Midterm Review

Integration Tips

- Try to algebraically manipulate integrand and use reverse Power Rule
- Check for Inverse Trig
- U-Substitution
- Consider Partial Fractions
- Integration By Parts (Tabular??)

Summary of Limit Forms

Determinate

$$\frac{0}{nonzero} \; or \; \frac{0}{\pm\infty} = 0$$

$$\frac{\pm\infty}{nonzero} = \pm\infty$$

$$\frac{nonzero}{0} = \pm\infty$$

$$nonzero \cdot \pm\infty = \pm\infty$$

Indeterminate

$$\frac{0}{0}, \frac{\infty}{\infty}$$

$$0 \cdot \infty$$

$$\infty - \infty$$

$$1^{\infty}, 0^{0}, \infty^{0}$$

Summary of Techniques

$\frac{0}{0}, \frac{\infty}{\infty}$	Use L'Hopital directly
$0 \cdot \infty$	Rewrite as division, then use L'Hopital
$\infty - \infty$	Find a common denominator then use L'Hopital.
$1^{\infty}, 0^{0}, \infty^{0}$	Use *ln* to pull exponent out then use L'Hopital.

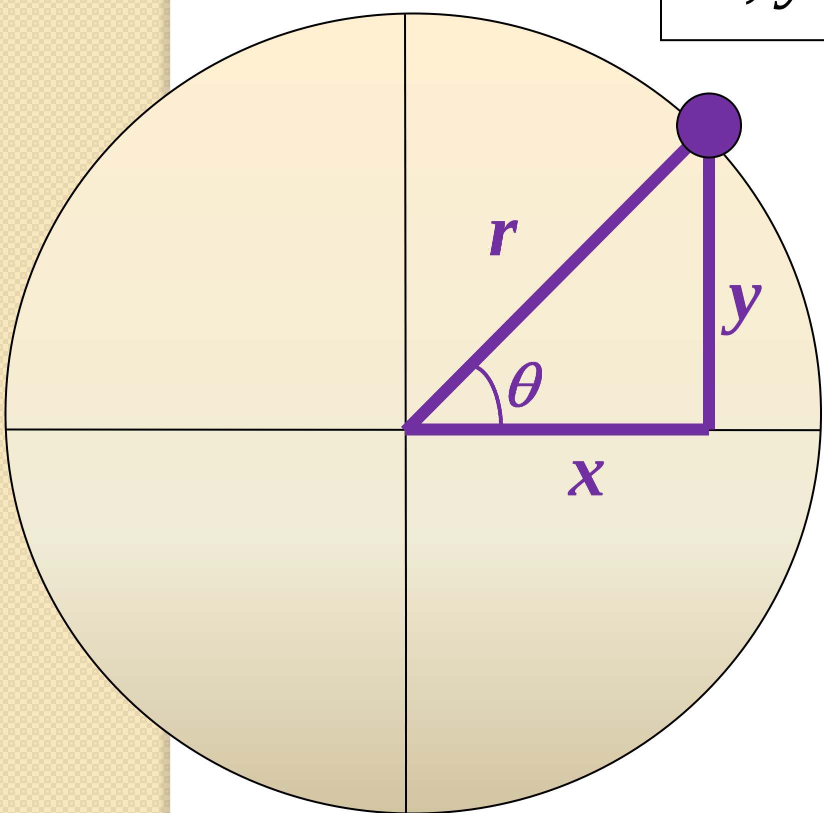
Polar vs Cartesian

Polar	*Cartesian*
$r\cos\theta = 2$	$x = 2$
$r^2\cos\theta\sin\theta = 4$	$xy = 4$
$r^2\cos^2\theta - r^2\sin^2\theta = 1$	$x^2 - y^2 = 1$
$r = 1 + 2r\cos\theta$	$y^2 - 3x^2 - 4x - 1 = 0$
$r = 1 - \cos\theta$	$x^4 + y^4 + 2x^2y^2 + 2x^3 + 2xy^2 - y^2 = 0$

