① $\sin^2x(1+\tan^2x) = \tan^2x$ Start

$\sin^2x(\sec^2x) = \tan^2x$ Pyth. ID.

$\sin^2x \cdot \frac{1}{\cos^2x} = \tan^2x$ Rec. ID.

$\frac{\sin^2x}{\cos^2x} = \tan^2x$ Multiply

$\tan^2x = \tan^2x$ Simplify (Quotient ID)

Q.E.D. End ⇓

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② $(\cos x)(\tan x + \sin x \cot x) = \sin x + \cos^2x$ Start

$\cos x \tan x + \cos x \sin x \cot x = \sin x + \cos^2x$ Distribute

$\cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}} + \cos x \cancel{\sin x} \cdot \frac{\cos x}{\cancel{\sin x}} = \sin x + \cos^2x$ $\sin x \& \cos x$ only

$\sin x + \cos^2x = \sin x + \cos^2x$ Divide

Q.E.D. End ⇓

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③ $\cot^2x - \cos^2x = \cos^2x \cot^2x$ Start

$\frac{\cos^2x}{\sin^2x} - \frac{\cos^2x}{1} = \cos^2x \cot^2x$ Quotient ID

$\frac{\cos^2x - \cos^2x \sin^2x}{\sin^2x} = \cos^2x \cot^2x$ Criss-Cross Applesauce

$\frac{\cos^2x(1-\sin^2x)}{\sin^2x} = \cos^2x \cot^2x$ Factor GCF

$\frac{\cos^2x(\cos^2x)}{\sin^2x} = \cos^2x \cot^2x$ Pyth. ID.

$\cos^2x \cot^2x = \cos^2x \cot^2x$ Quotient ID.

Q.E.D. End ⇓

④ $\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$ Start

$\sec^2 x(\sec^2 x - 1) = \tan^2 x(\tan^2 x + 1)$ Factor GCF

$\sec^2 x(\tan^2 x) = \tan^2 x(\sec^2 x)$ Pyth. ID.

$\tan^2 x \sec^2 x = \tan^2 x \sec^2 x$ Comm. Prop. of Mult.

Q.E.D. End ‼

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⑤ $\tan x + \cot x = \sec x \csc x$ Start

$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{1}{\cos x} \cdot \frac{1}{\sin x}$ sinx & cosx only

$\frac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \frac{1}{\cos x \sin x}$ Criss-Cross Applesauce

$\frac{1}{\cos x \sin x} = \frac{1}{\cos x \sin x}$ Pyth. ID.

Q.E.D. End ‼

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⑥ $\frac{\sec x}{\csc x} + \frac{\sin x}{\cos x} = 2\tan x$ Start

$\frac{\sec x \cos x + \sin x \csc x}{\csc x \cos x} = 2\tan x$ Criss-Cross Applesauce

$\frac{\frac{1}{\cos x} \cdot \frac{\cos x}{1} + \frac{\sin x}{1} \cdot \frac{1}{\sin x}}{\frac{1}{\sin x} \cdot \frac{\cos x}{1}} = 2\tan x$ sinx & cosx only

$\frac{1 + 1}{\cot x} = 2\tan x$ Simplify

$2\tan x = 2\tan x$ Quotient ID.

Q.E.D. End ‼

$$-\frac{\sin^2\theta}{1+\cos\theta} = \cos\theta$$ Start

$$1-\frac{1-\cos^2\theta}{1+\cos\theta} = \cos\theta$$ Pyth. ID.

$$1-\frac{(1+\cos\theta)(1-\cos\theta)}{(1+\cos\theta)} = \cos\theta$$ Diff. of Squares

$1-(1-\cos\theta) = \cos\theta$ Simplify / Divide

$1-1+\cos\theta = \cos\theta$ Distribute

$\cos\theta = \cos\theta$ Subtract

Q.E.D. End

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(8) $\frac{1}{\sec^2\theta} + \frac{1}{\csc^2\theta} = 1$ Start

$\cos^2\theta + \sin^2\theta = 1$ Reciprocal ID.

$1 = 1$ Pyth. ID.

Q.E.D. End

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(9) $\frac{\tan x \cos x}{\sin x} = 1$ Start

$\frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x} = 1$ sinx & cosx only

$1 = 1$ Divide

Q.E.D. End

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(10) $\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$ Start

$\sin x \sin x + \cos x \cos x = 1$ Reciprocal ID.

$\sin^2 x + \cos^2 x = 1$ Simplify

$1 = 1$ Pyth. ID.

Q.E.D. End

(11) $\tan^2 x = \dfrac{1 + \tan^2 x}{\csc^2 x}$ Start

$\tan^2 x = \dfrac{\sec^2 x}{\csc^2 x}$ Pyth. ID.

$\tan^2 x = \dfrac{\sin^2 x}{\cos^2 x}$ Reciprocal IDs.

$\tan^2 x = \tan^2 x$ Quotient ID.

Q.E.D. End

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(12) $\sin^2 x + \cot^2 x + \dfrac{1}{\sec^2 x} = \csc^2 x$ Start

$\sin^2 x + \cot^2 x + \cos^2 x = \csc^2 x$ Reciprocal ID.

$\cot^2 x + 1 = \csc^2 x$ Pyth. ID.

$\csc^2 x = \csc^2 x$ Pyth. ID.

Q.E.D. End

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(13) $\tan\theta(\tan\theta + \cot\theta) = \sec^2\theta$ Start

$\tan^2\theta + \tan\theta\cot\theta = \sec^2\theta$ Distribute

$\tan^2\theta + 1 = \sec^2\theta$ Reciprocal ID.

