PreCalculus - FINAL EXAM Review

1. Find the sum of $\sum_{k=1}^{17}(-9k+7)$.

[A] -36 [B] -1258 [C] -146 [D] -34

2. Find the kth partial sum of the arithmetic sequence $\{u_n\}$ with a common difference d. $k = 18$, $u_1 = -4$, $d = 4$

[A] 240 [B] 576 [C] 540 [D] none of these

3. Determine whether the sequence is arithmetic, geometric, or neither.
$-4, 20, -100, 500, -2500, \ldots$

[A] arithmetic [B] geometric[C] neither

4. Find the common ratio for geometric sequence $4\left(\frac{1}{2}\right)^{n-1}$.

[A] 8 [B] $\frac{1}{2}$ [C] 4 [D] $\frac{3}{2}$

5. There are 11 students participating in a spelling bee. How many ways can the students who go first, second, third, fourth and fifth be chosen?

[A] 1,663,200 ways [B] 55,440 ways

[C] 332,640 ways [D] 462 ways

6. Find all local maxima and minima of the function
$f(x) = -x^3 - 3x^2 + 9x - 3$.

[A] x = -3 and x = 1are local maxima.

[B] x = 1 is a local minimum, x = -3 is a local maximum.

[C] x = -3 is a local minimum, x = 1 is a local maximum.

[D] x = -3 and x = 1 are local minima.

7. Given $f(x) = x^3$ and $g(x) = -4 + 5x^2$, find $(g \circ f)(x)$ and its domain.

[A] $-4+5x^2,\ x \neq \sqrt{\frac{4}{5}}$ [B] $\frac{-4+5x^2}{x^3},\ x \neq 0$

[C] $-4+5x^6$, all real numbers [D] none of these

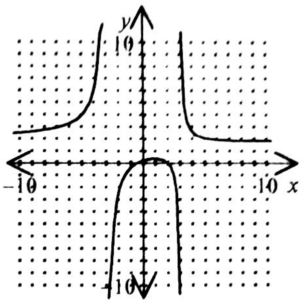
8. Find the inverse of the function.
$f(x) = \frac{3x-7}{8}$

[A] $g(x) = \frac{8x+7}{3}$ [B] $g(x) = \frac{8x-7}{3}$

[C] $g(x) = \frac{8}{3x-7}$ [D] $g(x) = 3x - 8$

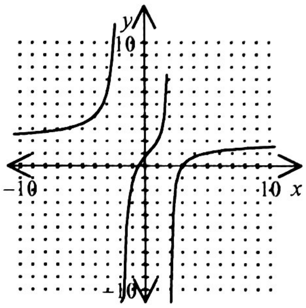
9. Which shows the graph and lists the asymptotes of the rational function
$f(x) = \frac{2x^2 - 5x - 3}{x^2 - 4}$?

[A]



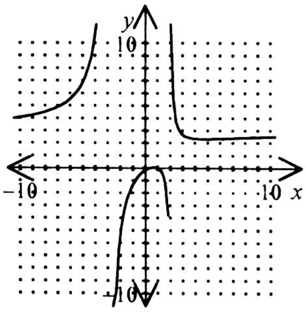
$x = -3,\ x = 3,\ y = 2$

[B]



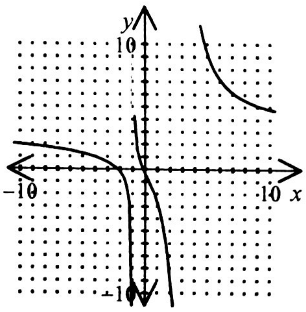
$x = -2,\ x = 2,\ y = 2$

[C]



$x = -3,\ x = 2,\ y = 3$

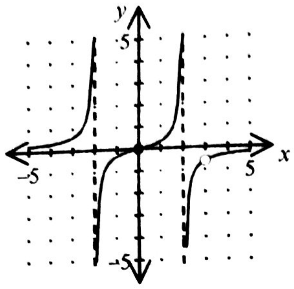
[D]



$x = -1,\ x = 3,\ y = 3$

10. Which graph is correct, including all x-intercepts, holes, and asymptotes of the function $y = \dfrac{x^2 - x}{(x+2)(x-2)(x-1)}$?

