Textbook Scavenger Hunt

Read pages 416-419 in your textbook and find the following.

pg. 417

1. Find the definition the book gives for **half-life** and write it here. Also, what is the half-life of Radium?

"number of yrs required for half of the atoms in a sample of radioactive material to decay" 1599 yrs

pg. 419

2. Find **Newton's Law of Cooling** and write a general differential equation that could model it. Be sure to define any variables and/or constants.

$\frac{dy}{dt} = -k(y - y_s)$ y: current temp. y_s: surrounding temp.

change in temp. over time

proportionality constant

pg. 416

3. Find the general **differential equation** for a quantity that increases or decreases at a rate proportional to the amount present. Write it here:

rate of change of y $\frac{dy}{dt} = ky$ proportionality constant

Answer #21-29, 34, 37, and 40 on pages 420-421.

$\boxed{21}$ $\frac{dy}{dx} = ky$

$y = Ce^{kx}$

$(0,6): 6 = Ce^{k(0)} = C$

$(4,15): 15 = 6e^{k(4)} = K$

$k = \frac{1}{4}\ln(5/2)$

$y = 6e^{[1/4 \ln(5/2)]x}$

$\approx 6e^{0.2291x}$

$x = 8: y = 6e^{1/4 \ln(5/2)(8)}$

$= 6e^{\ln(5/2)^2} = 6(25/4)$

$= \boxed{75/2}$

$\boxed{22}$ $\frac{dN}{dt} = kN$

$N = Ce^{kt}$

$(0,250): C = 250$

$(1,400): 400 = 250e^{k}$

$k = \ln\left(\frac{400}{250}\right) = \ln(8/5)$

$N = 250e^{\ln(8/5)t} \approx 250e^{0.4700t}$

when $t = 4, N = 250e^{4\ln(8/5)}$

$= 250e^{\ln(8/5)^4} = 250(8/5)^4$

$= \boxed{\frac{8192}{5}}$

$\boxed{23}$

$\frac{dV}{dt} = kV$

$V = Ce^{kt}$

$(0, 20,000): C = 20,000$

$(4, 12500): 12500 = 20,000e^{4k}$

$k = \frac{1}{4}\ln(5/8)$

$V = 20000e^{[1/4 \ln(5/8)]t} \approx 20000e^{-0.1175t}$

$t = 6: V = 20000e^{\ln(5/8)^{(3/2)}}$

$= 20000(5/8)^{3/2}$

$\approx \boxed{9882.118}$

$\boxed{24}$ $\frac{dP}{dt} = kP$

$P = Ce^{kt}$

$(0,5000): C = 5000$

$(1,4750): 4750 = 5000e^{k}$

$t = \ln(19/20)$

$P = 5000e^{\ln(19/20)t}$

$\approx 5000e^{-0.0513t}$

$t=5: P = 5000e^{\ln(19/20)(5)}$

$= 5000(19/20)^5 \approx \boxed{3868.905}$

$\boxed{25}$ $y = Ce^{kt}, (0, 1/2), (5,5)$

$C = 1/2$

$y = 1/2 e^{kt}$

$5 = \frac{1}{2}e^{5k}$

$k = \frac{\ln(10)}{5}$

$y = \frac{1}{2}e^{[\ln(10)/5]t}$

$= \frac{1}{2}(10^{t/5})$

$\boxed{y \approx \frac{1}{2}e^{0.4605t}}$

$\boxed{26}$ $y = Ce^{kt}, (0,4), (5, 1/2)$

$C = 4$

$y = 4e^{kt}$

$\frac{1}{2} = 4e^{5k}$

$k = \frac{\ln(1/8)}{5} \approx -0.4159$

$\boxed{y = 4e^{-0.4159}}$

$\boxed{27}$ $y = Ce^{kt}, (1,5), (5,2)$

$5 = Ce^{k} \Rightarrow 10 = 2Ce^{k}$

$2 = Ce^{5k} \Rightarrow 10 = 5Ce^{k}$

$2Ce^{k} = 5Ce^{5k}$

$2e^{k} = 5e^{5k}$

$\frac{2}{5} = e^{4k}$

$k = \frac{1}{4}\ln(\frac{2}{5}) = \ln(2/5)^{1/4}$

$C = 5e^{-k} = 5e^{-1/4\ln(2/5)} = 5(2/5)^{-1/4} = 5(5/2)^{1/4}$

$y = 5(5/2)^{1/4}e^{[1/4\ln(2/5)]t} \approx \boxed{6.2872e^{-0.2291t}}$

$\boxed{28}$ $y = Ce^{kt}, (3, 1/2), (4,5)$

$1/2 = Ce^{3k} \Rightarrow 1 = 2Ce^{3k}$

$5 = Ce^{4k} \Rightarrow 1 = \frac{1}{5}Ce^{4k}$

$2Ce^{3k} = \frac{1}{5}Ce^{4k}$

$10e^{3k} = e^{4k}$

$10 = e^{k}$

$k = \ln(10) \approx 2.3026$

$y = Ce^{2.3026t}$

$5 = Ce^{2.3026(4)} \Rightarrow C \approx 0.0005$

$\boxed{y = 0.0005e^{2.3026t}}$

$\boxed{29}$ C: initial value of y (when $t=0$)

