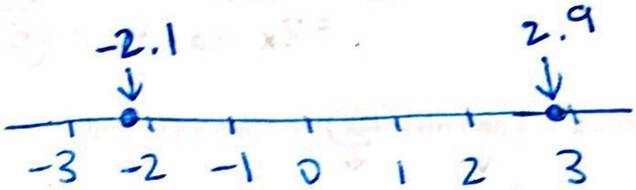
Day 4 Notes

I. THE TWELVE BASIC FUNCTIONS:

Please review The Twelve Basic Functions posted on the class website.
You ARE responsible for knowing the general equation and look of the graphs for ALL TWELVE.

RECALL functions such as the GREATEST INTEGER FUNCTION.
This function works by taking the number inside and ROUNDING DOWN.
Notation: [x] However, the calculator notation is int(x).

Ex: [3.2] means "find the integer that is closest to 3.2 without going over."
Answer: 3
Ex: [4.7] = 4
Ex: [2.999999999] = 2



Negatives are tricky...
Ex: [-2.1] = -3
Ex: [-4.3] = -5

The graph looks like a staircase because the y values have all been rounded down to nice integers.
Please note that one end of each "stair" is an open circle. This is sometimes called the "Step Function."

II. COMPOSITIONS:

1: If $g(x) = x^2 - 4$, then find the following: [It helps to rewrite the function as $g(\ \ ) = (\ \ )^2 - 4$]

a. $g(3) = (3)^2 - 4 = 9 - 4 = \boxed{5}$

b. $g(-4) = (-4)^2 - 4 = 16 - 4 = \boxed{12}$

c. $g(d+1) = (d+1)^2 - 4 = \boxed{d^2 + 2d - 3}$

d. $g(2x) = (2x)^2 - 4 = \boxed{4x^2 - 4}$

We will look at multiple functions at one time. We *always* start INside the ( ) and work our way OUT!!

Ex 2: $f(z) = -z + 4$ and $h(z) = 2z$. Find the following:

a. $f(h(3)) = f(6) = -(6) + 4 = \boxed{-2}$
$\hookrightarrow h(3) = 2(3) = 6$

b. $h(f(-5)) = h(9) = 2(9) = \boxed{18}$
$\hookrightarrow -(-5) + 4 = 5 + 4 = 9$

c. $h(h(8)) = 2(16) = \boxed{32}$
$\hookrightarrow 2(8) = 16$

d. $h(f(2)) = 2(2) = \boxed{4}$
$\hookrightarrow -(2) + 4 = -2 + 4 = 2$

NOTATION: $f(g(\_\_)) = (f \circ g)(\_\_)$

"f of g of x" whatever is in ( )

Ex 3: $g(x) = x^2 + 3$ and $j(x) = x - 1$. Find the following:

a. $(g \circ j)(4) = g(j(4)) = (3)^2 + 3 = \boxed{12}$
$4 - 1 = 3$

b. $(j \circ g)(4) = j(g(4)) = (19) - 1 = \boxed{18}$
$4^2 + 3 = 19$

c. $(g \circ j)(5) = (4)^2 + 3 = \boxed{19}$
$j(5) = 5 - 1 = 4$

d. $j(g(j(0))) = (4) - 1 = \boxed{3}$
$j(0) = -1$
$g(-1) = 4$

A ***Composition Function*** is the result of substituting one function into another.

Ex 4: Find the composition function, $f(g(x))$, for $f(x) = x + 1$ and $g(x) = x^2$

This really means $f(g(x)) = (x^2) + 1$.

Think of it this way: given $f(x) = x + 1$, find $f(x^2)$. $f(x^2) = (x^2) + 1$, so $f(g(x)) = x^2 + 1$.

outside inside

Ex 5: $f(x) = 3x + 5$ and $g(x) = x - 2$. Find $(f \circ g)(x)$. This means $f(\boxed{g(x)})$.

Rewrite the problem: $f(x-2) = 3(x-2) + 5$. What goes in the blanks? $x - 2$

$= 3x - 6 + 5 = \boxed{3x - 1}$

Ex 6: $h(x) = x^2 - 1$ and $p(x) = 2x$. Find $(h \circ p)(x)$ and $(p \circ h)(x)$.

