Logarithmic Functions

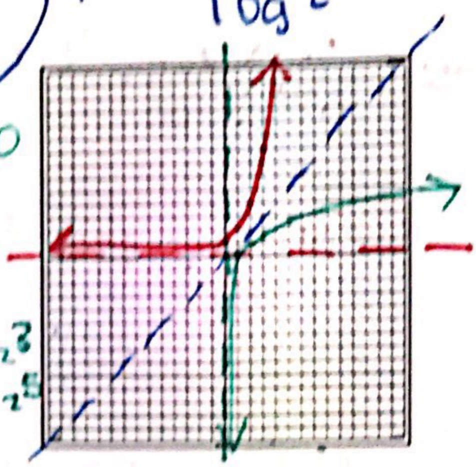
$y_2 = \frac{\log x}{\log 2}$

Make a table & then graph these 2 functions:

y=0 →

$y_1 = 2^x$		$y_2 = \log_2 x$	
x	y	x	y
0	1	0	—
1	2	1	0
2	4	2	1
3	8	3	1.59 ← $\log_2 3$
5	32	5	2.32 ← $\log_2 5$

x=0



So 2^x and $\log_2 x$ are inverses

A LOGARITHM is an exponent.

If $y = b^x$, then $\log_b(y) = x$.

Note: 1. You can't take the log of 0 or a negative number (y>0)
2. $b \neq 1$ $b > 0$

Ex) $\log_2(16) = x$ $\quad 2^x = 16 \rightarrow 2^x = 2^4$ $\boxed{x=4}$ "2 times itself ? times will =16?"

Ex) $\log_5 \sqrt[3]{25} = x$ $\quad 5^x = \sqrt[3]{25^1} \rightarrow 5^x = \boxed{25}^{\frac{1}{3}} \rightarrow 5^x = 5^{2 \cdot 1/3}$ $\boxed{x = 2/3}$

Writing in different forms:

Exponential Form	Logarithmic Form
$2^3 = 8$	$\log_2 8 = 3$
$4^2 = 16$	$\log_4 16 = 2$
$5^{-2} = 1/25$	$\log_5(1/25) = -2$
$3^{1/2} = \sqrt{3}$	$\log_3 \sqrt{3} = 1/2$

Basic Properties of Logarithms

For y>0, b≠1, b>0, for any real number x...	
1. $\log_b 1 = 0$ because	$b^0 = 1$
2. $\log_b b^1 = 1$ because	$b^1 = b$
3. $\log_b(b^y) = y$ because	$b^y = b^y$
4. $b^{(\log_b x)} = x$ because	$\log_b x = \log_b x$

Ex) $6^{\log_6(11)} = \boxed{11}$

Ex) $\log_0 8 = \boxed{F}$

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A common logarithm is a log in base 10

If $y = 10^x$, then $\log y = x$ (same as $\log_{10} y = x$).

Basic Properties of Common Logarithms

Let x & y be real numbers with x>0...	
1. $\log_{10} 1 = 0$ because	$10^0 = 1$
2. $\log_{10} 10 = 1$ because	$10^1 = 10$
3. $\log_{10} 10^y = y$ because	$10^y = 10^y$
4. $10^{\log x} = x$ because	$\log_{10} x = \log_{10} x$

Ex) $\log_{10} \sqrt[5]{10} = \cancel{\log_{10}} \cancel{10}^{1/5} = \boxed{1/5}$

Ex) $\log \frac{1}{1000} = \log_{10}\boxed{\left(\frac{1}{1000}\right)} = \log_{10} 1000^{-1} = \cancel{\log_{10}} \cancel{10}^{-3} = \boxed{-3}$

*These can also be done easily in the calculator.

Use the $\boxed{\text{LOG}}$ button!!

A natural logarithm is a log in base "e"

If $y = e^x$, then $\ln y = x$. $\rightarrow \log_e y = x$

Basic Properties of Natural Logarithms

Let x & y be real numbers with x>0...		
1. $\ln 1 = 0$ because	$\log_e 1 = 0$	$e^0 = 1$
2. $\ln e = 1$ because	$\cancel{\log_e} \cancel{e}^1 = 1$	$e^1 = e$
3. $\ln e^y = y$ because	$\cancel{\log_e} \cancel{e}^y = y$	$e^y = e^y$
4. $e^{\ln x} = x$ because	$e^{\log_e x} = x$	$e^x = e^x$

Ex) $\ln \sqrt{e} = \log_e \sqrt{e} = \cancel{\log_e} \cancel{e}^{1/2} = \boxed{1/2}$

