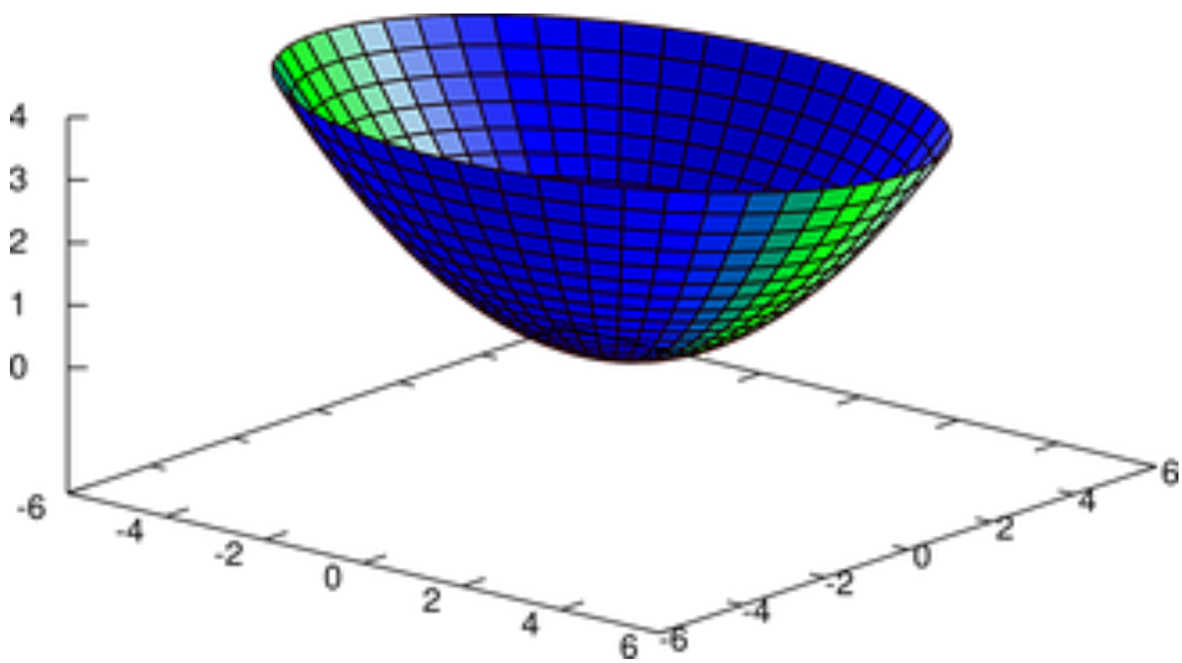
Pre-Calculus Indicators



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PUBLIC SCHOOLS OF NORTH CAROLINA
State Board of Education | Department of Public Instruction

Pre-Calculus Objectives

Pre-Calculus provides students an honors-level study of trigonometry, advanced functions, analytic geometry, and data analysis in preparation for calculus. Applications and modeling should be included throughout the course of study. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites

- Describe phenomena as functions graphically, algebraically and verbally; identify independent and dependent quantities, domain, and range, input/output, mapping.
- Translate among graphic, algebraic, numeric, tabular, and verbal representations of relations.
- Define and use linear, quadratic, cubic, exponential, rational, absolute value, and radical functions to model and solve problems.
- Use systems of two or more equations or inequalities to solve problems.
- Use the trigonometric ratios to model and solve problems.
- Use logic and deductive reasoning to draw conclusions and solve problems.

Strands: Number & Operations, Geometry & Measurement, Data Analysis & Probability, Algebra

COMPETENCY GOAL 1: The learner will describe geometric figures in the coordinate plane algebraically.

Objectives

1.01 Transform relations in two dimensions; describe the results algebraically and geometrically.

1.02 Use the quadratic relations (parabola, circle, ellipse, hyperbola) to model and solve problems; justify results.

a) Solve using tables, graphs, and algebraic properties.
b) Interpret the constants and coefficients in the context of the problem.

1.03 Operate with vectors in two dimensions to model and solve problems.

COMPETENCY GOAL 2: The learner will use relations and functions to solve problems.

Objectives

2.01 Use functions (polynomial, power, rational, exponential, logarithmic, logistic, piecewise-define, and greatest integer) to model and solve problems; justify results.

a) Solve using graphs and algebraic properties.
b) Interpret the constants, coefficients, and bases in the context of the problem.