k: proportionality constant

$\boxed{34}$ half-life = 1599 yrs $\Rightarrow$

$\frac{1}{2} = 1 \cdot e^{k(1599)} \Rightarrow k = \frac{\ln(1/2)}{1599}$

1.5 g after 1000 yrs $\Rightarrow$

$1.5 = Ce^{\ln(1/2)/1599)(1000)}$

$\Rightarrow C \approx 2.314$

initial quantity ≈ 2.314 g

$t = 10000: y = 2.314e^{\ln(1/2)/1599(10000)}$

$\approx \boxed{0.03\text{ g}}$

$\boxed{37}$ initial quantity = 5 g $\Rightarrow C = 5$

half-life = 5715 yrs $\Rightarrow$

$2.5 = 5e^{k(5715)} \Rightarrow k = \frac{\ln(1/2)}{5715}$

$t = 1000 \Rightarrow y = 5e^{[\ln(1/2)/5715](1000)}$

≈ 4.43 g

$t = 10000 \Rightarrow y = 5e^{[\ln(1/2)/5715](10000)}$

$\approx \boxed{1.49\text{ g}}$

$\boxed{40}$ half-life = 24,100 yrs $\Rightarrow$

$\frac{1}{2} = 1 \cdot e^{k(24100)} \Rightarrow k = \frac{\ln(1/2)}{24100}$

0.4 g after 10000 yrs

$\Rightarrow 0.4 = Ce^{[\ln(1/2)/24100](10000)}$

$\Rightarrow C \approx 0.533$ g ← initial quantity

$t = 1000: y = 0.533e^{[\ln(1/2)/24100] \cdot 1000}$

$\approx \boxed{0.52\text{ g}}$

1. ***Growth of Cholera Bacteria***

Suppose that the cholera bacteria in a colony grows unchecked according to the Law of Exponen... Change. The colony starts with 1 bacterium and doubles in number every half hour. How many bacte... will the colony contain at the end of 24 h?

$y \approx 2.8 \times 10^{14}$

2. ***Bacteria Growth***

A colony of bacteria is grown under ideal conditions in a laboratory so that the population increases exponentially with time. At the end of 3 hours, there are 10,000 bacteria. At the end of 5 h there are 40,000 bacteria. How many bacteria were present initially?

$y_0 = 1250$ bacteria

3. ***Radon- 222***

The decay equation for radon – 222 gas is known to be $y = y_0 e^{-0.18t}$, with t measured in days. About how long will it take the amount of radon in a sealed sample of air to decay to 90% of its original value?

$t \approx 0.585$ days

4. Polonium-210

The number of radioactive atoms remaining after t days in a sample of polonium -210 that starts with y_0 radioactive atoms is $y = y_0 e^{-0.005t}$

(a) Find the element's half-life.

$t \approx 138.629$ days

(b) A sample is not useful after 95% of the radioactive nuclei present on the day the sample ... disintegrated. For about how many days after the sample arrives will you be able to use the p...

$t \approx 599.146$ days

1 $y = y_0 e^{kt}$

$2 = 1 \cdot e^{\frac{1}{2}k}$

$2 = e^{k/2}$

$\ln 2 = k/2$

$2 \cdot \ln 2 = k$

$k \approx 1.386$ STORE!!

initial info: $y_0 = 1$, $y = 2 \cdot y_0 = 2$, $t = 1/2$

$y = e^{1.386t}$ 24 given

$\boxed{y \approx 2.8 \times 10^{14}}$

$\boxed{2}$ $y = \boxed{y_0} e^{kt}$

given: (t, y): $(3, 10000)$, $(5, 40000)$

$[10000 = y_0 e^{3k}] \cdot 4$

$40000 = y_0 e^{5k}$

$4 \cancel{y_0} e^{3k} = \cancel{y_0} e^{5k}$

$4 = e^{5k-3k}$

$4 = e^{2k}$

$\ln 4 = 2k$

$k = \frac{\ln 4}{2}$

$y = y_0 e^{kt}$

$10000 = y_0 e^{3k}$

$10000 = y_0 e^{3/2 \cdot \ln 4}$

$10000 = 8y_0$

$\boxed{y_0 = 1250 \text{ bacteria}}$

$\boxed{3}$ $y = 0.9y_0$

$y = y_0 e^{-0.18\boxed{t}}$

$0.9 \cancel{y_0} = \cancel{y_0} e^{-0.18t}$

$\ln(0.9) = -0.18t$

$\frac{\ln(0.9)}{-0.18} = t$

$\boxed{t \approx 0.585 \text{ days}}$

$\boxed{4}$ $y = y_0 e^{-0.005t}$

Half-Life

$\frac{1}{2} y_0 = y$

a) $0.5 = e^{-0.005t}$

$\ln(0.5) = -0.005t$

$\frac{\ln(0.5)}{-0.005} = t$

$\boxed{t \approx 138.629 \text{ days}}$

b) After 95% disintegrated $\Rightarrow$ when will 5% be left?

$y = .05y_0$

$y = y_0 e^{-0.005t}$

$0.05 \cancel{y_0} = \cancel{y_0} e^{-0.005t}$

$0.05 = e^{-0.005t}$

$\ln(0.05) = -0.005t$

$\boxed{t \approx 599.146 \text{ days}}$