[A]



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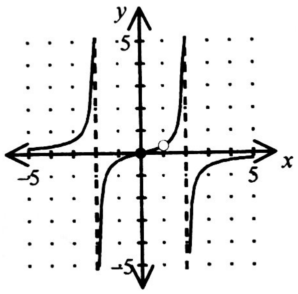

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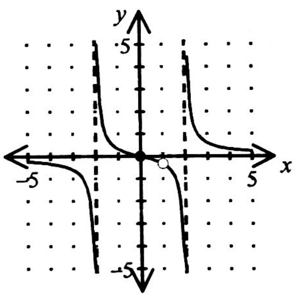
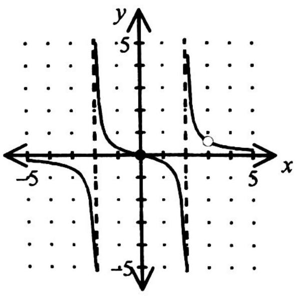


[D]



11. Determine the domain of the function $f(x) = \dfrac{x^2 + 13x + 40}{x^2 + 4x - 12}$.

[A] $(-\infty,5)\cup(5,8)\cup(8,\infty)$ [B] $(-\infty,-6)\cup(-6,2)\cup(2,\infty)$

[C] $(-\infty,-8)\cup(-8,-5)\cup(-5,\infty)$

[D] $(-\infty,-2)\cup(-2,6)\cup(6,\infty)$

12. Which of the following is *not* a polynomial?

[A] $x^2 - 2$ [B] $x^2 - 2 + 2x$ [C] -1 [D] $x^{-2} + 2$

13. Solve the equation and express each solution in $a + bi$ form.

$x^4 - 18x^2 - 243 = 0$

[A] $x = -3i,\ 3i,\ 3\sqrt{3}$, or $-3\sqrt{3}$

[B] $x = -3i,\ 3i,\ 3\sqrt{3}i$, or $-3\sqrt{3}i$

[C] $x = -3,\ 3,\ 3\sqrt{3}i$, or $-3\sqrt{3}i$

[D] $x = -3,\ 3,\ 3\sqrt{3}$, or $-3\sqrt{3}$

14. Determine the domain of the function.

$h(x) = \dfrac{7x}{x(x^2 - 25)}$

[A] $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

[B] $(-\infty,-5)\cup(-5,0)\cup(0,5)\cup(5,\infty)$

[C] $(-\infty,5)\cup(5,\infty)$

[D] $(-\infty,-25)\cup(-25,0)\cup(0,25)\cup(25,\infty)$

15. Perform the indicated operation and write the result in the form $a + bi$.

$-7i(3i+9) - 3(3+2i)$

[A] $30 - 57i$ [B] $72 - 15i$

[C] $-12 - 57i$ [D] $12 - 69i$

16. Find all the complex zeros of the polynomial function.

$f(x) = x^4 + 8x^3 + 16x^2 - 8x - 17$

[A] $1,\ -1,\ -4-i,\ -4+i$ [B] $4,\ -4,\ 1-2i,\ 1+2i$

[C] $1,\ -1,\ -4-2i,\ -4+2i$ [D] $4,\ -4,\ 1-i,\ 1+i$

17. Find the exact value of $\ln \sqrt[3]{e}$.

[A] $3e$ [B] 3 [C] $\dfrac{1}{3e}$ [D] $\dfrac{1}{3}$

18. If \$3500 is invested at an interest rate of 7%, compounded continuously, determine the balance in the account after 6 years. Use the formula $A = Pe^{rt}$.

[A] \$5326.87 [B] \$5713.11

[C] \$25,861.70 [D] \$5252.56

19. Solve for x.

$10^x = 30$

[A] $x = 3$ [B] $x = 1.48$ [C] $x = 2.30$ [D] $x = 1.10$

20. Which is $2\log x + 4\log(x+4)$ written as a single logarithm?

[A] $\log x^2(x+4)^4$ [B] $\log x(x+4)^8$

[C] $8\log x(x+4)$ [D] none of these

21. Which is the solution to $8^x = 7^{x+3}$?

[A] $x = 1.4502$ [B] $x = 21.8590$

[C] $x = 43.7180$ [D] none of these

22. The number of bacteria present in a culture after t minutes is given as $B = 1000e^{kt}$, where k is a constant. If there are 1673 bacteria present after 8 minutes, find k.

