

Day 2 Notes - Finding Limits Graphically

WHAT IS A LIMIT?

A **limit** describes how the output values of a function behave as input values approaches some given number "c"

NOTATION: $\lim_{x \rightarrow c} f(x) = L$

"The limit as x approaches c of f(x) is L."

Different Kinds of Limits

$\lim_{x \rightarrow c^-} f(x) = L$

Left-Hand Limit

$\lim_{x \rightarrow c^+} f(x) = L$

Right-Hand Limit

$\lim_{x \rightarrow c} f(x) = L$

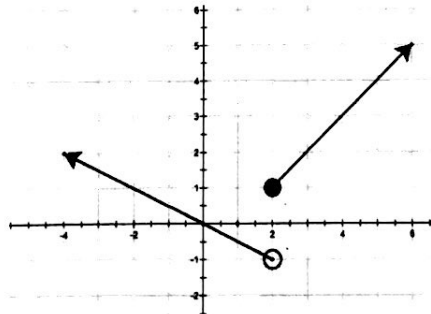
THE Limit
Left \neq Right
 $\hookrightarrow L = \text{DNE}$

Examples:

$\lim_{x \rightarrow 4^-} f(x) = 3$

$\lim_{x \rightarrow 4^+} f(x) = 3$

$\lim_{x \rightarrow 4} f(x) = 3$



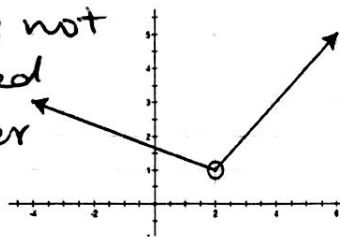
$\lim_{x \rightarrow 2^-} f(x) = -1$

$\lim_{x \rightarrow 2^+} f(x) = 1$

$\lim_{x \rightarrow 2} f(x) = \text{DNE}$

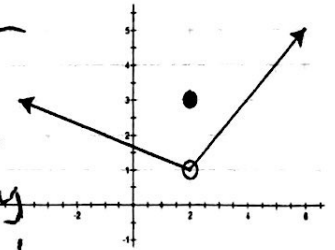
Misconception #1

A function does not have to be defined @ "c" in order for the limit to exist.



Misconception #2

If a function is defined @ "c", f(c) does not necessarily have to equal L.



