Logistic Functions:

Do you think it is reasonable for a population to grow exponentially indefinitely? no

Logistic Growth Functions ... functions that model situations where exponential growth is limited.

An equation of the form $f(x)=\dfrac{c}{1+a\boxed{b}^{x}}$ or $f(x)=\dfrac{c}{1+ae^{-kx}}$.

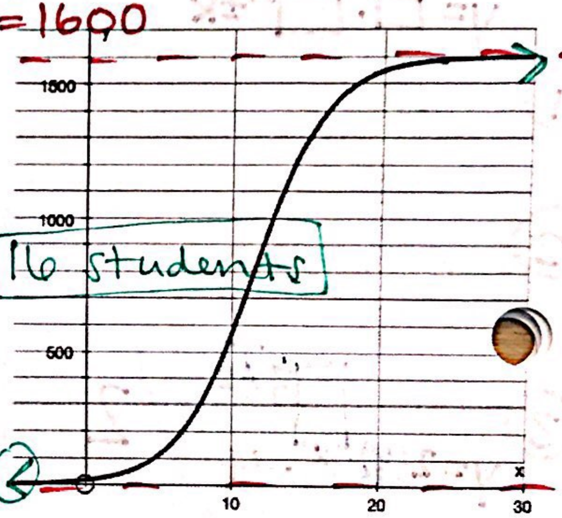
where c = growth limit ← A.K.A. "maximum capacity" or "carrying capacity"

The graph of a logistic function looks like an exponential function at first, but then "levels off" at y = c. The logistic function has two HA: $\boxed{y = 0 \text{ and } y = c.}$

Example of modeling with the logistic function:

The number of students infected with flu after t days at Springfield High School is modeled by the following function:

$$P(t)=\frac{1600}{(1+99e^{(-0.4t)})}$$



a) What was the initial number of infected students t = 0?

$P(0)=\dfrac{1600}{1+99e^{-.4(0)}}=\dfrac{1600}{1+99}=\dfrac{1600}{100}=\boxed{16 \text{ students}}$

b) After 5 days, how many students will be infected?

t=5: $P(5)\approx\boxed{111 \text{ students}}$ $=111.13\ldots$

c) What is the maximum number of students that will be infected?

$c=\boxed{1600 \text{ students}}$

d) According to this model, when will the number of students infected be 800?

t=? P(t)=800

$800=\dfrac{1600}{1+99e^{-0.4t}}$ swap!

$1+99e^{-.4t}=\dfrac{1600}{800}$ Divide

$1+99e^{-.4t}=2$

$-1 \qquad -1$

$\dfrac{99e^{-.4t}}{99}=\dfrac{1}{99}$

$\cancel{\ln e}^{-.4t} \quad \ln\left(\frac{1}{99}\right)$

$\dfrac{-.4t}{-.4}=\dfrac{\ln(1/99)}{-.4}$

$t\approx\boxed{11.49 \text{ days}}$ 4

Analyzing Logistic Functions:

Green: same for all logistic functions

$f(x)=\dfrac{9}{1+2(0.6)^{x}}$ 9 = c

D: $(-\infty,\infty)$
R: $(0, 9)$
Inc.: $(-\infty,\infty)$
Dec.: ——
Boundedness: Bounded
Extrema: ——
Asy.: y=0 & y=9
End Behavior: $\lim_{x\to-\infty} f(x)=0$
$\lim f(x)=9$

$f(x)=\dfrac{8}{1+4e^{-x}}$

D:
R:
Inc.:
Dec.:
Boundedness:
Extrema:
Asy.:
End Behavior:

Exponential Growth $y = a(1+r)^t$ Exp Decay $y = a(1-r)^t$

MODELING CONT.

Example 1: The population of Glenbrook in the year 1910 was 4200. Assume the population increased at the rate of 2.25% per year.

$r = 0.0225$ % → dec. $2.25 \div 100$ $t = 0$ $a = 4200$ $b > 1$ growth (+)

a) Write an exponential model for the population of Glenbrook. Define your variables.

$y = 4200(1 + .0225)^t$ y = population, t = yrs. after 1910

b) Determine the population in 1930 and 1900.

$t = 20$ $t = -10$ $\boxed{y(20) \approx 6554}$ $\boxed{y(-10) \approx 3362}$

c) Determine when the population is double the original amount.

$t = ?$ $y = 2 \cdot 4200 = 8400$ $Y_1 =$ equation, $Y_2 = 8400$ $t = 31.15$ $1910 + 31 = \boxed{1941}$

Example 2: The half-life of a certain radioactive substance is 14 days. There are 10 grams present initially.

$b = 1/2$ $t \div 14$ $a = 10$

t: time (days)
y: amount of substance remaining

$y = 10(1/2)^{t/14}$

a) Express the amount of substance remaining as an exponential function of time. Define your variables.

b) When will there be less than 1 gram remaining?

$t = ?$ t $y < 1$

Table:

X	Y_1
46	1.0254
t = 47	0.97589

$\boxed{\text{After 46 days}}$

Example 3: Find a logistic equation of the form $\boxed{y = \frac{c}{1 + ae^{-bx}}}$ ($c = 500$, $a = 99$) that fits the graph below, if the y-intercept is (0, 5) and the point (24, 135) is on the curve.

↳ Replace all constants! (a, b, c)

① Use (0, 5):

$5 = \frac{500}{1 + a\cancel{e^{-b(0)}}}$ → 1

$\frac{5}{1} = \frac{500}{1 + a}$

$5(1 + a) = 500$

$5 + 5a = 500$
$-5 \qquad -5$

$\frac{5a}{5} = \frac{495}{5}$

$\boxed{a = 99}$

② Use (24, 135): — $y = 500$

$135 = \frac{500}{1 + 99e^{-b(24)}}$

$b = 0.15$

$$\boxed{y = \frac{500}{1 + 99e^{-0.15x}}}$$