Polar vs Cartesian

$$r, \theta$$

$$x, y$$



Cartesian to Polar

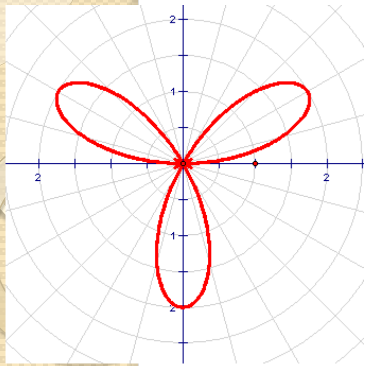
$$r^2 = x^2 + y^2$$

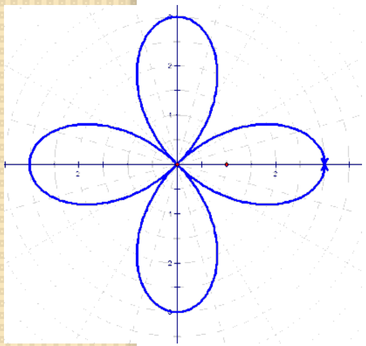
$$\tan\theta = \frac{y}{x}$$

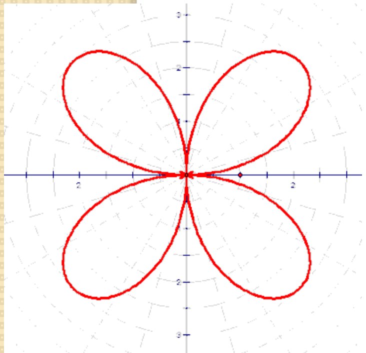
Polar to Cartesian

$$\cos\theta = \frac{x}{r} \quad \Leftrightarrow \quad x = r\cos\theta$$

$$\sin\theta = \frac{y}{r} \quad \Leftrightarrow \quad y = r\sin\theta$$







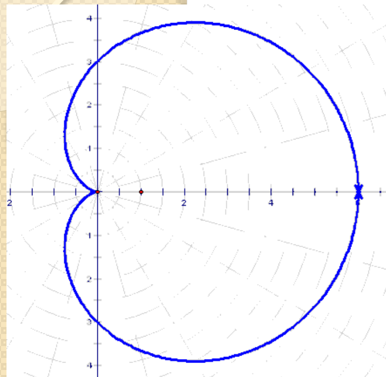
Rose Curves:

$$r = a\sin(n\theta) \quad r = a\cos(n\theta)$$

~Pick cosine if a leaf is split by the x-axis

~*a*: length of petal from the pole
(maximum r for the function)

n	# petals	Domain (Period)
Even	2n	$[0,2\pi]$
Odd	n	$[0,\pi]$

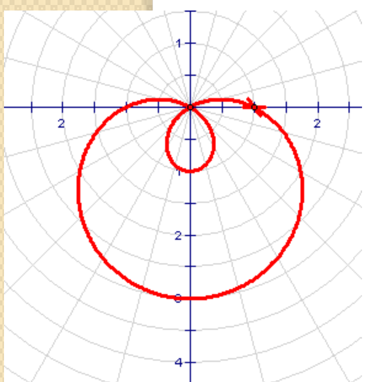


Limacon Curves:

$$r = a \pm b\sin(\theta) \quad r = a \pm b\cos(\theta)$$

y-axis symmetry—pick sine

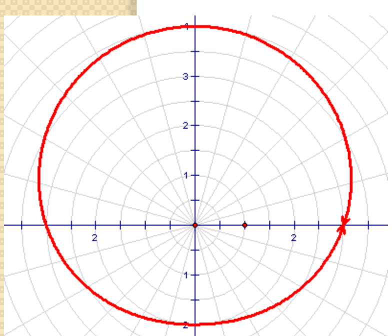
x-axis symmetry—pick cosine



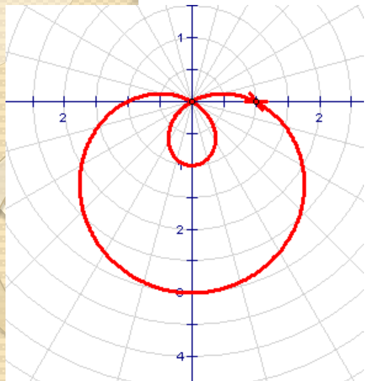
± indicates the orientation on the axis

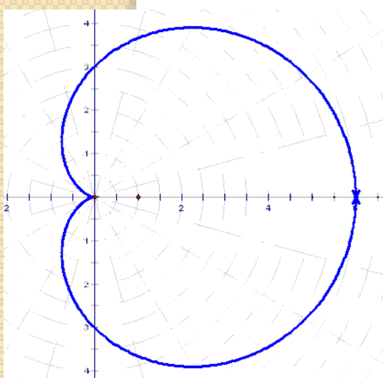
+ the orientation is on the positive axis

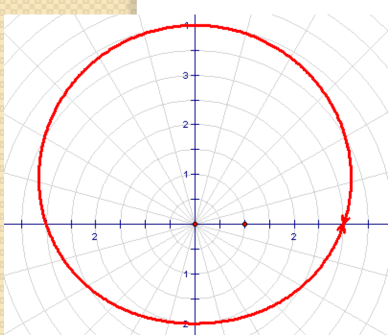
- the orientation is on the negative axis



Type of Limacon depends on the value of $\frac{a}{b}$







$\frac{a}{b}$	Type of Limacon Curve
$\frac{a}{b} < 1$ (which means $a < b$)	Inner Loop
$\frac{a}{b} = 1$ (which means $a = b$)	Cardioid
$1 < \frac{a}{b} < 2$ (which means $b < a < 2b$)	Dimpled
$2 \leq \frac{a}{b}$ (which means $2b \leq a$)	Convex

Domain for all types is $[0,2\pi]$.

Writing equations:

Calculate $\frac{\max + \min}{2}$ *and* $\frac{\max - \min}{2}$

Lengths and Areas

$$L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$$

$$A_{enclosed\_by\_polar\_curve} = \int_{\alpha}^{\beta} \frac{1}{2} r^2\ d\theta$$

$$A = \frac{1}{2} \int_{\alpha}^{\beta} R^2 - r^2 d\theta$$

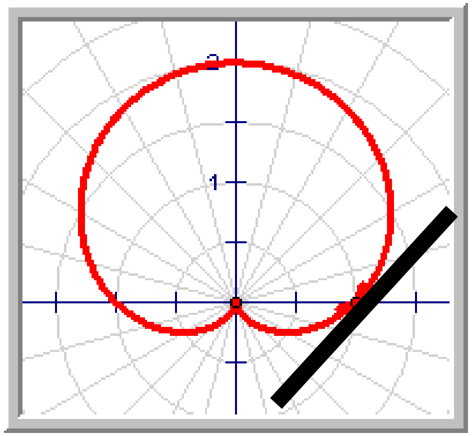
Example Problem . . .

For the curve, $r = 1 + \sin\theta$

A) Find the EQUATION for the slopes of the tangent lines

B) Find where there are horizontal tangents

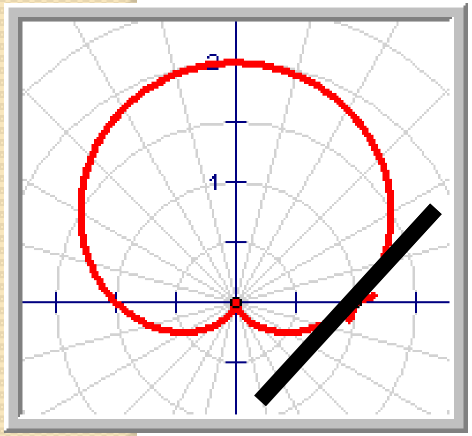
C) Find the slope when $\theta = 0$



Example Problem …

For the curve, $r = 1 + \sin\theta$

A) Find the EQUATION for the slopes of the tangent lines

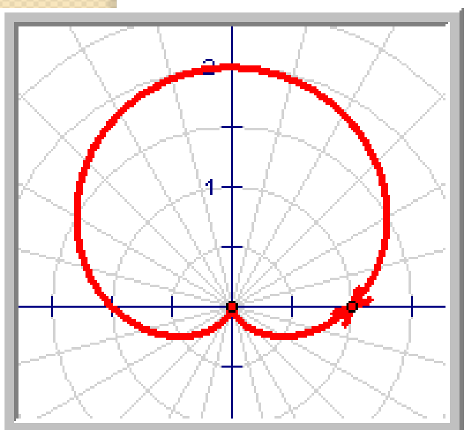


$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$$A) \qquad \frac{dy}{dx} = \frac{\cos\theta + 2\sin\theta\cos\theta}{-\sin\theta - \sin^2\theta + \cos^2\theta}$$