Practice #1

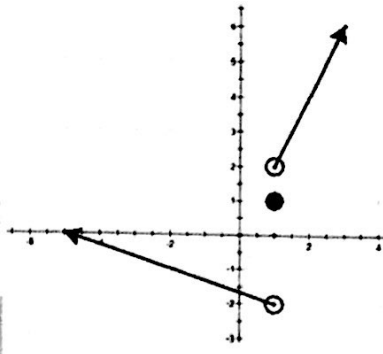
$$\lim_{x \rightarrow 2} f(x) = -1$$

$$\lim_{x \rightarrow 1^-} f(x) = -2$$

$$\lim_{x \rightarrow 1^+} f(x) = 2$$

$$\lim_{x \rightarrow 1} f(x) = \text{DNE}$$

$$f(1) = 1$$



Practice #2

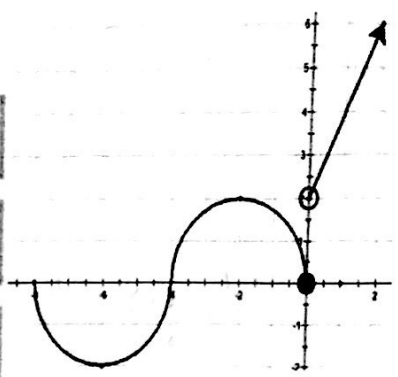
$$\lim_{x \rightarrow 2} f(x) = 2$$

$$\lim_{x \rightarrow 0^-} f(x) = 0$$

$$\lim_{x \rightarrow 0^+} f(x) = 2$$

$$\lim_{x \rightarrow 0} f(x) = \text{DNE}$$

$$f(0) = 0$$



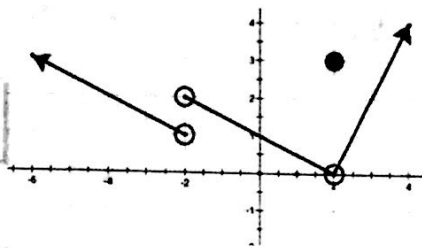
Practice #3

$$\lim_{x \rightarrow 2^-} f(x) = 0$$

$$\lim_{x \rightarrow 2^+} f(x) = 0$$

$$\lim_{x \rightarrow 2} f(x) = 0$$

$$f(2) = 3$$



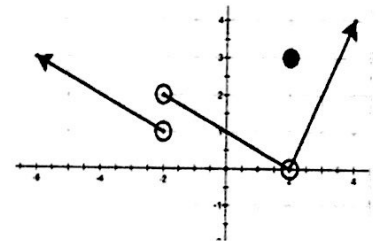
Practice #4

$$\lim_{x \rightarrow -2^-} f(x) = 1$$

$$\lim_{x \rightarrow -2^+} f(x) = 2$$

$$\lim_{x \rightarrow -2} f(x) = \text{DNE}$$

$$f(-2) = \text{DNE}$$

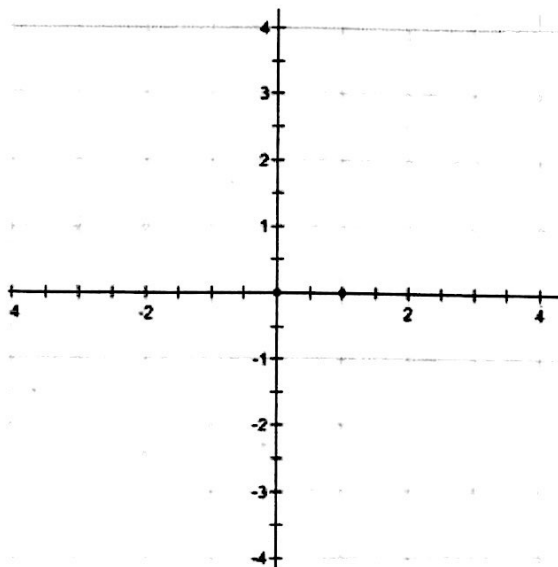


DRAW A GRAPH SUCH THAT

$$\lim_{x \rightarrow 0^+} f(x) = 2$$

$$\lim_{x \rightarrow 0^-} f(x) = 4$$

$$\lim_{x \rightarrow 2} f(x) = 6$$



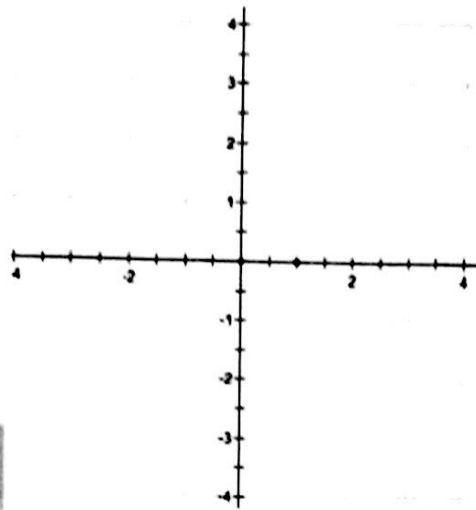
DRAW A GRAPH SUCH THAT

$$\lim_{x \rightarrow 3^+} f(x) = 4$$

$$\lim_{x \rightarrow 3^-} f(x) = 2$$

$$\lim_{x \rightarrow 1^+} f(x) = 0$$

$$\lim_{x \rightarrow 1^-} f(x) = -2$$



How to find limits in the calculator:

1. Change your TBLSET to:

TblStart = (x-value of limit statement)