[A] 0.074 [B] 0.064 [C] 0.515 [D] 4.117

23. Find the value of sec 34°.

[A] 0.6745 [B] 0.829 [C] 1.2062 [D] 1.4826

24. A tree casts a shadow of 27 meters when the angle of elevation of the sun is 28°. Find the height of the tree to the nearest meter.

[A] 13 m [B] 31 m [C] 14 m [D] 24 m

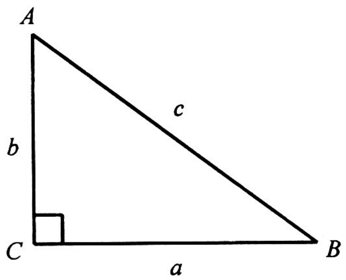
25. Which angle is *not* coterminal with 374°?

[A] 194° [B] 14° [C] 734° [D] −346°

26. Convert 219° to radians.

[A] $\frac{73}{120}\pi$ [B] $\frac{73}{30}\pi$ [C] $\frac{73}{60}\pi$ [D] $\frac{73}{90}\pi$

27. Given that $m\angle A = 37°$ and $c = 10$, find b in the right triangle below.



[A] 7.99 [B] 6.02 [C] 12.52 [D] 7.65

28. What are the values of sin θ and cos θ for the acute angle θ in standard position if $\tan\theta = \frac{1}{3\sqrt{7}}$?

[A] $\sin\theta = 8, \cos\theta = \frac{3\sqrt{7}}{8}$

[B] $\sin\theta = \frac{3\sqrt{7}}{8}, \cos\theta = \frac{1}{8}$

[C] $\sin\theta = \frac{1}{8}, \cos\theta = \frac{3\sqrt{7}}{8}$

[D] $\sin\theta = \frac{1}{8}, \cos\theta = \frac{8}{3\sqrt{7}}$

29. On a Ferris wheel, you travel through a central angle of $\frac{128\pi}{9}$ before stopping. If the radius of the Ferris wheel is76 feet, how many feet have you traveled?

[A] 3395.7 feet [B] 3385.7 feet

[C] 15,290.7 feet [D] 15,280.7 feet

30. Find $\cos\theta$ if $(-8, -15)$ is a point on the terminal side of θ.

[A] $\frac{15}{8}$ [B] $-\frac{15}{17}$ [C] $\frac{8}{15}$ [D] $-\frac{8}{17}$

31. Which single expression is equivalent to $\sin(\theta + 2\pi)$?

[A] $\sin\theta$ [B] $-\cos\theta$ [C] $-\sin\theta$ [D] $\cos\theta$

32. θ is an angle in standard position with point $P(1, -8)$ on the terminal side. Which statement is *not* correct?

[A] $\cos\theta = \frac{-8\sqrt{65}}{65}$ [B] $\tan\theta = -8$

[C] $\cot\theta = -\frac{1}{8}$ [D] $\sin\theta = \frac{-8\sqrt{65}}{65}$

33. A water wave is created in a wave tank. It has an amplitude of 5 and a period of $\frac{3\pi}{5}$. Find the equation of this wave as a sine function.

[A] $f(t) = 5\sin\frac{3\pi t}{5}$ [B] $f(t) = \frac{10}{3}\sin 5t$

[C] $f(t) = 5\sin\frac{10t}{3}$ [D] $f(t) = \frac{3\pi}{5}\sin\frac{t}{5}$

34. What is the amplitude and period of $f(t) = -6\cos(7t)$?

[A] amplitude: 6; period: $\frac{2}{7}\pi$ [B] amplitude: 12; period: $\frac{2}{7}\pi$

[C] amplitude: 6; period: $\frac{7}{2}\pi$ [D] amplitude: −6; period: $\frac{7}{2}\pi$