2.02 Use trigonometric and inverse trigonometric functions to model and solve problems; justify results.

a) Solve using graphs and algebraic properties.
b) Create and identify transformations with respect to period, amplitude, and vertical and horizontal shifts.
c) Develop and use the law of sines and the law of cosines.

2.03 For sets of data, create and use calculator-generated models of linear, polynomial, exponential, trigonometric, power, logistic, and logarithmic functions.

a) Interpret the constants, coefficients, and bases in the context of the data.
b) Check models for goodness-of-fit; use the most appropriate model to draw conclusions or make predictions.

2.04 Use the composition and inverse of functions to model and solve problems.

2.05 Use polar equations to model and solve problems.

a) Solve using graphs and algebraic properties.
b) Interpret the constants and coefficients in the context of the problem.

2.06 Use parametric equations to model and solve problems.

2.07 Use recursively-defined functions to model and solve problems.

a) Find the sum of a finite sequence.
b) Find the sum of an infinite sequence.
c) Determine whether a given series converges or diverges.
d) Translate between recursive and explicit representations.

2.08 Explore the limit of a function graphically, numerically, and algebraically.

Pre-Calculus Objective 1.01
Transformations

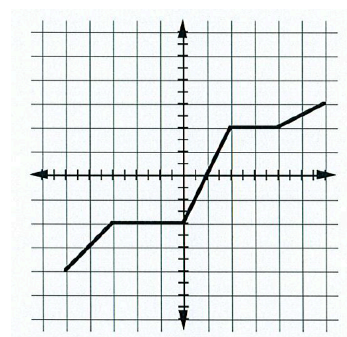
Vocabulary/Concepts/Skills:

- Effects of a, b, c and d in $y = af(bx + c)d$
- Coefficients
- Translation
- Reflection
- Dilation
- Even/Odd
- Symmetries

Example 1: Let $f(x) = x^2 - 4x$. Graph $f(x)$ and $g(x)$. Identify similarities and explain differences between $f(x)$ and $g(x)$.

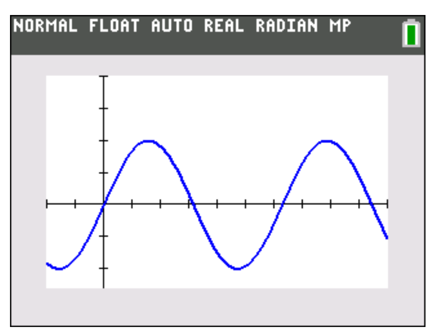
1. $g(x) = 2f(x)$
2. $g(x) = f(2x)$
3. $g(x) = -f(x)$
4. $g(x) = f(-x)$
5. $g(x) = f(x + 2)$
6. $g(x) = f(x) - 4$
7. $g(x) = |f(x)|$
8. $g(x) = f(|x|)$

Example 2: If the graph of $f(x)$ is given, sketch the graph of $g(x)$. Use the transformations from *Example 1* above.



Example 3: Describe the circle with equation $x^2 + y^2 - 6x + 4y - 36 = 0$ in terms of a transformation of the unit circle whose center is at the origin.

Example 4: The figure to the right shows the graph of $y = f(x)$. Sketch $y = |f(x)|$; $y = |f(x)| + 3$; $y = -2 \cdot |f(x)|$.



Example 5: Given $f(x) = \frac{1}{x}$ and $f\ (x + 4)$.

a. Graph the function and the transformation.
b. Compare the domain, range, and asymptotes of the two functions.
c. What are the domain, range, and asymptotes of $f^{-1}(x)$?

Pre-Calculus Objective 1.02
Conics

Vocabulary/Concepts/Skills:

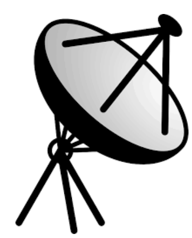
- Parabola
- Circle
- Ellipse
- Hyperbola
- Conic Sections
- Standard Form
- Center
- Focus
- Major/Minor Axes
- Vertices
- Focal axis
- Lines of Symmetry
- Directrix
- Asymptotes
- Transformations
- Parametric Forms
- Solve Equations and Inequalities Justifying Steps Used

Example 1: According to Kepler's first law of planetary motion, each planet moves in an ellipse with the sun at one focus. Assume that one focus (the Sun) has coordinates (0,0) and the major axis of each planetary ellipse is the *x*-axis on a cosmic coordinate system (one unit = one billion kilometers). The minimum and maximum distances for Neptune are 4.456 and 4.537 billion kilometers, respectively. And the minimum and maximum distances for Pluto are 4.425 and 7.375 billion kilometers, respectively.

a. For each planet determine the coordinates of the center and second focus.
b. Graph and describe the orbits.
c. Write an equation that represents the orbit. (As an extension, determine the eccentricity.)