$(h \circ p)(x)$: $h(2x) = (2x)^2 - 1$ $= \boxed{4x^2 - 1}$

out in

$(p \circ h)(x)$: $p(x^2 - 1) = 2(x^2 - 1)$ $= \boxed{2x^2 - 2}$

out in

Ex 7: $f(x) = 2x^2$ and $g(x) = x + 3$. Find the following:

a. $f(g(x)) = f(x+3) = 2(x+3)^2 = 2(x^2 + 6x + 9)$ $= \boxed{2x^2 + 12x + 18}$

b. $f \cdot g(3) = f(6) = 2(6)^2 = 2(36) = \boxed{72}$

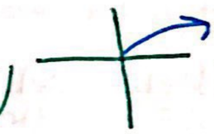
c. $(g \cdot f)(x) = g(2x^2) = (2x^2) + 3$ $= \boxed{2x^2 + 3}$

d. $g(f(-2)) = g(8) = 8 + 3 = \boxed{11}$

<u>When the functions get complicated, finding the DOMAIN of the composition is tricky.</u>

Ex: $f(x) = x^2 + 1$ and $g(x) = \sqrt{x}$

a) Find the domain of $f(x)$: $(-\infty, \infty)$



b) Find the domain of $g(x)$: $[0, \infty)$

c) Find $f(g(x))$: $(\sqrt{x})^2 + 1 = x + 1$ (linear)

d) Find $g(f(x))$: $\sqrt{x^2 + 1}$

<u>Because the domain of g(x) is restricted (it isn't "all real numbers"), it will restrict your answer for the composition:</u>

e) Find the DOMAIN of $f(\boxed{g(x)})$: $[0, \infty)$ (it isn't "all reals")

III. **DE-COMPOSING:**

This is essentially working backwards. I will give you the result of f(g(x)) and you will have to find f(x) and g(x). ***You can do it!***

Ex: Decompose: $f(g(x)) = \sqrt{x-1}$

outside inside

** *Try to think to yourself, "Self, what is the mother function here?" Once you identify that... you have identified f(x). Then ask yourself, "Self, what was plugged into the mother function to produce this result?"*

Answer: mother function would be $\sqrt{x}$, so $f(x) = \sqrt{x}$. g(x) would therefore $= x - 1$.

Ex: $f(g(x)) = \frac{2}{3x+1}$ Find f(x) and g(x).

$\rightarrow g(x) = 3x+1$

$f(x) = \frac{2}{x}$

IV. **IMPLICITLY DEFINED FUNCTIONS:**

An implicitly defined function is one that in and of itself FAILS the vertical line test. However it can be re-written as 2 separate expressions that are functions individually!

To do these problems, you need to $\boxed{\text{get y by itself}}$ as if you were going to graph it in the calculator.

Ex: Rewrite as implicitly defined functions: $x^2 + y^2 = 4$.

$-x^2 \qquad -x^2$

This is a CIRCLE (not a function).

Process is to *get y by itself*: $y^2 = -x^2 + 4$

$y = \pm\sqrt{-x^2+4}$

So, to graph this in the calculator, we would graph $y_1 = \sqrt{-x^2+4}$ and $y_2 = -\sqrt{-x^2+4}$ where individually, those are each functions!

Ex: $x^2 + 2xy + y^2 = 1$ (hint: factor the left side first)

$(x+y)^2 = 1$

$x+y = \pm 1 \rightarrow y = -x \pm 1 \rightarrow y_1 = -x+1$

$y_2 =$ [redacted] $-x-1$

Graph your answer(s) and see what the original equation represents! parallel lines

Ex: Find the implicitly defined functions for: $x^2 + \left(y - \sqrt[3]{x^2}\right)^2 = 1$.

$\left(y - \sqrt[3]{x^2}\right)^2 = 1 - x^2 \rightarrow y - \sqrt[3]{x^2} = \pm\sqrt{1-x^2}$

$y = \sqrt[3]{x^2} \pm \sqrt{1-x^2}$

Graph to see what this equation represents! heart ♡

V. INVERSE FUNCTIONS:

The inverse of a function has all of the same points as the original function, EXCEPT all of the x and y values have been switched.