$\sec^2\theta = \sec^2\theta$ Pyth. ID.

Q.E.D. End

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(14) $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$ Start

$\cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$ Pyth. ID.

$\cos^2 x - 1 + \cos^2 x = 2\cos^2 x - 1$ Distribute

$2\cos^2 x - 1 = 2\cos^2 x - 1$ Combine Like Terms

Q.E.D. End

$\sec^2x + \csc^2x = \sec^2x \csc^2x$ Start

$\frac{1}{\cos^2x} + \frac{1}{\sin^2x} = \frac{1}{\cos^2x \sin^2x}$ Reciprocal IDs.

$\frac{\sin^2x + \cos^2x}{\cos^2x \sin^2x} = \frac{1}{\cos^2x \sin^2x}$ Criss-cross Applesauce

$\frac{1}{\cos^2x \sin^2x} = \frac{1}{\cos^2x \sin^2x}$ Pyth. ID.

Q.E.D. End ‼

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(16) $\sec x - \cos x = \sin x \tan x$ Start

$\frac{1}{\cos x} - \frac{\cos x}{1} = \sin x \cdot \frac{\sin x}{\cos x}$ sinx & cosx only

$\frac{1 - \cos^2x}{\cos x} = \frac{\sin^2x}{\cos x}$ Criss-cross Applesauce

$\frac{\sin^2x}{\cos x} = \frac{\sin^2x}{\cos x}$ Pyth. ID.

Q.E.D. End ‼

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(17) $\sin^4x - \cos^4x = \sin^2x - \cos^2x$ Start

$(\sin^2x - \cos^2x)(\sin^2x + \cos^2x) = \sin^2x - \cos^2x$ Diff. of Squares

$(\sin^2x - \cos^2x)(1) = \sin^2x - \cos^2x$ Pyth. ID.

$\sin^2x - \cos^2x = \sin^2x - \cos^2x$ Simplify

Q.E.D. End ‼

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(18) $\frac{1}{1 - \sin x} - \frac{1}{1 + \sin x} = 2 \tan x \sec x$ Start

$\frac{1 + \sin x - 1 + \sin x}{(1 - \sin x)(1 + \sin x)} = 2 \tan x \sec x$ Criss-Cross Applesauce

$\frac{2 \sin x}{1 - \sin^2x} = 2 \tan x \sec x$ Diff. of Squares & combine LTs.

$\frac{2 \sin x}{\cos^2x} = 2 \tan x \sec x$ Pyth. ID.

$2 \tan x \sec x = 2 \tan x \sec x$ Quot. & Rec. IDs.

Q.E.D. End ‼

(19) $\sec\theta + \csc\theta = (\tan\theta + \cot\theta)(\cos\theta + \sin\theta)$ Start

$\sec\theta + \csc\theta = \tan\theta\cos\theta + \tan\theta\sin\theta + \cot\theta\cos\theta + \cot\theta\sin\theta$ BOX FOIL

$\sec\theta + \csc\theta = \frac{\sin\theta}{\cos\theta}\cdot\cos\theta + \frac{\sin\theta}{\cos\theta}\cdot\sin\theta + \frac{\cos\theta}{\sin\theta}\cdot\cos\theta + \frac{\cos\theta}{\sin\theta}\cdot\sin\theta$ sin & cos only

$\sec\theta + \csc\theta = \sin\theta + \frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\sin\theta} + \cos\theta$ Divide

$\sec\theta + \csc\theta = \frac{\sin^2\theta\cos\theta + \sin^3\theta + \cos^3\theta + \cos^2\theta\sin\theta}{\cos\theta\sin\theta}$ LCD

$\sec\theta + \csc\theta = \frac{\cos\theta(\sin^2\theta + \cos^2\theta) + \sin\theta(\sin^2\theta + \cos^2\theta)}{\cos\theta\sin\theta}$ GCF

$\sec\theta + \csc\theta = \frac{\cos\theta + \sin\theta}{\cos\theta\sin\theta}$ Pyth. ID.

$\sec\theta + \csc\theta = \csc\theta + \sec\theta$ Reciprocal ID.

$\sec\theta + \csc\theta = \sec\theta + \csc\theta$ Comm. Prop. of Addition

Q.E.D. End ‖

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(20) $\frac{1+\tan x}{1+\cot x} = \frac{\sin x}{\cos x}$ CHALLENGE... omit.

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(21) $\frac{\sec x}{\sin x} - \frac{\sin x}{\cos x} = \cot x$ Start

$\frac{\sec x\cos x - \sin^2 x}{\sin x\cos x} = \cot x$ criss-cross Applesauce

$\frac{1 - \sin^2 x}{\sin x\cos x} = \cot x$ Reciprocal ID.

$\frac{\cos^2 x}{\sin x\cos x} = \cot x$ Pyth. ID.

$\frac{\cos x}{\sin x} = \cot x$ Divide

$\cot x = \cot x$ Quotient ID.

Q.E.D. End ‖

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(22) $\cos^2 x + \tan^2 x\cos^2 x = 1$ Start

$\cos^2 x(1 + \tan^2 x) = 1$ Factor GCF

$\cos^2 x(\sec^2 x) = 1$ Pyth. ID.

$\cos^2 x \cdot \frac{1}{\cos^2 x} = 1$ Reciprocal ID.

$1 = 1$ Simplify

Q.E.D. End ‖