Example 2: Suppose a satellite is in an elliptical orbit with the center of the Earth as one of its foci. The orbit has a major axis of 8910 miles with a minor axis of 8800 miles.

a. Write an equation to model the path of the satellite.
b. How far is the Earth from the center of the elliptical path?



Example 3: A parabolic satellite dish is modeled by the equation $y = \frac{1}{12}x^2$ and is measured in feet. In order to receive optimal signals, a satellite company must construct the receiver to be the focus of the parabolic dish.

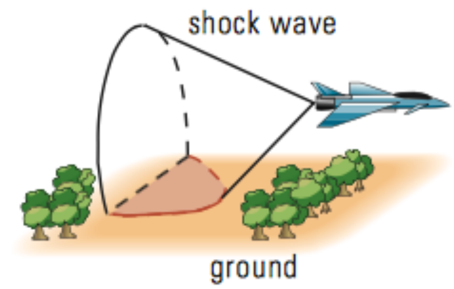
a. How far from the vertex of the dish should the receiver be placed?

Example 4: Given the following equation: $4y^2 - 2x - 16y = -13 - x^2$

a. Describe the type of conic section that is represented by the equation. Justify your response.
b. Sketch the graph that models the equation of the conic section.

Example 5: When an airplane travels faster than the speed of sound, the sound waves form a cone behind the airplane. If the airplane is flying parallel to the ground, the sound waves intersect the ground in a hyperbola with the airplane directly above its center. A sonic boom is heard along a hyperbola. If you hear a sonic boom that is audible along a hyperbola with the equation $\frac{x^2}{100} - \frac{y^2}{4} = 1$ where *x* and *y* are measured in miles.

a. What is the shortest horizontal distance you could be to the airplane?



Example 6: The shape of a roller coaster loop in an amusement park can be modeled by $\frac{y^2}{4225} + \frac{x^2}{2500} = 1$ where *x* and *y* are measured in feet.

a. What is the width of the loop along the horizontal axis?
b. Determine the height of the roller coaster from the ground when it reaches the top of the loop, if the lower rail is 25 feet from ground level.

Pre-Calculus Objective 1.03
Vectors

Vocabulary/Concepts/Skills:

- Magnitude
- Direction
- Addition/Subtraction of vectors
- Scalar multiplication
- Resultant vector

Example 1: A pilot flies a plane due west for 150 miles, then turns 42° north of west for 70 miles. Find the plane's resultant distance and direction from the starting point.

Example 2: A ferry shuttles people from one side of a river to the other. The speed of the ferry in still water is 25 mi/h. The river flows directly north at 9 mi/h. If the ferry heads directly west, what are the ferry's resultant speed and direction?

Resulting speed = ______________

Describe the direction (include angle and compass direction):

Example 3: To find the distance between two points A and B on opposite sides of a lake, a surveyor chooses a point C which is 720 feet from A and 190 feet from B. If the angle at C measures 68°, find the distance from A to B.

Example 4: A baseball is thrown at a 22.5° angle with an initial velocity of 70 m/s. Assume no air resistance and remember that the acceleration due to gravity is $-9.8\ {}^{m}/_{s^2}$.

a. What is the initial vertical component of the ball's velocity?
b. What is the horizontal component of the ball's velocity?
c. How long until the ball hits the ground?
d. How high did the ball travel?
e. How far did the ball travel horizontally when it hit the ground?

Example 5: Without the wind, a plane would fly due east at a rate of 150 mph. The wind is blowing southeast at a rate of 50 mph. The wind is blowing at a 45° angle from due east. How far off of the due east path does the wind blow the plane?

