Parametrics Free Response - SET 1

1 (a) $x(4) = \int_0^4 3 + \cos(t^2)\,dt$ ① integral

$= \underbrace{x(2)}_{\text{given: } 1} + \underbrace{\int_2^4 3 + \cos(t^2)\,dt}_{\approx 6.133}$ ① handles initial condition $\approx \boxed{7.133}$ ① answer

(b) Equation of a Line: Need SLOPE + POINT!

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-7}{3+\cos 4} \approx -2.983$ * store in calc. for more accuracy! *

① finds $dy/dx\,|_{t=2}$

pt. given: $\langle x(t), y(t)\rangle|_{t=2} = (1, 8)$

$\boxed{y - 8 = -2.983(x - 1)}$ ① equation

(c) Speed $= \sqrt{(-7)^2 + (3+\cos 4)^2} \approx \boxed{7.383}$ ① answer

(d) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \Rightarrow 2t+1 = \frac{dy/dt}{3+\cos(t^2)}$

$\frac{dy}{dt} = (2t+1)(3+\cos(t^2))$ ① $\frac{dy}{dt}$

$\frac{d^2y}{dt^2} = (2t+1)(-2t\sin(t^2)) + 2(3+\cos(t^2))\,|_{t=4}$

≈ 24.814

$\frac{d^2x}{dt^2} = -2t\sin(t^2)|_{t=4} \approx 2.303$ ① $x''(4)$

$\boxed{\langle 2.303, 24.814\rangle}$ ① answer

2 (a) $\text{speed} = \sqrt{(dx/dt)^2 + (dy/dt)^2}$

$= \sqrt{\underbrace{(2t-4)}_{x'(t)}^2 + \underbrace{(te^{t-3}-1)}_{y'(t)\text{ given!}}^2}\Big|_{t=3}$ ① answer

$= \sqrt{[2(3)-4]^2 + [3e^0 - 1]^2} = \sqrt{8} = \boxed{2\sqrt{2}\ \text{m/s}}$ 1/9

(b) $\text{distance} = \int_0^4 \sqrt{(2t-4)^2 + (te^{t-3}-1)^2}\,dt$ ① integral

$\approx \boxed{11.587\ \text{m}}$ ① answer 2/9

(c) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 0$ ... when $te^{t-3} - 1 = 0$ and $2t - 4 \neq 0$

① considers $\frac{dy}{dx} = 0$

$\boxed{t \approx 2.20794\ \text{sec}}$ ① 2.207 or 2.208 3/9

$\downarrow$

$x'(2.20794) > 0$ ① Direction w/ Reason.

$\therefore$ The particle is moving to the **RIGHT**!

(d) $x(t) = 5 = t^2 - 4t + 8 \Rightarrow \boxed{t = 1, 3}$ ← (i)

① $t = 1, 3$

$\frac{dy}{dx}\Big|_{t=1} = \boxed{0.432}$ ← (ii)

$\frac{dy}{dx}\Big|_{t=3} = \boxed{1}$ ① slopes 3/9

$y(1) = y(3) = \underbrace{3 + \frac{1}{e}}_{y(2)} + \int_2^3 \underbrace{te^{t-3} - 1}_{dy/dt}\,dt = \boxed{4}$ ← (iii)

① y-coord.

3 (a) speed $= \sqrt{(dx/dt)^2 + (dy/dt)^2}\,\Big|_{t=4}$

$= \sqrt{(x'(4))^2 + (y'(4))^2} \approx \boxed{2.912 \text{ m/s}}$

1/9 ① speed @ $t=4$

(b) distance $= \int_0^4 \sqrt{\left(\underbrace{dx/dt}_{\text{given}}\right)^2 + \left(\underbrace{dy/dt}_{\text{given}}\right)^2}\, dt \approx \boxed{6.423 \text{ m}}$

2/9 ① integral ① answer

(c) $x(4) = \underbrace{x(0)}_{\text{given: } -3} + \underbrace{\int_0^4 dx/dt\, dt}_{\approx 2.10794} \approx \boxed{-0.892}$

3/9 ① uses $x(0) = -3$ ① uses integrand ① answer

(d) slope $= 2 \Rightarrow \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = 2$ ① $\dfrac{dy/dt}{dx/dt} = 2$

3/9 [OR $\ln(t^2+1) = 2\arctan\left(\frac{t}{t+1}\right)$]

$t > 0 \Rightarrow$ pick $\boxed{t \approx 1.35766 \text{ sec}}$ ① t-value

At this time ...

$\vec{a}(t) = \langle x''(t), y''(t) \rangle\,\Big|_{t=1.357\ldots}$

$= \boxed{\langle 0.135, 0.955 \rangle}$

① values for x'' + y''