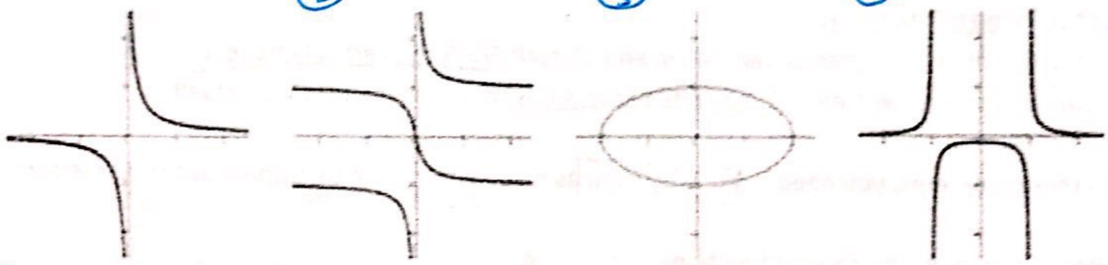
Notation: $f^{-1}(x)$ is the inverse function of $f(x)$.

Given a graph, its inverse is a reflection across the line y=x. Therefore, a relation with ordered pairs (a, b) would have an inverse containing ordered pairs in the form (b,a).

Graphically: We use the **VERTICAL Line Test** to see if the graph of a relation is a function.
Now, we will use the **HORIZONTAL Line Test** to see if the graph of a function has an inverse.

Example 1: The graph of a relation is shown. a) Is the relation a function? b) Does the relation have an inverse that is a function?

1 2 3 4



1: a) yes b) yes
2: a) no b) yes
3: a) no b) no
4: a) yes b) no

A function whose inverse is also a function (meaning it passes both the vertical and horizontal line tests) is called one-to-one, because each x is paired with a unique y and each y is pair with a unique x.

Which of the above graphs is one-to-one? the 1st one

Finding an Inverse Function Algebraically:

1 If the function has f(x), change it to y.

2. Switch all the x's to y's and all the y's to x's.

3. Solve for y.

4. Replace y with $f^{-1}(x)$, which means "the inverse of f(x)".

Example 2: Find an equation $f^{-1}(x)$ if $f(x) = \frac{x}{x+1}$. Give the domain of $f^{-1}(x)$, including any restrictions "inherited" from f.

1. $y = \frac{x}{x+1}$
2. $x = \frac{y}{y+1}$
3. $\frac{y+1}{1} = \frac{y}{x}$

Divide by y

$\frac{y+1}{y} = \frac{1}{x}$

Break up fraction

$\frac{y}{y} + \frac{1}{y} = \frac{1}{x}$

$\frac{y}{y} = 1$

$1 + \frac{1}{y} = \frac{1}{x}$

subtr. 1

$\frac{1}{y} = \frac{1}{x} - 1$

$\frac{1 \cdot x}{\frac{1 \cdot x}{x} - 1 \cdot x} = y$

$\frac{x}{1-x} = y$

4. $\boxed{f^{-1}(x) = \frac{x}{1-x}}$

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Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

Inverses of Functions

Determine whether the functions are inverses. $f(g(x)) = -1$

1) $f(w) = \frac{5}{3}w + 6$

$g(w) = \frac{3(w-6)}{5}$ yes

2) $f(r) = -4 + 1r$

$g(r) = \frac{r-4}{2}$ no

3) $f(n) = -11n + 5$

$g(n) = \frac{n-5}{-11}$ yes

4) $f(s) = (s + 6)^3$

$g(s) = s^{\frac{1}{3}} - 6$ yes

5) $f(y) = 6(y + 5)^4$

$g(y) = 6y^{\frac{1}{4}} - 5$ no

6) $f(d) = 6d$

$g(d) = \frac{d}{6}$ yes

Find the inverse of each function.

7) $f(b) = \frac{2}{5}b - 10$

$f^{-1}(b) = \frac{5(b+10)}{2}$

8) $f(q) = 3 + 5q$

$f^{-1}(q) = \frac{q-3}{5}$

9) $f(x) = 5x - 11$

$f^{-1}(x) = \frac{x+11}{5}$

10) $f(z) = (z - 7)^3$

$f^{-1}(z) = z^{1/3} + 7$

11) $f(m) = -2(m - 8)^2$

$f^{-1}(m) = \left(\frac{m}{2}\right)^{1/2} + 8$

12) $f(k) = -8k$

$f^{-1}(k) = \frac{k}{-8}$

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