Pre-Calculus Objective 2.01
Modeling Functions

Vocabulary/Concepts/Skills:

- Independent Variables
- Dependent Variables
- Domain
- Range
- Interval notation
- Set notation
- Zeros
- Intercepts
- Effects of a, b, c, & d in $y = af(bx + c) + d$
- Asymptotes
- Minimum
- Maximum
- Intersections
- End Behavior Models
- Increasing/Decreasing
- Global versus Local Behavior
- System of Equations
- Piece-wise defined
- Greatest integer
- Power
- Rational
- Exponential
- Logarithmic
- Logistic

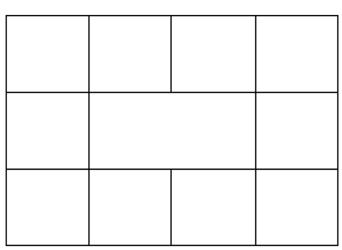
Example 1: A study showed that the function $M(t) = 6.5ln(1.4t + 4)$ approximates the population of mice in an abandoned building where *t* is the number of months since the building was abandoned five years before.

a. Identify the 12-month interval when the mice population grew the most and the 12-month interval in which it grew the least. Write your answer in both set notation and interval notation.

Example 2: Each orange tree in a California grove produces 600 oranges per year if no more than 20 trees are planted per acre. For each additional tree planted per acre, the yield per tree decreases by 15 oranges.

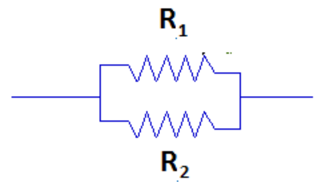
a. Describe the orange tree yield algebraically.
b. Determine how many trees per acre should be planted to obtain the greatest number of oranges.

Example 3: A real estate developer is planning to build a small office building with a center courtyard. There will be ten rooms, all of the same dimensions, and each rectangular office will have 180 square feet of floor space. The floor plan for the building is shown. All walls are made of cinder block. What dimensions should the rooms have to minimize the total length of the walls to be built? Give answers to the nearest tenth of a foot.



Example 4: When two resistors with resistances R_1 and R_2 are connected in parallel, their combined resistance R is given by the formula:

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

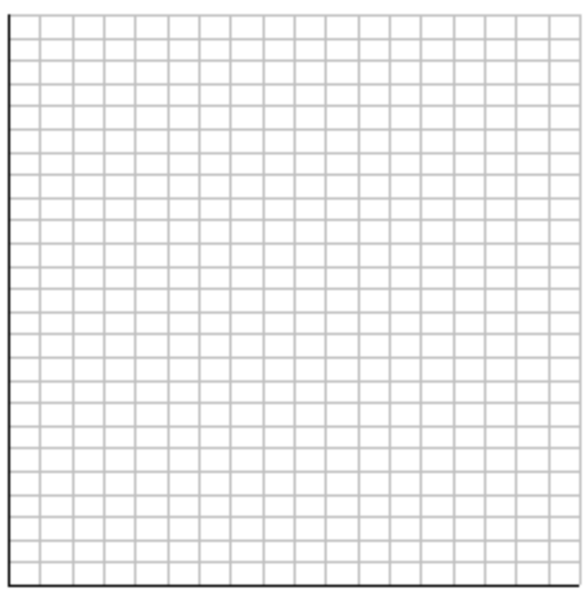


Suppose that a fixed 8-ohm resistor is connected in parallel with a variable resistor denoted by x.

a. Define the combined resistance as a function of x.
b. Graph the function $R(x)$.
c. At what value will the combined resistance max out? Explain why this occurs.

Example 5: The function $C(t) = \dfrac{5t}{0.01t^2 + 3.3}$ describes the concentration of a drug in the blood stream over time. In this case, the medication was taken orally. C is measured in micrograms per milliliter and t is measured in minutes.

a. Sketch a graph of the function over the first two hours after the dose is given. Label axes.
b. Determine when the maximum amount of the drug is in the body and the amount at that time.
c. Explain within the context of the problem the shape of the graph between taking the medication orally ($t = 0$) and the maximum point. What does the shape of the graph communicate between the maximum point and two hours after taking the drug?
d. What are the asymptotes of the rational function $C(t) = \dfrac{5t}{0.01t^2 + 3.3}$?
   What is the meaning of the asymptotes within the context of the problem?
e. Expand the window of the graph to include negative values for t.
   Discuss the asymptotes.