35. What are the amplitude, period, and phase shift of the given function?

$f(t) = \frac{1}{3}\sin(3t - 3\pi)$

[A] amplitude: 1
phase shift: -3π
period: $\frac{2}{3}\pi$

[B] amplitude: $-\frac{1}{3}$
phase shift: π
period: 3

[C] amplitude: $\frac{1}{3}$
phase shift: π
period: 3π

[D] amplitude: $\frac{1}{3}$
phase shift: π
period: $\frac{2}{3}\pi$

36. Which is $\sin(\tan^{-1} v)$ written as an algebraic expression in v?

[A] $\frac{1}{v+1}$ [B] $\frac{v}{\sqrt{v^2+1}}$ [C] $\sqrt{v^2+1}$ [D] $\sqrt{v}$

37. A weight attached to the end of a spring is pulled down 8 centimeters. It takes 10 seconds for it to complete one cycle of moving from its equilibrium position to 8 centimeters below, then rising to 8 centimeters above, and then finally returning to equilibrium. (This is assuming a spring with perfect elasticity and ignoring all other forces.)
a. Find a sinusoidal function to represent the motion of the moving spring.
b. Use the function from part (a) to predict the position of the weight after 15 seconds.

[A] $h(t) = 8\sin\frac{\pi}{5}t,\ \approx 1$ cm [B] $h(t) = 10\sin\frac{\pi}{4}t,\ \approx -7.1$ cm

[C] $h(t) = 10\sin\frac{\pi}{4}t,\ \approx 7.1$ cm

[D] $h(t) = 8\sin\frac{\pi}{5}t,\ \approx 0$ cm

38. What are all the exact t-values for which $\tan t = 0$?

[A] $\frac{\pi}{6} + n\pi$ [B] $0 + n\pi$ [C] $\frac{\pi}{4} + n\pi$ [D] $2\pi + n\pi$

39. Find all solutions of $\tan^2 x = \frac{\sqrt{2}}{2}\sec x$ in $(0, 2\pi)$.

[A] $x = \frac{\pi}{4}, \frac{7\pi}{4}$ [B] $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

[C] $x = \frac{5\pi}{6}, \frac{7\pi}{6}$ [D] none of these

40. Use factoring, the quadratic formula, or identities to solve $\cos x + 1 = \sin^2 x$. Find all solutions on the interval $[0, 2\pi)$.

[A] $x = \pi,\ x = \frac{\pi}{2},\ x = \frac{3\pi}{2}$ [B] $x = \pi,\ x = \frac{\pi}{2},\ x = \frac{2\pi}{3}$

[C] $x = \frac{\pi}{3},\ x = \frac{5\pi}{6},\ x = \frac{3\pi}{2}$

[D] $x = \frac{\pi}{3},\ x = \frac{5\pi}{6},\ x = \frac{2\pi}{3}$

Find all solutions of the equation on the interval $[0, 2\pi)$.

41. $2\cot^2 x + 3\csc x = 0$

[A] $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ [B] $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

[C] $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ [D] $\frac{7\pi}{6}, \frac{11\pi}{6}$

42. $2\csc^2\frac{x}{2} - 3\csc\frac{x}{2} - 2 = 0$

[A] $\frac{2\pi}{3}$ [B] $\frac{\pi}{3}, \frac{5\pi}{3}$ [C] $\frac{4\pi}{3}$ [D] π

43. Solve $\cos 2x + 5\sin x = -2$ on the interval $[-\pi, \pi]$.

[A] $x = -\frac{\pi}{6}, -\frac{5\pi}{6}$ [B] $x = -\frac{\pi}{6}, -\frac{5\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}$

