

DIRECT, INVERSE, & JOINT VARIATION

**Definition:** Variation is the relationship between variables that describes how they react to each other.

I. DIRECT VARIATION:

**Ex 1:** Suppose you make \$8 per hour at your job.

If you work 5 hours, you make \$ 40. If you work 7 hours, you make \$ 56.

As the hours you work increase, the money you make increases.

Write an equation using  $M$  for money made and  $h$  for hours worked:  $M = 8h$

\*\* This is called a direct relationship – as one variable increases, the other one increases.  
Likewise, if one variable decreases, the other variable decreases.

Direct Variation General Equation:  $y = kx$  Constant of Variation =  $k$

**Ex 2:** Find the missing variable. (Hint: Use one pair of variables to solve for "k" in the general equation. Then use "k" and your given variable to find the missing variable.)

a) If  $y$  varies directly as  $x$  and  $y = 12$  when  $x = -3$ , find  $y$  when  $x = 16$ .

$$\frac{12}{-3} = \frac{y}{16} \rightarrow -4 = \frac{y}{16} \rightarrow \boxed{y = -64}$$

b) If  $y$  varies directly as  $x$  and  $x = 15$  when  $y = 5$ , find  $x$  when  $y = 9$ .

$$\frac{5}{15} = \frac{9}{x} \rightarrow 5x = 135 \rightarrow \boxed{x = 27}$$

II. JOINT VARIATION:

**Definition:** When a variable varies directly with more than one variable, we call it Joint Variation.

For example, " $z$  varies jointly with  $x$  and  $y$ " can be written as  $z = kxy$ , meaning  $z$  varies directly with the product of  $x$  and  $y$ . As  $x$  increases,  $z$  would increase. As  $y$  decreases,  $z$  would decrease.

**Ex 3:** Find the missing variable:

a) If  $y$  varies jointly as  $x$  and  $z$  and if  $y = -24$  when  $x = 4$  and  $z = 3$ , find  $y$  when  $x = -6$  and  $z = 2$ .

$$\frac{-24}{4(3)} = \frac{y}{(-6)(2)} \rightarrow \frac{-24}{12} = \frac{y}{-12} \rightarrow \frac{288}{12} = \frac{y}{12} \rightarrow \boxed{y = 24}$$

b) Write an equation to model this situation:  $V$  varies jointly with  $w$  and  $h$ .

$$\boxed{V = kwh}$$

III. INVERSE VARIATION:

**Ex 4:** Suppose you take a road trip covering 210 miles on the highway.  
 If you travel at a constant rate of 60 mph, how long would it take?  
 If you travel at a constant rate of 70 mph, how long would it take?  
 As your rate increases, the time decreases

$\frac{210}{60} = 3.5$   
 $\frac{210}{70} = 3$   
 $t = \frac{210}{r}$

Write an equation using  $r$  for rate and  $t$  for time:  $t = \frac{k}{r} \rightarrow t = \frac{210}{r}$

This is called an inverse relationship – as one variable increases, the other decreases

Inverse Variation General Equation:  $y = k/x$  Constant of Variation =  $k$

**Ex 5:** Find the missing variable:

$y = k/x \rightarrow k = xy$

a) If  $y$  varies inversely as  $x$  and  $y = 22$  when  $x = -2$ , find  $y$  when  $x = -11$ .

$-2(22) = -11(y) \rightarrow -44 = -11y \rightarrow y = 4$

b) If  $y$  varies inversely as  $x$  and  $y = 30$  when  $x = 5$ , find  $y$  when  $x = -3$ .

$5(30) = -3y \rightarrow \frac{150}{-3} = \frac{-3y}{-3} \rightarrow y = -50$

IV. SUMMARY:

Variation Type	Equation
Direct $k = \frac{y}{x}$	$y = kx$
Inverse $k = xy$	$y = \frac{k}{x}$
Joint $k = \frac{y}{xz}$	$y = kxz$

**Ex 6:** Determine whether each is an example of direct, inverse or joint variation (or neither).  
 Then find the constant of variation.

	a) $d = 60t$	b) $y = \frac{5}{x}$	c) $y = \frac{x}{2}$	d) $y = 4x - 3$	e) $\frac{100}{r} = t$	f) $I = 1000rt$
Variation Type:	Direct	Inverse	Direct	Neither	Inverse	Joint
Constant:	60	5	1/2		100	1000

**Ex 7:** Time to travel 250 miles varies inversely with the rate (speed) of a car. Write an equation that shows this relationship and find the time the trip would take if traveling at 60 mph.

$t = \frac{k}{r}$   
 $t = \frac{250}{60} = 4.1\bar{6}$  hours