Example 6: What value of n would make the function continuous? Show both graphically and algebraically.

$$f(x) = \begin{cases} x^2 + 1 & x < -1 \\ n & -1 \le x \le 3 \\ 5 - x & x > 3 \end{cases}$$

Pre-Calculus Objective 2.02
Trigonometric Functions

Vocabulary/Concepts/Skills:

- Period
- Amplitude
- Phase shift
- Frequency
- Intercepts
- Sinusoidal
- Domain/Range
- Law of Sines
- Law of Cosines
- Dependent Variables
- Identities
- Trig Ratios (sin, cos, tan, sec, csc, cot)
- Effects of *a*, *b*, *c*, & *d* in $y = af(bx + c) + d$
- Unit Circle
- Radian Measure
- Degree measure
- Sine and Cosine of Special Angles (multiples of $\pi, \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}$)

Example 1: At a particular location on the Atlantic coast, a pier extends over the water. The height of the water on one of the supports is 5.4 feet, at low tide (2 AM) and 11.8 feet at high tide, 6.2 hours later.

a. Write an equation describing the depth of the water at this location t hours after midnight.
b. Use the form $h(t) = a \cdot cos\left[\frac{2\pi}{T}(t - b)\right] + c$. What will be the depth of the water at this support at 4 AM?

Example 2: Find the amplitude, period and phase shift of the function $f(x) = -\frac{1}{2}\sin\left(\frac{\pi}{6} - 3x\right)$.

Example 3: Solve $2 \cdot \sin(0.5x) = 0.75$ for $\pi \le x \le 2\pi$.

Example 4: In the interval $[0, 2\pi)$, find the exact solutions for $\sin 2x - \tan x = 0$ without a calculator.

Example 5: To find the distance between two points A and B on opposite sides of a lake, a surveyor chooses a point C which is 720 feet from A and 190 feet from B.

If the angle at C measures 68°, find the distance from A to B.

Example 6: From the deck of a Cape Fear steamboat, you watch a point on the blade of the paddlewheel as it rotates. The point's distance (d) from the surface of the water is a sinusoidal function of time. After three seconds the point on the wheel is at its highest, 16 feet above the surface of the water. The diameter of the wheel is 18 feet, and a complete revolution takes 12 seconds.

a. Sketch a graph of the sinusoidal model.
b. Write an equation for the model.
c. How long does the point remain under water
d. How far above the surface of the water was the point when the stopwatch read 11 seconds?

Example 7: Solve $2\sin^2\theta - \sin\theta - 3 = 0\ for\ 0 \leq \theta \leq 2\pi$.

Example 8: Using the formula, $\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$, find $\sin\left(\frac{7\pi}{12}\right)$ without a calculator.

Example 9: Verify the following trigonometric identity. $\frac{\cos(x)}{1+\sin(x)} = \sec(x) - \tan(x)$.

Example 10: A plane is flying from city A to city B, which is 115 mi due north. After flying 45 mi, the pilot must change course and fly 15° west of north to avoid a thunderstorm.

a. If the pilot remains on this course for 25 mi, how far will the plane be from city B?
b. How many degrees will the pilot have to turn to the right to fly directly to city B? How many degrees from due north is this course?

Example 11: Maximum and minimum average daily temperatures of two cities are given.

	January 15 (15th day)	July 16 (197th day)
Montreal, Quebec	−10°C	21°C
Orlando, Florida	15°C	28°C

a. On the same graph, sketch a sinusoidal curve (day of the year, temperature) for each city and create an equation to represent each curve.
b. Explain differences between the curves.

Pre-Calculus Objective 2.03
Calculator Models of Functions

Vocabulary/Concepts/Skills:

- Regression
- Residuals
- Correlation Coefficient (linear data)
- Interpret constants, coefficients, bases
- Interpolate
- Extrapolate
- Estimate
- Predict

Example 1: The data in the table shows the circulation in millions of USA Today from 1985 to 1993. Years are shown as the number of years since 1985.

Years since 1985	0	1	2	3	4	5	6	7	8
Circulation	1.418	1.459	1.586	1.656	1.755	1.843	1.867	1.957	2.001

a. Use the first and last data points to find an exponential model of the form $y = ae^{bx}$ for the data. How good is the fit?
b. According to this model, what is the estimated circulation in 2002?
c. Compare the estimate with the actual circulation. What other functions might model the data well?
d. Explain why another function is more reasonable in the context of the problem.