[C] $x = \frac{\pi}{3}, \frac{2\pi}{3}$ [D] $x = \frac{\pi}{6}, \frac{5\pi}{6}$

44. What is the simplified form of $\sin\left(x - \frac{3\pi}{2}\right)$?

[A] $-\sin x$ [B] $\sin x$ [C] $\cos x$ [D] $-\cos x$

45. Find the exact value of $\cos\left(\frac{5\pi}{12}\right)$.

[A] $2 + \sqrt{3}$ [B] $\sqrt{6} - \sqrt{2}$

[C] $\frac{\sqrt{6} - \sqrt{2}}{4}$ [D] $\sqrt{6} + \sqrt{2}$

46. Which expression is equal to the given expression? $\csc 2x - \cot 2x$

[A] $\tan x$ [B] $\cot x$ [C] $\tan\frac{x}{2}$ [D] $\cot\frac{x}{2}$

47. Use the half-angle identity to find the exact value of $\cos\left(\frac{11\pi}{8}\right)$.

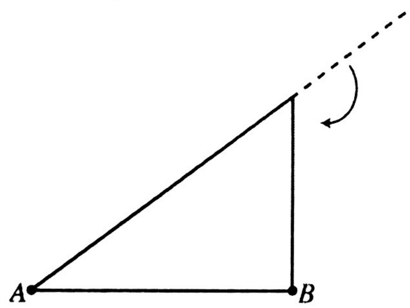
[A] $-\frac{\sqrt{2-\sqrt{2}}}{2}$ [B] $\sqrt{4+2\sqrt{2}} - 2\sqrt{2+\sqrt{2}}$

[C] $1 + \sqrt{2}$ [D] $-\sqrt{4-2\sqrt{2}} - 2\sqrt{2-\sqrt{2}}$

48. Given $\tan x = -\frac{2}{9}$ and $\frac{\pi}{2} < x < \pi$, find the exact value of $\tan 2x$.

[A] $\frac{73}{81}$ [B] $\frac{4\sqrt{77}}{81}$ [C] $-\frac{36}{77}$ [D] $-\frac{4\sqrt{77}}{81}$

49. Island A is 160 miles from Island B. A ship captain travels 280 miles from Island A and then finds that he is off course and 170 miles from Island B. What angle, in degrees, must he turn through to head straight for Island B?



[A] 149.11° [B] 120.89°

[C] 59.11° [D] none of these

50. Given a triangle with $a = 3$, $A = 25°$, and $B = 32°$, what is c?

[A] 3.8 [B] 0.3 [C] 2.4 [D] 6

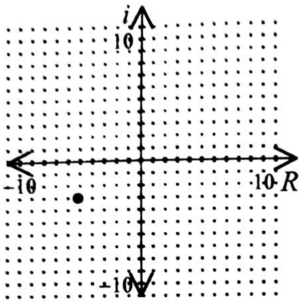
51. Find the area of the triangle ABC under the given conditions.

$A = 23°$, $b = 2$ feet, and $c = 3$ feet

[A] 2.8 ft^2 [B] 1.2 ft^2 [C] 2.3 ft^2 [D] none of these

52. Plot $-3\sqrt{3}-3i$ in the complex plane.

[A]



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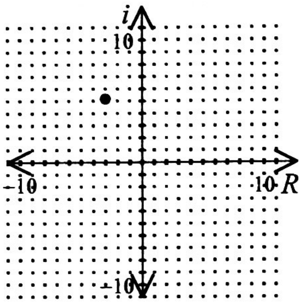

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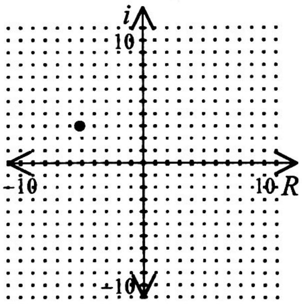

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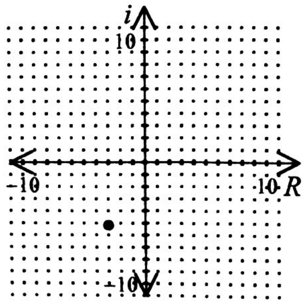


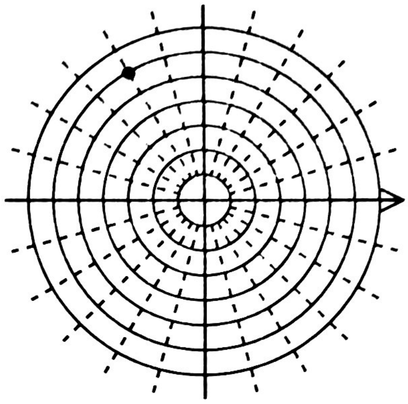
53. Find the absolute value of the complex number.