B) Find where there are horizontal tangents

$$\frac{dy}{dx} = \frac{\cos\theta + 2\sin\theta\cos\theta}{-\sin\theta - \sin^2\theta + \cos^2\theta}$$



$$\frac{dy}{d\theta} = 0 \quad \Rightarrow \cos\theta + 2\cos\theta\sin\theta = 0 \quad \Rightarrow$$

$$\cos\theta \;\; 1 + 2\sin\theta \;\; = 0$$

$$\cos\theta = 0 \qquad\qquad 1 + 2\sin\theta = 0$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2} \qquad\qquad \theta = \frac{11\pi}{6}, \frac{7\pi}{6}$$

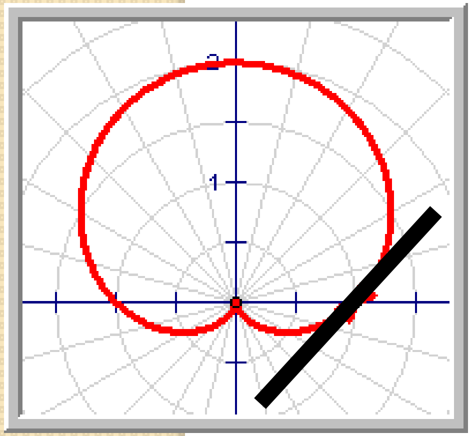
We discard $\frac{3\pi}{2}$ because it makes the derivative undefined

Example Problem . . .

For the curve, $r = 1 + \sin\theta$

C) Find the slope when $\theta = 0$

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$



$$\left.\frac{dr}{d\theta}\right|_{\theta=0} = \_\_\_\_\_\_\_\_$$

$$r(0) = \_\_\_\_\_\_$$

$$\sin(0) = \_\_\_\_\_\_$$

$$\cos(0) = \_\_\_\_\_\_$$

Finding SLOPE

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

Polar

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

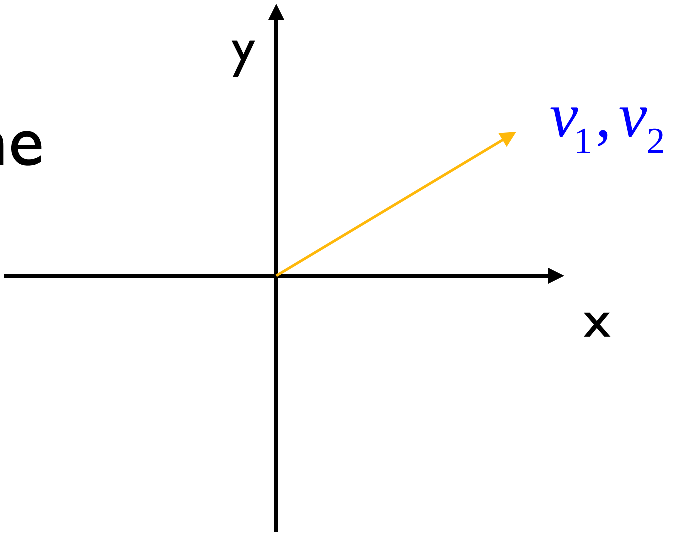
Parametric

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(y') = \frac{\frac{d\frac{dy}{dx}}{dt}}{\frac{dx}{dt}}$$

$$L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

A vector is in standard position if the initial point is at the origin.



The component form of this vector is: $\mathbf{v} = \langle v_1, v_2 \rangle$

Order matters!