Number of Years Since 1970	**Number of Subscribers (millions)**
0	4.5
5	9.8
10	16.0
14	29.0
16	37.5
18	44.0
20	50.0
22	53.0
24	55.3

Example 2: Shown is the number of cable television subscribers in the US for several years between 1970 and 1994. Years are expressed as number of years since 1970 and the number of subscriptions is given in millions.

a. Look at a scatter plot of the data and decide on an appropriate function to model the data.
b. The data appears to be leveling off. What in the context explains the leveling off?
c. Looking at your equation model, what is the number to which the data levels off- i.e. what is the carrying capacity?

Example 3: Given the data tables below:

Table 1:

Time (s)	**0**	**0.04**	**0.08**	**0.12**	**0.16**	**0.20**	**0.24**	**0.28**	**0.32**	**0.36**	**0.40**
Height (ft)	4.54	4.46	4.34	4.16	3.94	3.68	3.37	3.02	2.63	2.2	1.74

Table 2:

Time in minutes (x)	**1**	**4**	**7**	**9**	**13**	**17**	**20**	**23**	**25**
Number of bacteria (y)	3	21	46	65	108	158	198	240	270

Table 3:

Roll	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Number Cubes Remaining	252	207	170	146	123	100	85	67	56	48

Table 4:

***t* (mths)**	**0.5**	**1.5**	**2.5**	**3.5**	**4.5**	**5.5**	**6.5**	**7.5**	**8.5**	**9.5**	**10.5**	**11.5**	**12.5**	**13.5**
T (°F)	-10.1	-3.6	11.0	30.7	48.6	59.8	62.5	56.8	45.5	25.1	2.7	-6.5	-10.8	-5.0

Table 5:

Year	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**
Average Price (dollars per gallon)	3.26	2.35	2.78	3.52	3.64	3.52	3.36

1) Match the data table with the correct model.
   A. Exponential
   B. Power
   C. Quadratic
   D. Quartic
   E. Trigonometric
2) Find a regression/prediction equation for each data table.
3) Compare the actual and predicted values of the dependent variable. How closely does the model predict the given values of the domain?

Pre-Calculus Objective 2.04
Composition/Inverse Functions

Vocabulary/Concepts/Skills:

- Decomposition to Simpler Forms
- $f \circ f^{-1}(x) = x$
- $f^{-1} \circ f(x) = x$
- Reflection over $y = x$
- Domain/Range of Inverses
- One-to-One
- Domain Restrictions

Example 1: Write the inverse of $f(x) = (x-2)^2 + 3$ if $x \leq 2$.

Example 2: Let $f(x) = \frac{x}{x-3}$ and $g(x) = \frac{x+1}{2x}$. Find and simplify $f\big(g(x)\big)$ and identify its domain.

Example 3: If $f(x) = -4x + 2$ and $g(x) = \sqrt{x-8}$, when does $(f \circ g)(x) = -6$?

Example 4: Write the inverse of $y = 2^{3x} + 4$.

Example 5: Write the inverse of $h(x) = \sqrt[3]{x} - 3$.

Example 6: You are looking for a new refrigerator at a Kitchens-R-Us and will need to have it delivered. In order to stick to your budget, you must remember that the current tax rate is 7.8% and also a factor in the delivery fee.

a. Write a function $P(x)$ to model the total price of the refrigerator, including tax..
b. Since you need to have it delivered, you must include the $65 delivery fee. Write a function $D(x)$ that models the original price of the refrigerator plus the fee.
c. In order to budget a final price, you must include the price of the refrigerator, tax and delivery fee. Find the function $P(D(x))$ and then $D(P(x))$. Which option is cheaper?
d. If state law prohibits taxes to be added to a delivery fee, which of your functions is compliant with the law?

Pre-Calculus Objective 2.05
Polar Equations

Vocabulary/Concepts/Skills:

- Polar Coordinate System
- Pole
- Radius
- Magnitude
- Direction
- Argument
- Translate between Rectangular and Polar Coordinates
- Graphing Technology

Example 1: When recording live performances, sound engineers often use a cardioid microphone because it captures the singer's voice with limited outside noise from the audience. Suppose the boundary of the optimal pickup region is given by the equation $r = 2 + 2\sin\theta$, where r is measured in meters from the microphone on the mic stand.

a. What is the maximum distance a musician could stand away from the microphone and still be within this boundary?