$-8+2i$

[A] 60 [B] 8.25 [C] 7.75 [D] 68

54. Which is the graph of the polar coordinate $\left(-6, \dfrac{2\pi}{3}\right)$?

[A]



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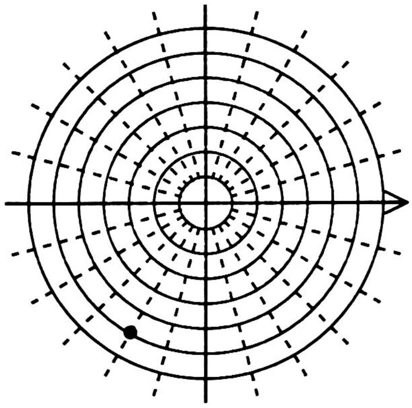

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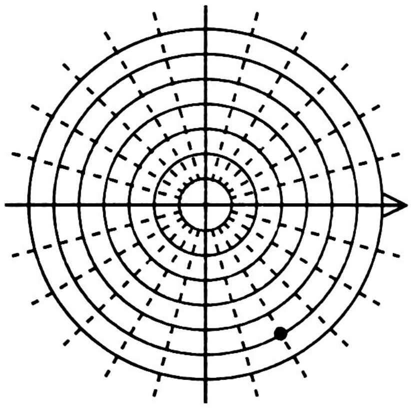
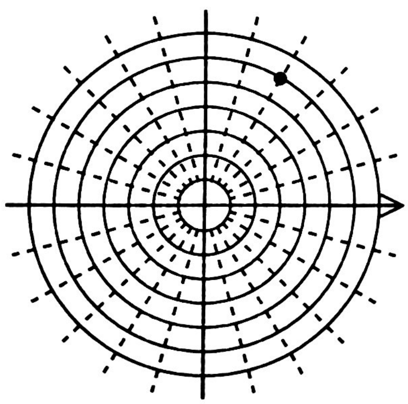


[D]



55. Express the number in polar form.
$2-2i$

[A] $2\sqrt{2}\left(\cos\dfrac{7\pi}{4}+i\sin\dfrac{7\pi}{4}\right)$

[B] $2\sqrt{2}\left(\cos\dfrac{7\pi}{4}-i\sin\dfrac{7\pi}{4}\right)$

[C] $2\sqrt{2}\left(\cos\dfrac{9\pi}{4}+i\sin\dfrac{9\pi}{4}\right)$ [D] none of these

56. Evaluate: $(-2+2i)^5$

[A] $128-128i$ [B] $128+128i$

[C] $-128+128i$ [D] none of these

57. Find the rectangular coordinates of $\left(3, \dfrac{5\pi}{3}\right)$.

[A] $\left[-\dfrac{3\sqrt{3}}{2}, \dfrac{3}{2}\right]$ [B] $\left[\dfrac{3\sqrt{5}}{2}, -\dfrac{3}{2}\right]$

[C] $\left[\dfrac{3}{2}, -\dfrac{3\sqrt{3}}{2}\right]$ [D] $\left[-\dfrac{3}{2}, \dfrac{3\sqrt{5}}{2}\right]$

58. Find the equation of the hyperbola centered at the origin that satisfies the given conditions: X-intercepts ± 4, asymptote $y=\dfrac{5}{4}x$.

[A] $25x^2-25y^2=400$ [B] $16x^2-25y^2=400$

[C] $25x^2-16y^2=400$ [D] $25x^2+16y^2=400$

59. Find the equation of the parabola with vertex at the origin that satisfies the given condition: directrix $x=5$

[A] $x=-20y^2$ [B] $x=\dfrac{1}{20}y^2$

[C] $x=5y^2$ [D] $x=-\dfrac{1}{20}y^2$

60. Find the center, vertices, and foci for the ellipse $4x^2+16y^2=64$.

[A] center $(0, 0)$
vertices $(\pm 4, 0)$
foci $(\pm 3.5, 0)$

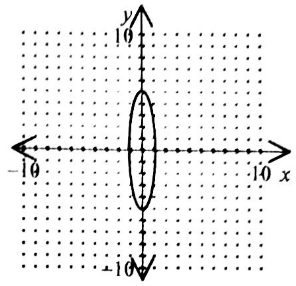
[B] center $(2, 4)$
vertices $(4, 0)$ and $(4, 8)$
foci $(4, -1.5)$ and $(4, 5.5)$