Terminal Point – Initial Point

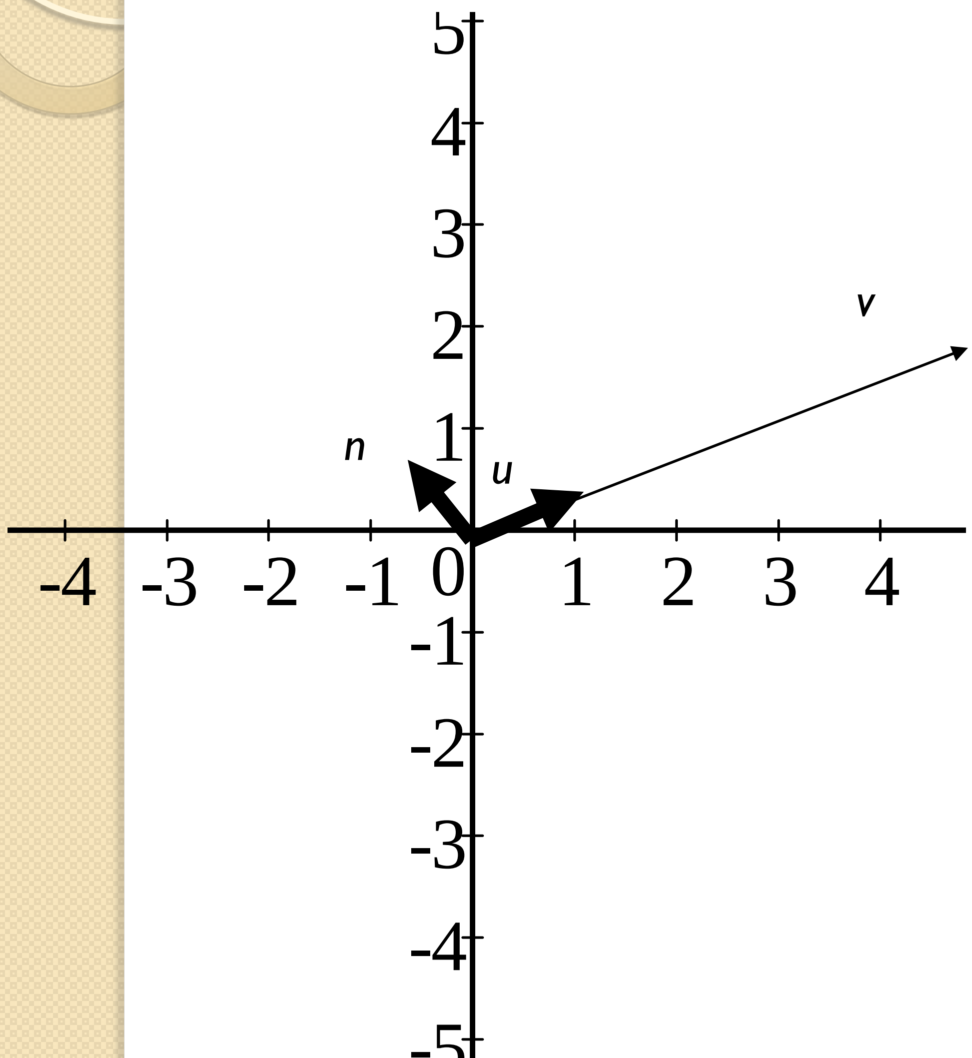
The magnitude (length) of $\mathbf{v} = \langle v_1, v_2 \rangle$ is $|\mathbf{v}| = \sqrt{{v_1}^2 + {v_2}^2}$

If $|\mathbf{v}| = 1$ then **v** is a <u>unit vector</u>.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{dy}{dx}$$

$$\text{Vector} = \langle x_2 - x_1, y_2 - y_1 \rangle = \langle dx, dy \rangle$$

Parallel and Normal Unit Vectors



Vector *u* is a unit vector parallel to vector *v*.

$$u = \frac{v}{|v|} = \left\langle \frac{v_1}{|v|}, \frac{v_2}{|v|} \right\rangle$$

