Parametrics Free Response Practice – Set 2

1992

AP Calculus BC
Free Response BC 3

4. At time t, $0 \le t \le 2\pi$, the position of a particle moving along a path in the xy-plane is given by the
parametric equations $x = e^t \sin t$ and $y = e^t \cos t$.

(a) Find the slope of the path of the particle at time $t = \frac{\pi}{2}$.

(b) Find the speed of the particle when $t = 1$.

(c) Find the distance traveled by the particle along the path from $t = 0$ to $t = 1$.

(a) $\frac{dy}{dx} = \left.\frac{e^t \cos t - e^t \sin t}{e^t \cos t + e^t \sin t}\right|_{t=\frac{\pi}{2}} = \frac{e^{\pi/2}(0) - e^{\pi/2}(1)}{e^{\pi/2}(0) + e^{\pi/2}(1)} = \frac{-e^{\pi/2}}{e^{\pi/2}} = \boxed{-1}$

(b) $\left.\frac{dy}{dx}\right|_{t=1} = \langle e^1\cos(1) + e^1\sin(1),\ e^1\cos(1) - e^1\sin(1)\rangle$

speed $= \sqrt{e^2[\cos^2(1) + \sin^2(1)] + e^2[\cos^2(1) + \sin^2(1)]} = \sqrt{2e^2} = \boxed{e\sqrt{2}}$

(c) $\int_0^1 \sqrt{(e^t\cos t + e^t \sin t)^2 + (e^t \cos t - e^t \sin t)^2}\, dt = \int_0^1 \sqrt{2e^{2t}}\, dt$

$= \sqrt{2}\int_0^1 e^t\, dt = \sqrt{2}\,(e^t)\Big|_0^1 = \boxed{\sqrt{2}\,(e-1)}$

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$x''(t) = 2$

5. A particle moves along the graph of $y = \cos x$ so that the x-coordinate of acceleration is always 2. At time $t = 0$, the particle is at point $(\pi, -1)$ and the velocity vector of the particle is $(0,0)$

$r(0) = \langle \pi, -1\rangle$ $\quad v(0) = \vec{0}$

(a) Find the x- and y-coordinates of the position of the particle in terms of t.

(b) Find the speed of the particle when its position is $(4, \cos 4)$

(a) $x''(t) = 2 \Rightarrow x'(t) = 2t + C.\ x'(0) = 0 + C \Rightarrow C = 0 \Rightarrow x'(t) = 2t$

$x(t) = t^2 + D.\ x(0) = \pi \Rightarrow 0 + D \Rightarrow D = \pi \Rightarrow x(t) = t^2 + \pi$

$r(t) = \langle t^2 + \pi,\ \cos(t^2 + \pi)\rangle$

(b) $v(t) = \langle 2t,\ -2t\sin(t^2 + \pi)\rangle$

$4 = t^2 + \pi \Rightarrow t = \sqrt{4 - \pi}$

speed $= \sqrt{(2(\sqrt{4-\pi}))^2 + (-2(\sqrt{4-\pi})\sin(4))^2} = \sqrt{4(4-\pi) + 4(4-\pi)\sin^2(4)}$

$= \sqrt{(16 - 4\pi)(1 + \sin^2(4))} \approx \boxed{2.324}$