[C] center $(0, 0)$
vertices $(0, \pm 4)$
foci $(0, \pm 3.5)$

[D] center $(4, 2)$
vertices $(0, 2)$ and $(4, 2)$
foci $(0.52\ )$ and $(7.52\ )$

61. Which graph matches the equation $x^2+16y^2=16$?

[A]



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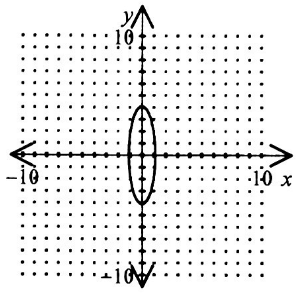

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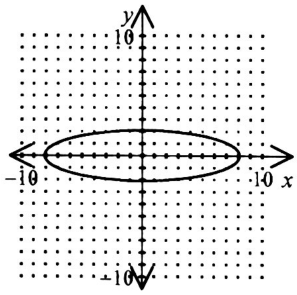

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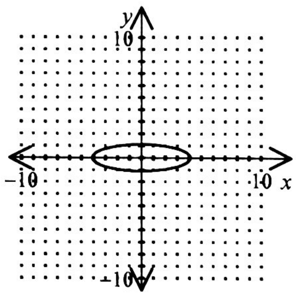
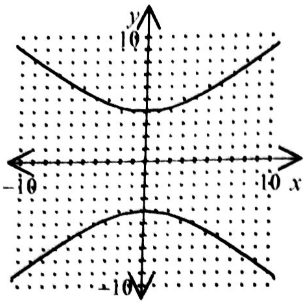
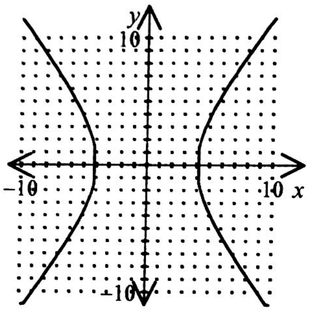


62. Which is the graph of the equation $25x^2 - 16y^2 = 400$?

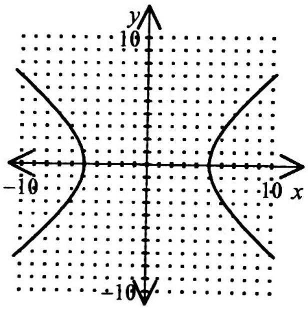
[A]



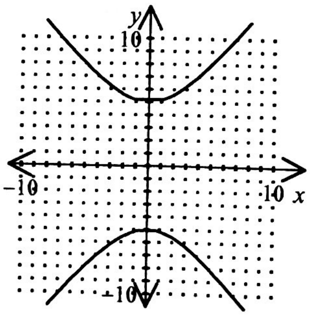
[B]



[C]

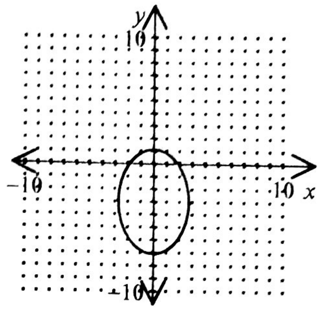


[D]

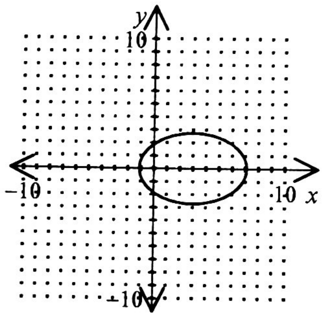


63. Sketch the graph of the polar equation $r = \dfrac{7}{4 + 3\cos\theta}$.

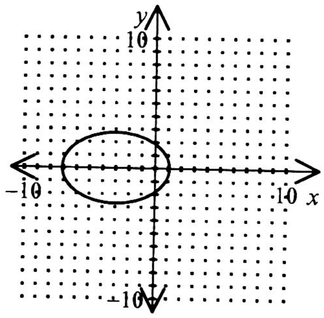
[A]