Indpnt: ASK

2. Explore x-values in the Table which are extremely close to the x-value of the limit statement.

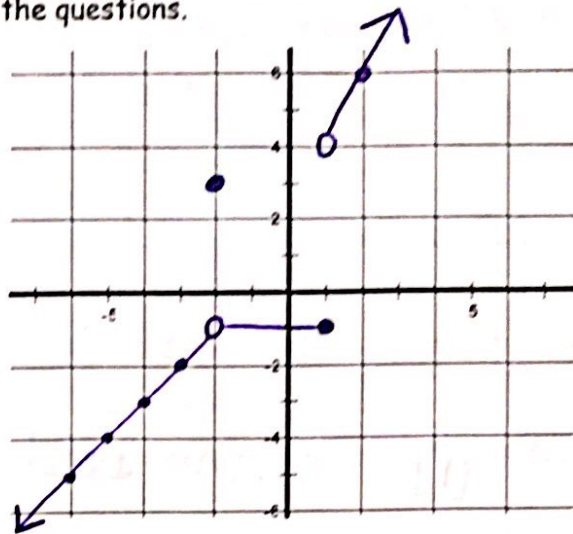
$$1. \lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1} = 2$$

$$2. \lim_{x \rightarrow -3} \frac{x}{x+3} = \text{DNE}$$

$$3. \lim_{x \rightarrow 0} \frac{x}{\sqrt{x+1}-1} = 2$$

Graph the piece-wise functions and then answer the questions.

$$1. f(x) = \begin{cases} x+1 & x < -2 \\ 3 & x = -2 \\ -1 & -2 < x \leq 1 \\ 2x+2 & 1 < x \end{cases}$$

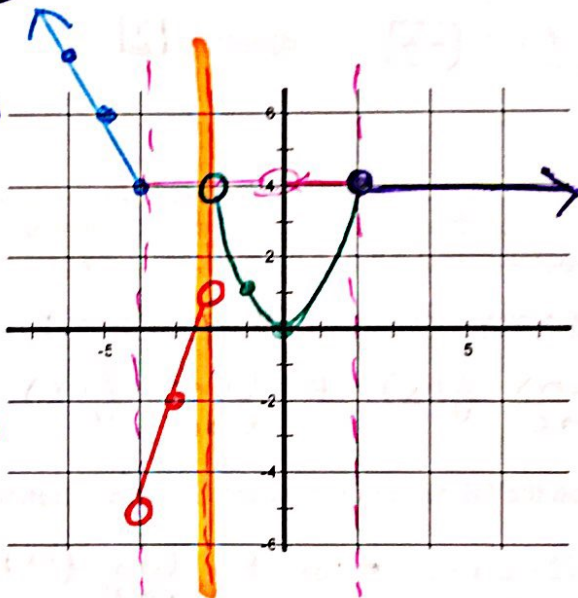


- $\lim_{x \rightarrow -2^+} =$
- $\lim_{x \rightarrow -2^-} =$
- $\lim_{x \rightarrow 1^+} =$
- $\lim_{x \rightarrow 5^+} =$

hole: $<$ $>$
point: \leq \geq

$$2. f(x) = \begin{cases} -2x-4 & x \leq -4 \\ 3x+7 & -4 < x < -2 \\ x^2 & -2 \leq x \leq 2 \\ 4 & 2 < x \end{cases}$$

x	f(x)
-6	8
-5	6
-4	4
-3	-2
-2	1
-1	1
0	0
1	1
2	4
2	4



- $\lim_{x \rightarrow -2^-} =$ DNE
 - $\lim_{x \rightarrow -2^+} =$ 4
 - $\lim_{x \rightarrow -2} =$ 4
 - $\lim_{x \rightarrow 2^-} =$ 4
- Right

Find each limit with the graphing calculator.