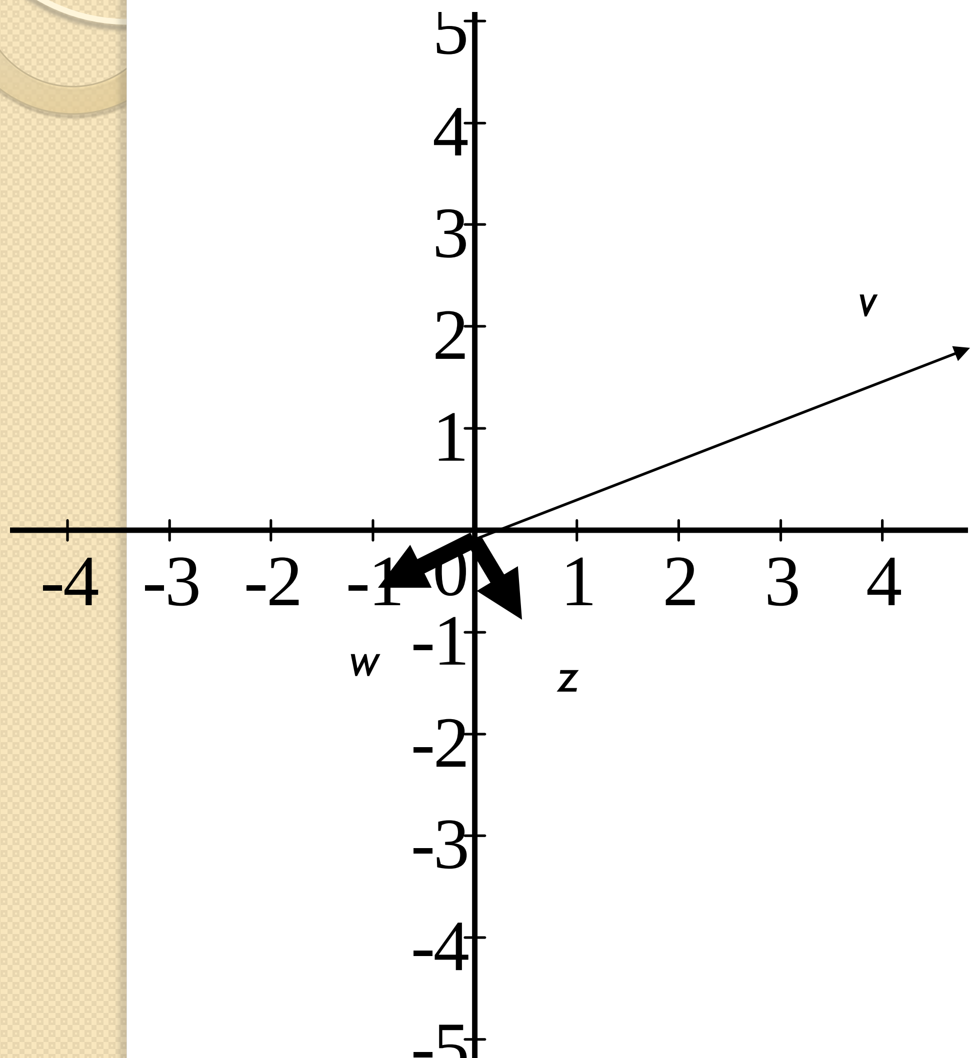
Vector *n* is a unit vector normal to vector *v*.

$$n = \langle -u_2, u_1 \rangle$$

Opposite

Reciprocal

Parallel and Normal Unit Vectors



w and z are also tangent and normal vectors to *v.*

Example Problem

Find the unit vectors that are tangent and normal to the following parametrized curve at the point where t=4.