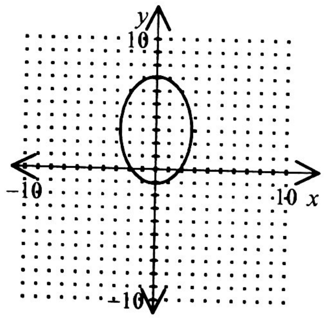
[B]



[C]

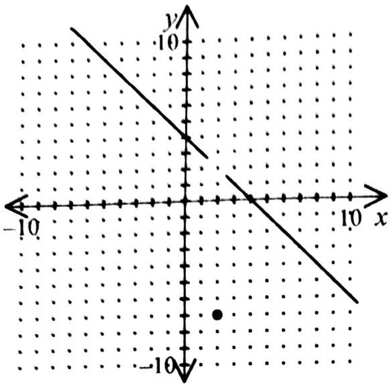


[D]



64. Use the graph of the function f to determine the given limit.

$\lim_{x \to 2} f(x)$



[A] -7 [B] 2 [C] 4 [D] does not exist

65. Use a calculator to find $\lim_{x\to 0} \dfrac{2\sec(4x)-2}{4x\sec(4x)}$.

[A] $\dfrac{1}{2}$ [B] 4 [C] 2 [D] none of these

Find the limit, if it exists.

66. $\lim_{x\to 0} \dfrac{\sqrt{x+6}-\sqrt{6}}{x}$

[A] 6 [B] $\sqrt{6}$ [C] $\dfrac{1}{2\sqrt{6}}$ [D] none of these

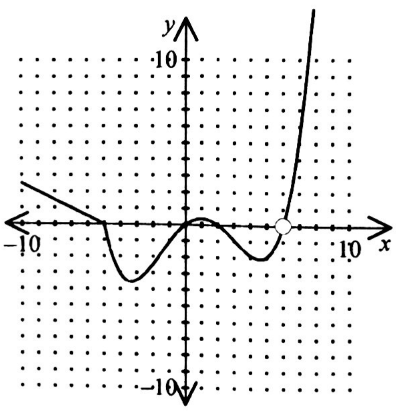
67. $\lim_{x\to 2} \left(3x^3+x^2+22\right)$

[A] 54 [B] 36 [C] 46 [D] none of these

68. Find $\lim_{h\to 0} \dfrac{f(3+h)-f(3)}{h}$ if $f(x)=x^3$.

[A] 27 [B] 140 [C] 0 [D] 152

69. At which of the given values is the graph discontinuous?



[A] 6 [B] 0 [C] 2 [D] -5

70. Determine all numbers at which the function is continuous.

$$f(x)=\begin{cases}\dfrac{x^2+3x-10}{x^2+2x-8} & \text{if } x\neq 2\\[2mm] \dfrac{7}{6} & \text{if } x=2\end{cases}$$

[A] Continuous at every real number except $x=-4$.

[B] Continuous at every real number except $x=2$ and $x=-5$.

[C] Continuous at every real number except $x=2$.

[D] Continuous at every real number except $x=-4$ and $x=2$.

71. Find the values of x (if any) at which $f(x)=\dfrac{x+6}{x^2-36}$ is not continuous.

If so, is the discontinuity removable?

[A] removable at -6, non-removable at 6

[B] non-removable at -6, removable at 6

[C] non-removable at -6 and 6 [D] none of these

72. If $z=8\left(\cos\dfrac{5\pi}{9}+i\sin\dfrac{5\pi}{9}\right)$ and $w=2\left(\cos\dfrac{\pi}{9}+i\sin\dfrac{\pi}{9}\right)$ find:

a. zw

b. $\dfrac{z}{w}$

[A] a. $zw=16\left(\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}\right)$, b. $\dfrac{z}{w}=4\left(\cos\dfrac{4\pi}{9}+i\sin\dfrac{4\pi}{9}\right)$

[B] a. $zw=16\left(\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}\right)$, b. $\dfrac{z}{w}=4\left(\cos\dfrac{\pi}{36}+i\sin\dfrac{\pi}{36}\right)$

[C] a. $zw=16\left(\cos\dfrac{100\pi}{9}+i\sin\dfrac{100\pi}{9}\right)$, b. $\dfrac{z}{w}=4\left(\cos\dfrac{\pi}{36}+i\sin\dfrac{\pi}{36}\right)$

[D] none of these