Example 2: Find the intersection of the following two polar graphs, without using a calculator.

$$r = 2 + 3\sin\theta \text{ and } r = \sin\theta$$

Example 3: Archaeologists want to create a map of a recent dig using polar coordinates. On their grid, they used a rectangular coordinate system and marked the king's tomb at $(-12, 5)$.

a. What would be the new coordinates of the king's tomb if it were marked on a polar grid?

Example 4: Convert the following equation from polar to rectangular form.

$$r = 6\cos\theta$$

Example 5: Convert the following equation from rectangular to polar form.

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

Example 6: Given the equations below:

1) $4x^2 + 8y^2 - 8x + 48y + 44 = 0$
2) $y^2 - 12x + 18y + 153 = 0$

a. Identify the conic and write each equation in rectangular form.
b. Write each equation in polar form.

Pre-Calculus Objective 2.06
Parametric Equations

Vocabulary/Concepts/Skills:

- Parameter
- Dependent/ Independent Variable
- Parametric Plots
- Motion over Time
- Translate between Parametric and Rectangular Forms

Example 1: The Sea Queen leaves the port at Nassau at 7:00 PM. She sails due east at 15 mph. The Ocean Princess leaves a small island that is 27 miles north of Nassau at the same time. She sails due south at 22 mph.

a. Assuming the ships continue in the same directions at the same speeds, write parametric equations that model the paths of the two ships as they would appear on a radar screen located at Nassau.
   - Let *d* represent the distance between the two ships.
   - Express *d* as a function of *t*, the number of hours elapsed since 7:00 PM.

Example 2: A batter at spring training camp hits a baseball with an initial velocity of $90\,\frac{ft}{sec}$ at an angle of 35° from the horizontal. Assume that the batter makes contact with the ball 2.5 ft above home plate.

Projectile Motion – Parametric Equations

$$x(t) = (v_0 \cdot \cos\theta)t \qquad y(t) = -\frac{1}{2}gt^2 + (v_0 \cdot \sin\theta)t + h$$

Note: $g = 32\,\frac{ft}{sec^2}$ or $g = 9.8\,\frac{m}{sec^2}$

a. Write parametric equations to model the motion of the ball.

   $x(t) =$

   $y(t) =$

b. How high is the ball after 1.5 seconds?
c. How far away is the ball after 0.7 seconds?
d. What is the maximum height reached by the ball?
e. What is the total horizontal distance the ball travels?
f. How much time elapsed between the ball being hit and landing on the ground/.’
g. The outfield fence is 12ft high and 225ft from home plate. Did the batter hit a homerun? Defend you answer.

Example 3: Two rockets are fired from a space station. The first rocket's path can be described using the parametric equations $x(t) = t + 4$ and $y(t) = 3t - 1$. The second rocket's path can be described using the parametric equations $x(t) = t + 4$ and $y(t) = 2t + 9$.

a. Eliminate the parameter for the equations given for the first rocket and express its path as a function.
b. Eliminate the parameter for the equations given for the second rocket and express its path as a function.
c. Will the two rockets collide? Defend your answer using mathematical reasoning.

Pre-Calculus Objective 2.07
Recursively-defined Functions

Vocabulary/Concepts/Skills:

- Linear Sequence
- Arithmetic Sequence
- Geometric Sequence
- Geometric Series
- Subscript Notation
- Summation Notation
- Converge
- Diverge
- Translate between recursive and explicit representations

Example 1: Mr. Smith has recently retired, and he will rely on his savings to supplement Social Security. He has an account in which the balance was \$25,000 on June 1, when he retired. The account earns interest at the rate of 4.8% annual interest, and the interest is compounded monthly. Mr. Smith will withdraw \$1100 from the account each month.

a. Write a recursive system to represent the account balance over time.
b. How much money will be in the account 12 months after Mr. Smith retired?
c. After how much time will the balance in the account have dropped to under \$5,000?