$$x = \frac{t}{2} + 1, \quad y = \sqrt{t} + 1, \quad t \geq 0$$

Review of position, velocity, and acceleration

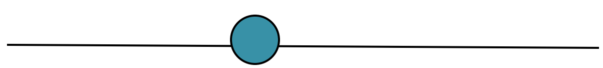
s(t) = Position at time *t*

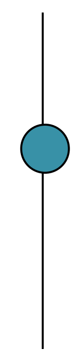
v(t) = Velocity at time *t*

a(t) = Acceleration at time *t*

$$v(t) = \frac{ds}{dt}$$

$$a(t) = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$





Models motion in 1-dimension

Position, velocity, and acceleration in 2-dimensional space

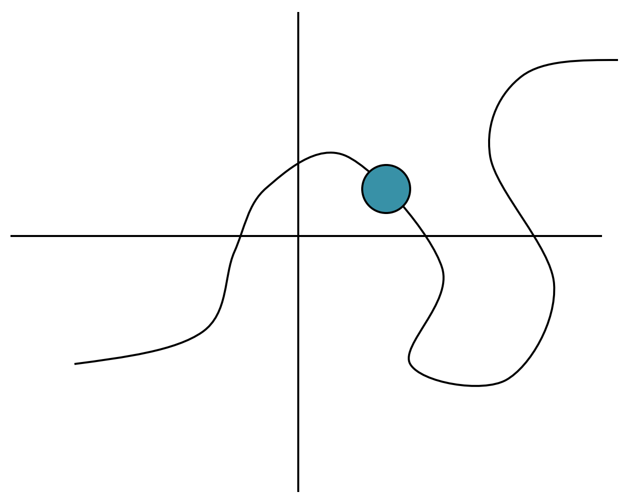
r(t) = Position at time *t*

v(t) = Velocity at time *t*

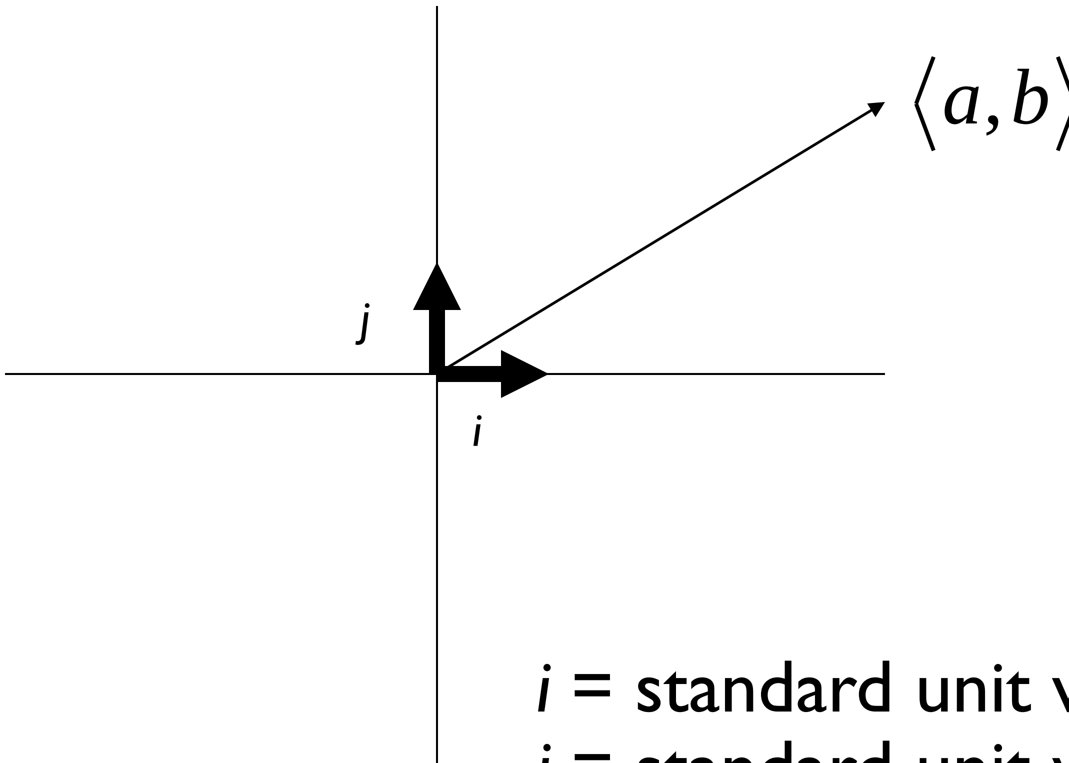
a(t) = Acceleration at time *t*

$$v(t) = \frac{dr}{dt}$$

$$a(t) = \frac{dv}{dt} = \frac{d^2 r}{dt^2}$$

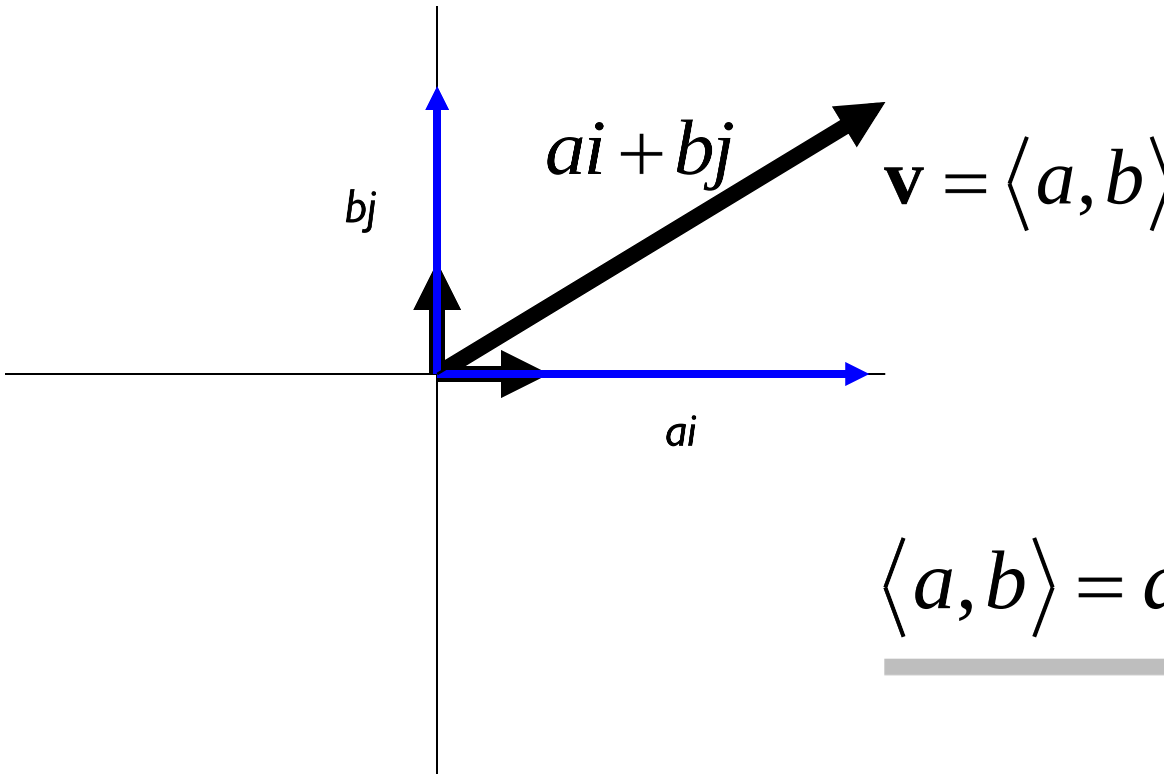


Standard Unit Vectors



i = standard unit vector <1,0>
j = standard unit vector <0,1>

Any vector $\mathbf{v} = \langle a, b \rangle$ can be written as a linear combination of the two standard unit vectors.



$$\langle a, b \rangle = ai + bj$$

SPEED vs VELOCITY

$$\text{Speed} = \left| \mathbf{v} \quad \mathbf{t} \quad \right|$$

"Speed" is magnitude of velocity.
Speed has no direction.
Velocity has direction.

$$\text{Direction of motion} = \frac{\text{velocity vector}}{\text{speed}} = \frac{\mathbf{v} \quad t}{\left| \mathbf{v} \quad t \quad \right|}$$

"Direction" is a unit vector that indicates direction but not magnitude.

→

Practice….

Solve the initial value problem.

$$\frac{d^2r}{dt^2} = -2\mathbf{i} - 2\mathbf{j}, \quad \left.\frac{dr}{dt}\right|_{t=1} = 4\mathbf{i}, \quad \mathbf{r}(1) = 3\mathbf{i} + 3\mathbf{j}$$