{\frac{\pi}{6}}\right)$ **to Cartesian coordinates** (x,y) **:**

$x=4\cos(\pi/6)=4(\sqrt{3}/2)=2\sqrt{3}$

$y=4\sin(\pi/6)=4(1/2)=2$

$(4,\pi/6)=\boxed{(2\sqrt{3},2)}$

Examp[illegible]

1) $\left(\overset{r}{4}.\right.$

2) $\left(\overset{r}{-3},\overset{\theta}{\frac{5\pi}{4}}\right)$

$x=-3\cos(5\pi/4)=-3\left(-\frac{\sqrt{2}}{2}\right)$

$y=-3\sin(5\pi/4)=-3\left(-\frac{\sqrt{2}}{2}\right)$

$\boxed{\left(\frac{3\sqrt{2}}{2},\frac{3\sqrt{2}}{2}\right)}$

3) $\left(\overset{r}{-5},\overset{\theta}{\frac{\pi}{6}}\right)$

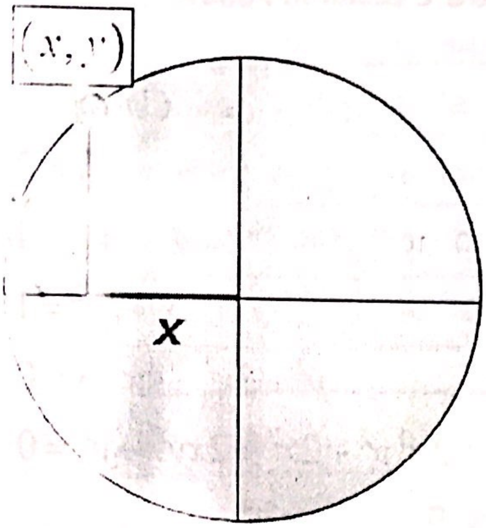
$x=-5\cos(\pi/6)=-5(\sqrt{3}/2)$

$y=-5\sin(\pi/6)=-5(1/2)$

$\boxed{\left(\frac{-5\sqrt{3}}{2},\frac{-5}{2}\right)}$

Conv… from Rectangular to Polar

$$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x}$$



Example:

Convert $\overset{x}{(-3}, \overset{y}{\sqrt{3})}$ **to Polar Coordinates**

Finding r: $r^2 = (-3)^2 + (\sqrt{3})^2$

$r^2 = 9 + 3 = 12$

$r = \pm\sqrt{12} = \pm 2\sqrt{3}$

Finding θ is more involved . . .

$\tan\theta = \frac{\sqrt{3}}{-3} = \frac{-1}{\sqrt{3}} = \frac{-1/2}{\sqrt{3}/2} = \frac{1/2}{-\sqrt{3}/2}$ ← y ← x

$\tan\theta$? $(\sqrt{3}/2, -1/2)$ + $(-\sqrt{3}/2, 1/2)$

$\theta = \underline{5\pi/6}$ or $11\pi/6$

Now w… have t… pair up the r and θ . . .

infinite options!!

The o… …rian coordinates place the point in the $\boxed{\text{2nd quadrant}}$. So . . . .

$$\boxed{(2\sqrt{3},\ 5\pi/6) \text{ or } (-2\sqrt{3},\ 11\pi/6)}$$

Ex.: Con… $\boxed{(\overset{}{-3}, -4)}$ ← QIII to Polar Coordinates. → $\boxed{(5, 233.13^\circ) + (-5, 53.13^\circ)}$

x y

$r^2 = (-3)^2 + (-4)^2$

$r^2 = 9 + 16$

$r^2 = 25 \rightarrow \underline{r = \pm 5}$

$\tan\theta = -4/-3$

$\theta \approx \underline{53.13}$ $+ 180^\circ = \underline{233.13^\circ}$

↑ Q1 ↑ Q3

More ex…ples!

1) $(1, \sqrt{\ }$…

2) $(1, -1)$ = $\boxed{(\sqrt{2}, \frac{7\pi}{4}), (-\sqrt{2}, \frac{3\pi}{4})}$ ← QIV

$r^2 = 1^2 + (-1)^2 = 2$

$r = \pm\sqrt{2}$

$\tan\theta = -1$

$\theta = 3\pi/4,\ 7\pi/4$

↑ QII ↑ QIV

3) $(\sqrt{2}, -\sqrt{2})$ = $\boxed{(2, \frac{7\pi}{4}), (-2, \frac{3\pi}{4})}$ ← QIV

$r^2 = \sqrt{2}^2 + (-\sqrt{2})^2 = 4$

$r = \pm 2$

$\tan\theta = -\sqrt{2}/\sqrt{2} = -1$

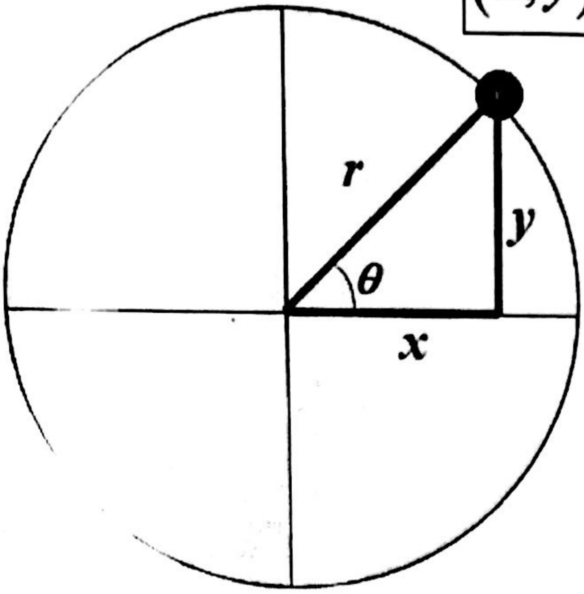
$\theta = 3\pi/4,\ 7\pi/4$

QII QIV

Recap

(r,θ)

(x,y)



Cartesian to Polar

$$r^2 = x^2 + y^2$$

$$\tan\theta = \frac{y}{x}$$

Polar to Cartesian

$$\cos\theta = \frac{x}{r} \quad \Leftrightarrow \quad x = r\cos\theta$$

$$\sin\theta = \frac{y}{r} \quad \Leftrightarrow \quad y = r\sin\theta$$

Converting EQUATIONS from Polar to Cartesian

1. $r = -3\sec\theta$

$r = \frac{-3}{\cos\theta}$

$r\cos\theta = -3$

$\boxed{x = -3}$

vertical line !!

2. $r = \frac{5}{\sin\theta - 2\cos\theta}$

$\underbrace{r\sin\theta} - 2\underbrace{r\cos\theta} = 5$

$y - 2x = 5$

$\boxed{y = 2x + 5}$

line with slope of 2 and y-int. of (0,5)