Example 2: After a person takes pain medication, his kidneys filter the medicine out of his blood stream. During any four-hour time period, his kidneys will remove 35% of the medication that remains in the bloodstream.

a. Suppose a patient takes 800 mg of ibuprofen at 8 AM on Tuesday morning. If he does not repeat the dosage, how much medicine will remain his blood stream at 12 midnight on the same day?
b. Suppose a patient takes 800 mg of ibuprofen at 8 AM on Tuesday morning, and repeats the same dosage every 4 hours. How much medicine will remain his blood stream at 12 midnight on the same day (immediately before he takes his 5th dosage)?
c. This patient continues taking 800 mg of ibuprofen every 4 hours for several days. The amount of ibuprofen in his bloodstream (in mg) varies between two amounts. What are they?

Example 3: A sequence of numbers is defined as $a_1 = 7, a_2 = \sqrt[3]{a_1} + 1, a_3 = \sqrt[3]{a_2} + 1$ and in general $a_{n+1} = \sqrt[3]{a_n} + 1$ for all $n \geq 1$.

a. What is the value of a_5 to the nearest hundredth?

Example 4: The Collatz Conjecture is a famous, unproven conjecture involving recursive relationships. Collatz claims that for every positive integer greater then zero, this sequence will always result in 1.

The conjecture is defined by the rule: $c(n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ 3n+1, & 3n+1, \end{cases}$

a. If we let $n_0 = 5$, how many terms will it take before the sequence becomes 1?
b. If we let $n_0 = 12$, how many terms will it take before the sequence becomes 1?

Example 5: How do you know when a series converges or diverges?

Example 6: Use the following series to answer the questions below: $\sum_{n=4}^{8}(3n-2)$

a. Write the series in expanded form.
b. Evaluate the series through the given terms.

Example 7: Find the sum of the first 20 terms of the arithmetic sequence with $a_{12} = 63$ and $a_{19} = 7$. Write an explicit formula for the sequence.

Example 8: James knows that a certain convergent series has the sum of 2.5 and its first term is 2.

a. Find the "r" value for this series.

Example 9: There are 15 rows of seats on a concert hall with 25 seats in the 1st row, 27 seats in the 2nd row, 29 seats in the 3rd row, and so on.

a. Write an explicit form for the sequence.
b. How many seats are in the concert hall?
c. If the price per ticket is $12, how much will be the total sales for a one-night concert if it is sold out?

Example 10: Suppose that $a_1 = 7$ and $a_n = 5 \cdot a_{n-1}$ when $n > 1$.

a. Expand the recursive sequence through term 6.
b. Write an explicit formula for the sequence.

Pre-Calculus Objective 2.08
Limits of Functions

Vocabulary/Concepts/Skills:

- Table of Values
- Limits
- Approaching
- Existence
- Asymptotes
- Continuous
- Infinity
- End Behavior

Example 1: Let $v_0 = 4$ and $v_n = cos(v_{n-1})$.

a) Graphically determine how the value behaves as *n* approaches infinity.

b) Verify that the limiting value of v_{n} satisfies the equation $cos(x) = x$.

Example 2: Let $w_0 = 100$ and $w_n = 0.95w_{n-1} + 80$.

a) Graphically determine how the value of w_n behaves as *n* approaches infinity.

b) Verify that the limiting value of w_n satisfies the equation, $0.95x + 80 = x$.

Example 3: The value of the expression $200\left(1+\frac{0.05}{k}\right)^{10k}$ gives the account balance if \$200 is invested for 10 years in an account that pays 5% annual interest that is compounded *k* times a year.

a) Find the account balance if *k* = 4, 12, 52, 356, 8760, or 525,600 compounded for each quarter, month, week, day, hour, minute.

b) What value does $200\left(1+\frac{0.05}{k}\right)^{10k}$ approach as k approaches infinity?

Example 4: Let $f(x) = \frac{x+1}{(x-2)(x+3)}$.

a) Use the table of values to determine what happens to $f\ (x)$ as $x \to -3$.

Example 5: Describe the behavior of $f(x) = \frac{1}{x}\sin(x)$ for large, positive values of *x*.

Example 6: Describe the behavior of $f(x) = \sin\left(\frac{\pi}{x}\right)$ as $x \to 0$.

Example 7: Find the following limits without using a calculator:

a) $\lim_{x\to 2} \frac{2x^2-4x}{x^2-4}$

b) $\lim_{x\to 5} \frac{\frac{1}{x}-\frac{1}{5}}{x-5}$

c) $\lim_{x\to\infty} \frac{4n^2-5n}{3n^2+4}$