Ex) $\ln e^5 = \cancel{\log_e e^5} = \cancel{\log_e} \cancel{e}^5 = \boxed{5}$

Ex) $e^{\ln 4} = \cancel{e}^{\cancel{\log_e} 4} = \boxed{4}$

Properties of Logarithmic Functions

Properties of Logarithms

Condensed Form	Expanded Form
M,N, b are positive numbers where b≠1…	
1. Product Rule: $\log_b(M \cdot N) =$	$\log_b M + \log_b N$
2. Quotient Rule: $\log_b(M \div N)$	$= \log_b M - \log_b N$
3. Power Rule: $\log_b M^N =$	$N \cdot \log_b M$

Expanding Logs: "Bring everything to the "ground" & separate!"

EX) $\log(x y^3)$

$= \log(x) + \log(y^3)$

$= \boxed{\log x + 3\log y}$

EX) $\ln\left(\frac{x^2 \cdot \sqrt[3]{q}}{y \sqrt{t}}\right)$ Top ⊙ → ⊕ Bottom ⊖

$= \ln x^2 + \ln q^{1/3} - \ln y - \ln t^{1/2}$

$= \boxed{2\ln x + \frac{1}{3}\ln q - \ln y - \frac{1}{2}\ln t}$

Condensing Logs: "Send to the "sky" & only write log once!"

EX) $3\ln x - \frac{1}{2}\ln y + 5\ln z$

$= \ln x^3 - \ln y^{1/2} + \ln z^5$ (TOP, Bottom, TOP)

$= \boxed{\ln\left(\frac{x^3 \cdot z^5}{\sqrt{y}}\right)}$

EX) $\frac{4}{3}\log_2 27 - 2\log_2 9$

$= \log_2 27^{4/3} - \log_2 9^2$ (top, Bottom)

$= \log_2\left(\frac{\sqrt[3]{27^4}}{9^2}\right) = \log_2\left(\frac{81}{81}\right) = \log_2(1$

$= \boxed{0}$

Your calculator (unless you have the new TI-84+ operating system) will only evaluate a logarithm with base 10 or *e*. If you need to evaluate a logarithm with a different with a different base……..

To change base (for those not in common log):
$\log_b x = \frac{\log x}{\log b}$ *and* $\log_b x = \frac{\ln x}{\ln b}$

More examples:

3.5 EQUATION SOLVING AND MODELING

Learning Targets:
1. Solve exponential and logarithmic equations.

When you solve an equation, you "undo" what has been done … addition to undo subtraction, multiplication to undo division. Since exponents and logarithms are inverses of each other, it follows that in order to solve a logarithmic equation, you can write it as an exponent to "undo" the logarithm, and if you are solving for an exponent, you write the equation as a logarithm.

NOTE: You can only switch between exponential and logarithmic forms when you have $\log_b y = x$ or $b^x = y$

① Isolate / Condense ② Rewrite ③ Solve ④ CYA!!

Example 1: Solve the following equations:

a) $e^{x-2} = 19$

$\log_e 19 = x - 2$
$+2 \quad +2$
$\ln(19) + 2 = x$
$\boxed{x = 4.94}$

b) $3 - 5\boxed{(2)^{3x}} = -14$ -3

-3
$\frac{-5(2)^{3x}}{-5} = \frac{-17}{-5}$
$2^{3x} = 17/5$
$\frac{\log_2(17/5)}{3} = \frac{3x}{3}$
$\boxed{x = 0.589}$

c) $\log_2(x) = 4$

$2^4 = x$
$\boxed{x = 16}$

d) $7 - 3\boxed{\log(x)} = -5$ -7

$\frac{-3\log x}{-3} = \frac{-12}{-3}$
$\log_{10} x = 4$
$10^4 = x$
$\boxed{x = 10000}$

e) $\log x + \log(x + 21) = 2$

$\log_{10}(x(x+21)) = 2$
$10^2 = x(x+21)$
$100 = x^2 + 21x$
$0 = 1x^2 + 21x - 100$
$0 = (x-4)(x+25)$
$x - 4 = 0 \qquad x + 25 = 0$
$\boxed{x = 4} \qquad \cancel{\boxed{x = -25}}$ ← NOT ALLOWED!!

⊙	⊕
−100	21
−4, 25	

f) $\log_3(x + 4) - \log_3(x - 5) = 2$

$\log_3\left(\frac{x+4}{x-5}\right) = 2$
$3^2 = \frac{x+4}{x-5}$
$\frac{9}{1} = \frac{x+4}{x-5}$
$9(x-5) = x+4$
$9x - 45 = x + 4$
$8x = 49$
$\boxed{x = 49/8}$