$$3. \lim_{x \rightarrow 4} \frac{2x-8}{x^2-x-12} \approx \boxed{0.2857}$$

$$4. \lim_{x \rightarrow 1} \frac{(x+1)}{(\sqrt{x+1})} = \boxed{1}$$

$$5. \lim_{x \rightarrow 1} \frac{x-1}{\ln x} = \boxed{1}$$

$$6. \lim_{x \rightarrow -4} \sqrt{x^2-16} = \boxed{DNE}$$

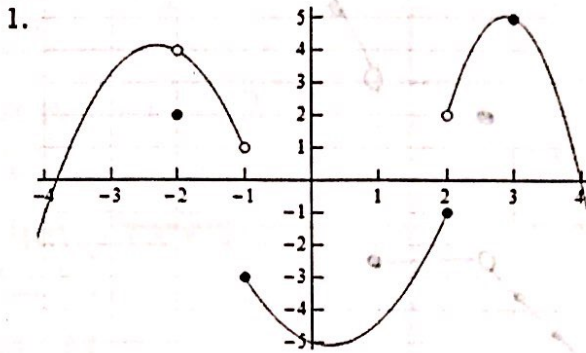
$$\lim_{x \rightarrow -4^-} f(x) = 0$$

$$\lim_{x \rightarrow -4^+} f(x) = DNE$$

Pre-Calculus Honors
Finding Limits Graphically – Day 2 Homework

Name: _____

DIRECTIONS: USE THE FOLLOWING GRAPHS TO ESTIMATE THE LIMITS AND FUNCTIONAL VALUES, OR EXPLAIN WHY THE LIMITS DO NOT EXIST.

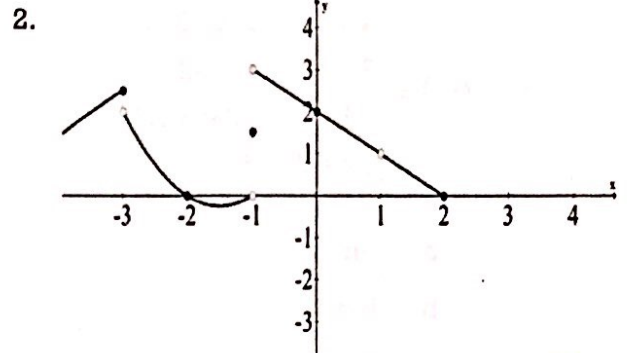


a) $\lim_{x \rightarrow -1^-} f(x) = \boxed{1}$

c) $\lim_{x \rightarrow -1} f(x) = \boxed{\text{ONE}}$

b) $\lim_{x \rightarrow -1^+} f(x) = \boxed{-3}$

d) $f(-2) = \boxed{2}$



a) $\lim_{x \rightarrow -3} h(x) = \boxed{\text{ONE}}$

b) $\lim_{x \rightarrow 1} f(x) = \boxed{1}$

c) $\lim_{x \rightarrow 2} f(x) = \boxed{\text{ONE}}$

d) $f(1) = \boxed{\text{DNE}}$

3. Your friend did not do homework and was on his/her cell phone during the lesson. While looking at a graph they found that the right sided limit equaled 4 and the left sided limit equaled negative 4. They said that since 4 was a closed circle, it was the limit. Are they correct? Why or why not?

(Answers vary) The limit DNE because the $\lim_{x \rightarrow c^-} f(x) \neq \lim_{x \rightarrow c^+} f(x)$. AKA: $4 \neq -4$.

4. Graph the following piece-wise function. Then find the limits.

$$f(x) = \begin{cases} 2-x; & x < 2 \leftarrow \text{use for } \lim_{x \rightarrow 2^-} f(x) \\ -3; & x = 2 \leftarrow f(2) \\ x^2 - 4; & x > 2 \leftarrow \text{use for } \lim_{x \rightarrow 2^+} f(x) \end{cases}$$

a) $\lim_{x \rightarrow 2} f(x) = \boxed{0}$

b) $\lim_{x \rightarrow 2^+} f(x) = \boxed{4}$

c) $\lim_{x \rightarrow 2^-} f(x) = \boxed{0}$

d) $f(2) = \boxed{-3}$

